A VMD and FAN Based Hybrid Model for Air Quality Index Forecasting

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A VMD and FAN Based Hybrid Model for Air Quality Index Forecasting

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Abstract

The time series data of Air Quality Index (AQI) is very complex and nonstationary, therefore the forecasting and accurate prediction of AQI is challenging. This study propose a novel VMD-FAN hybrid model using Variational Mode Decomposition (VMD) for handling noise and Fourier Analysis Networks (FAN) for handling periodicity, the hybrid model is good at predicting short term time series AQI data on a air quality dataset of Taiwan. The cleaned AQI timeseries extracted from the original dataset is decomposed into individual Intrinsic Mode Functions (IMFs) using VMD and each IMF is predicted using a FAN model subsequently aggregated to form a final forecast of AQI values.The proposed hybrid model predicts the AQI of Annan district in Taiwan with a MAE, MSE, RMSE and MAPE as 0.717643, 1.352704, 1.163058, 1.495354% respectively and is better than the compared base model. The generalizability of the model is further validated with extension analysis on different cities in Taiwan. The proposed hybrid model showcase high performance and its ability to predict complex AQI data and contributes to the research in the domain.

1 Introduction

1.1 Background

Breathing poisonous air is one of the greatest threats to life and the environment, having an impact on millions of people's lives. PM2.5, NO2 and O3 are known to cause severe respiratory and cardiovascular diseases and reduced life span. Air quality forecasting is relevant to prevent and reduce these impacts and to guide legal actions in order to protect the population's health. However, air quality data is highly detailed, and it contains non-stationary properties such as non-linearity, seasonality and abnormal changes due to meteorological and anthropogenic variations. Such challenges raise the need of ensuring the existence of sound forecasting models that can manage high complexity levels.

The air quality index (AQI) is an index used by environmental protection agencies for indicating the outdoor air quality, this represents various pollutants levels as a individual quantitative representation of the quality of air. Even though traditional air quality forecasting methods like ARIMA and exponential smoothing have created a strong foundation for advanced air quality prediction and analytics, they are limited by assumptions and limitations of linearity and cannot model non-stationary and stochastic features of air quality data efficiently. Other machine learning approaches mainly using LSTM networks, which are sorts of deep learning methods, have shown great improvements in learning complicated temporal structures. However, even these models have issues in dealing with noise, periodicity and a sudden shift in data behavior.

To address these limitations, This paper uses a hybrid model combining signal decomposition techniques and advanced forecasting networks. To decompose the original AQI series we use Variational Mode Decomposition (VMD), introduced by Dragomiretskiy and Zosso (2014), this is an effective tool for isolating intrinsic mode functions (IMFs), filtering noise, and capturing significant features from non-stationary time series. If the decomposed IMFs are predicted individually and the integrated to obtain the final forecast, which improves the efficiency and accuracy of the training algorithm. The neural networks which are commonly used exhibit problems in the modeling and reasoning of periodicity and instead of understanding the periodicity they tend to model based on memorizing periodicity. As a alternative Fourier Analysis Networks (FAN), proposed by Dong et al. (2024), leverage Fourier series to model periodic behaviors, making them particularly suitable for cyclical data like AQI, which exhibits daily and seasonal patterns.

By integrating VMD for data decomposition and FAN for periodicity modeling, this study aims to create a robust and accurate AQI forecasting system.Through experiments this study aim to demonstrate the effectiveness of VMD-FAN hybrid model

1.2 Research Questions

The key research questions addressed in this study are as follows:

How effectively does a hybrid model combining Variational Mode Decomposition (VMD) with Fourier Analysis Network (FAN) enhance the accuracy of time-series forecasting for air quality data, compared to traditional methods?

1.3 Research Objectives

The objectives of this study are as follows:

- To design a hybrid forecasting system by integrating VMD and FAN to address the non-linearity, noise, and periodicity in AQI data.
- To optimize VMD and FAN parameters (e.g., number of modes and penalty parameter) for efficient decomposition and forecasting of AQI time series.
- To evaluate the performance of the VMD-FAN model against a baseline forecasting approach.

1.4 Structure of the Report

The report is structured as follows. Section 2 reviews and summarizes the relevant researches on air quality forecasting. Section 3 discusses the methodology of the research and implementation of the proposed hybrid VMD-FAN model. Section 4 analyses the results obtained from the study and evaluates the proposed model performance. Section 5 summarizes the overall results of the study and discusses the limitations and proposes the future work.

2 Related Work

Before discussing the methodology of this research, it is important to critically review existing research related to the domain. This will help understand shortcomings and improve upon existing methodologies.

Air quality forecasting that include prediction of Air Quality Index (AQI) can help mitigate impacts of pollution. Traditional methods like ARIMA have already served as foundation of the time-series forecasting but limited by the non linearity and complexity of the data. whereas the research and advancement in machine learning methods have improved in better capture dynamic patterns and data complexity. Also studies have demonstrated the effectiveness of hybrid models which combine different approaches for time-series forecasting.This section summarizes recent studies in the domain.

2.1 Traditional Forecasting Approaches

Traditional time-series forecasting (TSF) methods like ARIMA and SARIMA have been employed since the early stages to predict air quality data. These methods assume linearity and stationarity. This limits their effectiveness and are not suitable in modeling the non-linear and stochastic nature of air quality data.

Pant et al. (2023) used SARIMA on AQI data for Dehradun, India and achieved an RMSE of 15.2 and a MAPE of 8.3%. While SARIMA was able to capture seasonal patterns effectively, but it has failed in handling with non-linearities and sudden changes in pollutants. Marinov et al. (2022) used ARIMA for air quality forecasting in Sofia City, this demonstrated the acceptable performance of ARIMA model for short-term predictions, however the model showed low flexibility for highly dynamic time series data. Atoui et al. (2022) analyzed exponential smoothing methods for AQI forecasting and this highlights their inability to model sudden variations in pollutant levels. While these methods have been the foundation for TSF, their limitations demands the exploration of non-linear and data-driven approaches.

2.2 Machine Learning Approaches

Machine learning models have improved the air quality forecasting by addressing the limitations of traditional techniques. These models are good at capturing the non-linear trends and temporal dependencies in the data. This section discusses some neural network approaches for forecasting and also the hybrid machine learning models.

2.2.1 Neural Networks

Feed-Forward Neural Networks (FFNNs):

Corani (2005) compared FFNNs with Lazy Learning models for AQI data in Milan. the study shows the FFNNs superiority over traditional methods, but FFNNs required extensive tuning and large datasets where required to generalize the trained models.

Long Short-Term Memory Networks (LSTMs):

Du et al. (2021) applied LSTMs to Beijing AQI data, reducing RMSE by 12% compared to ARIMA. LSTMs are good at capturing long-term temporal dependencies but they struggle with noise and periodicity. Another study by Chang et al. (2020) introduced aggregated LSTMs to integrate data from multiple stations for improving the forecasting accuracy for PM2.5 levels.

Convolutional Neural Networks (CNNs):

The study by Lim et al. (2019) conducted in South Korea demonstrates CNNs' ability to identify spatial-temporal patterns in pollutant concentrations, thus improving predictions across multiple regions.

2.2.2 Hybrid Machine Learning Models

Hybrid models combine different machine learning techniques to utilize their individual strengths in forecasting. Yi et al. (2018) developed DeepAir, a hybrid CNN-LSTM model for AQI forecasting, this study demonstrate the ability of Hybrid models in achieving accurate predictions across multiple monitoring stations. The study by Arnaudo et al. (2020) used Random Forests (RF) to integrate traffic and meteorological data, achieving short-term prediction accuracy with an RMSE of 8.7 in Milan. While these models captured non-linearity and temporal dependencies, they fail to perform well in handling noise and capturing periodic trends.

2.3 Variational Mode Decomposition (VMD)

VMD is a signal decomposition technique introduced by Dragomiretskiy and Zosso (2014). While compared to Empirical Mode Decomposition (EMD), VMD minimizes mode mixing and isolates intrinsic mode functions (IMFs), which can handle high noise and works better for non- stationary time series data.

Wang et al. (2024) the authors used VMD to decompose PM2.5 data in Beijing with BiLSTM for forecasting. The model reduced noise and achieved an RMSE of 10.5 and a MAPE of 6.1%. In a similar study by Lv et al. (2022) VMD was used in combination with LSTM for power grid load forecasting, demonstrating its capability to capture multi-scale temporal patterns. Showing its adaptability to various non-linear time series Zhou et al. (2024) extended VMD with multifractal analysis for rainfall prediction.Chen et al. (2023) demonstrated the integration of VMD with LSTM for AQI forecasting, achieving significant accuracy improvements over standalone models. Lim et al. (2019) integrated VMD with optimization algorithms for AQI predictions, highlighting the importance of decomposition in improving forecasting performance.

VMD is important in preprocessing since it decomposing the AQI data,this enhance the level of accuracy in the forecast results by better handling noise.

2.4 Fourier Analysis Networks (FAN)

FAN, presented by Dong et al. (2024) integrates Fourier series into neural network architectures to explicitly model periodic components in time-series data. This makes FAN particularly relevant for forecasting of air quality data which contains daily and seasonal cyclic patterns.

The authors in Dong et al. (2024) showed that FAN can reveal oscillating and repetitive features in synthetic air quality data with an RMSE of 9.1 and better than traditional neural networks.The study suggest that FAN outperforms LSTMs and CNNs with regard to generalizing across datasets and overcomes problems related to inherent periodicity.

2.5 Summary

The literature review identifies the shift from statistical models to active machine learning and hybrid models in air quality prediction. Although traditional methods such as ARIMA and SARIMA formed the foundation, they struggle with non-linear and non-stationary behaviors. These approaches are overcome by using other approaches like LSTMs and CNNs but they don't perform well when there is a lot of noise or periodicity and require extensive training and large datasets. Other models which have been used in combination with VMD include neural networks have exhibited better performance by providing both noise reduction and feature extraction improving forecasting.

This study builds on these advancements by proposing a VMD-FAN hybrid model, aiming to improve AQI forecasting accuracy.The Table 1 summarizes the literature review studies.

Table 1: Summary of Reviewed Studies

Study/Authors	**Methods**	**Key Findings**
Pant et al. (2023)	SARIMA	RMSE: 15.2, MAPE: 8.3%. Effective for seasonal trends but limited in handling non-linearity.
Marinov et al. (2022)	ARIMA	Good for short-term predictions; ineffective for dynamic, non-stationary time series.
Du et al. (2021)	LSTM	Reduced RMSE by 12% compared to ARIMA; effective for temporal dependencies but sensitive to noise.
Chang et al. (2020)	Aggregated LSTMs	Improved PM2.5 forecasting by integrating data from multiple sources.
Arnaudo et al. (2020)	Random Forest (RF)	RMSE: 8.7. Robust short-term predictions by incorporating traffic and meteorological data.
Wang et al. (2024)	VMD-BiLSTM	RMSE: 10.5, MAPE: 6.1%. VMD reduced noise and improved forecasting accuracy.
Lv et al. (2022)	VMD-LSTM	Demonstrated VMD's capability to capture multi-scale patterns in time series data.
Zhou et al. (2024)	VMD with multifractal analysis	Extended VMD for non-linear and multi-frequency behaviors; suitable for complex datasets.
Dong et al. (2024)	Fourier Analysis Networks (FAN)	RMSE: 9.1. Superior in capturing periodicity compared to MLPs and Transformers.
Chen et al. (2023)	VMD-LSTM	Improved AQI forecasting accuracy by isolating periodic components with VMD.
Lim et al. (2019)	VMD with optimization	Highlighted the importance of decomposition in improving forecasting performance.

3 Methodology

This section explains the systematic approach employed in this research to address the forecasting of air quality indices. It begins with data collection and preprocessing steps to clean and segment the data, followed by the decomposition of time series using Variational Mode Decomposition (VMD). This is integrated with the Fourier Analysis Network (FAN) to form a hybrid model, combining their strengths to improve prediction accuracy and address complexities in AQI data. Figure 1 illustrates the proposed hybrid VMD-FAN model for AQI time-series forecasting.

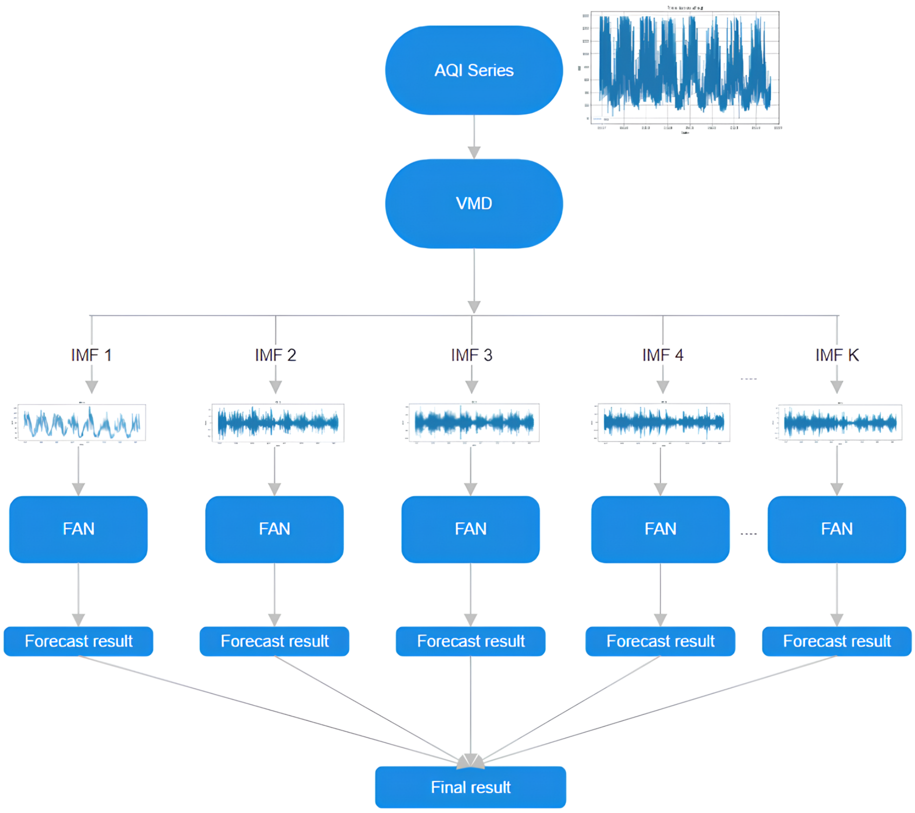


Figure 1: VMD-FAN Framework for AQI Forecasting

3.1 Study Area

The study area for this research focuses on Taiwan, located in East Asia between latitude 21° 53' N to 25° 18' N and longitude 119° 18' E to 124° 34' E. Taiwan has a subtropical climate characterized by four distinct seasons: It is divided into spring season ranging from March to May, summer season ranging from June to August, autumn season ranging from September to November and winter ranging from December to February. Seasons have a great effect on air quality in Taiwan; the concentration of pollutants is higher during winter caused by extra stability of the air and pollution transportation from neighbouring territories. The data set used for this study ranges from the year 2016 to 2024 with air quality parameters like PM2.5, PM 10, NOx, CO levels for different parts of Taiwan. Hence, this diverse and comprehensive database offers a strong foundation on which to assess the performance of the proposed hybrid VMD-FAN model relative to conventional forecasting models.

3.2 Data Collection

3.2.1 Data Source

The data set used in this study is identified as the Taiwan Air Quality Data 2016-2024 as found in Kaggle[1]. This dataset is composed by daily air quality measurements in various sites in Taiwan, thus contain both spatial and temporal resolution. The columns of the dataset used in the study is described in the data dictionary Table 2

Table 2: Taiwan Air Quality Data 2016-2024 data dictionary.

Column	**Description**	**Data Type**
date	Date and time of the reading	Text
sitename	Station name	Text
aqi	Air Quality Index	Numeric

The dataset includes various additional variables other than those used in this study which include SO2, CO, O3, PM10, PM2.5, NO2, NOx and NO and measure different pollutants like sulfur dioxide, carbon monoxide, ozone, and particulate matter in the air, provided in various units like ppb or ppm. Columns like o3-8hr, co-8hr, pm2.5-avg, pm10-avg, and so2-avg denote smoothed averages of pollutant concentrations, Also geographic details like longitude, latitude are included . The dataset contains features like the site name that permits city level analysis. The hybrid model proposed in this study is both built and evaluated using this dataset.

3.3 Data Preprocessing

This study focuses on the univariate time series forecasting, although the dataset contains various factors that contribute to air quality the study is limited to Air Quality Index and its forecasting using hybrid VMD-FAN model. The dataset was pre-processed following a structural approach in order to optimize it for time series analysis of the Air Quality Index (AQI). The main data preprocessing steps focused on cleaning and organizing the data to ensure it was ready for accurate decomposition and forecasting detailed process is discussed in the below sections.

3.3.1 Initial Inspection and Column Selection

The first step is to get an initial understanding of the dataset and its structure, more specifically any missing values and the data type of the columns. The dataset contains **5882208** rows and **25** columns. To simplify the analysis and focus on key variables, only the '**date**', '**sitename**' and '**aqi**' columns were selected. These columns were chosen to create time-series data for air quality index while reducing unnecessary complexity.

[1]Dataset url: `https://www.kaggle.com/datasets/taweilo/taiwan-air-quality-data-20162024`

3.3.2 Missing and Invalid entries

For missing values, the rows which contain incomplete details in the chosen columns were dropped. This was important in order to ensure that data is clean to allow for analysis in further steps. The 'date' and site' columns does not contain any missing values whereas 'aqi' had **43020** missing values also the values with AQI that were either zero or less than zero were omitted since the AQI only has positive values.

3.3.3 City-wise Data Segmentation

The dataset was segmented by splitting them based on the location, separate city wise AQI time series was formed. This enables us to evaluate the performance of the proposed model in different parts of the study area. This is used for the extension analysis to understand the generalization ability of the proposed model. For the initial development and evaluation of the model the time series of site '**Annan**' is used this contains **67144** rows. The district of Annan was selected based on the fact that it represents typical seasonal and periodic patterns of air quality in Taiwan, also the data was clean and had complete time series with minimal missing values Figure 2 shows the cleaned time-series data for 'Annan' city

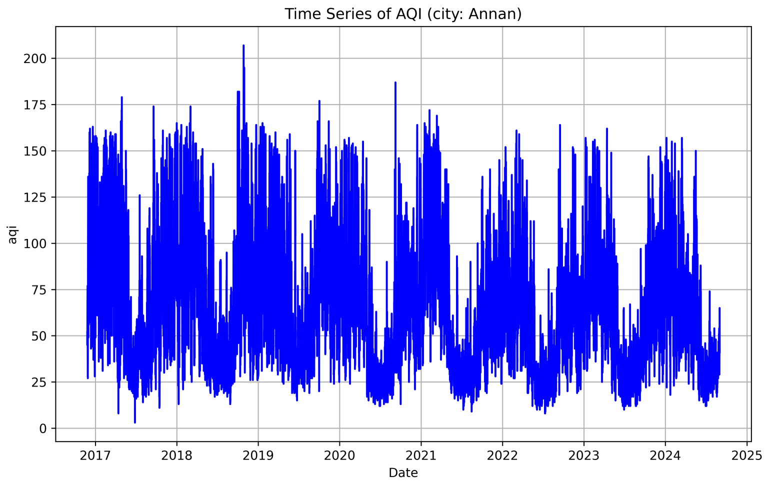


Figure 2: Time Series of Air Quality Index (AQI) for Annan (2016-2024)

3.3.4 Finalization and Saving

Following preprocessing, cleaned data for each of the cities was output separately to files for further processing. The grouped data for each city is saved as a separate csv file. This organization ensure that dataset obtained is clean and organized for further decomposition by means of the VMD followed by forecasting using the FAN.

These preprocessing steps help in cleaning out the dataset and obtaining the best format suitable for our analysis that aims to forecast timeseries of AQI.

3.4 Data Decomposition Using VMD

Variational Mode Decomposition (VMD) is used to decompose the Air Quality Index (AQI) time-series data into a series of intrinsic mode functions (IMFs). This step ensures the extracting of distinct frequency components from the time-series data, this enables effective forecasting by isolating underlying patterns.

VMD is a modern signal processing technique that decomposes a time-series signal into a predefined number of IMFs. Each IMF represents a specific frequency band, capturing components such as trends, periodic patterns, and noise. By isolating these components, VMD provides a structured representation of the data, each of these IMF are further processed using the FAN to obtain the forecasting results. The VMD can be represented as equation 1 (Dragomiretskiy and Zosso; 2014)

$$\min_{\{u_k\},\{\omega_k\}} \left\{ \sum_{k=1}^{K} \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * \left(u_k(t) e^{-j\omega_k t} \right) \right] \right\|_2^2 \right\}, \tag{1}$$

subject to the constraint:

$$\sum_{k=1}^{K} u_k(t) = f(t),$$

where:

- $u_k(t)$: the k-th mode (sub-signal).
- ω_k: the center frequency of the k-th mode.
- $f(t)$: the original signal.
- ∂_t: time derivative.
- $*$: convolution operator.
- $\delta(t) + \frac{j}{\pi t}$: the impulse response of the Hilbert transform.

3.4.1 Parameter Tuning of vmd

To find the optimal decomposition which will lead to better forecasting, decomposition with every combination of these parameters in Table 3 where conducted:

Table 3: Parameters for Variational Mode Decomposition (VMD)

Parameter	**Description**	**Tested Values**
K	Number of IMFs	3, 5, 7
Alpha	Bandwidth Constraint	1000, 3000
Tau	Noise-tolerance threshold	1e-6
Tol	Convergence tolerance	1e-5, 1e-7

3.4.2 Decomposition of AQI Data

For each city, the cleaned AQI data was subjected to VMD decomposition. The AQI time-series data was input into the VMD algorithm with the predefined parameter grid. The data was decomposed into multiple IMFs, each representing a specific frequency range. The decomposed IMFs were saved in structured files for each city, organized based on

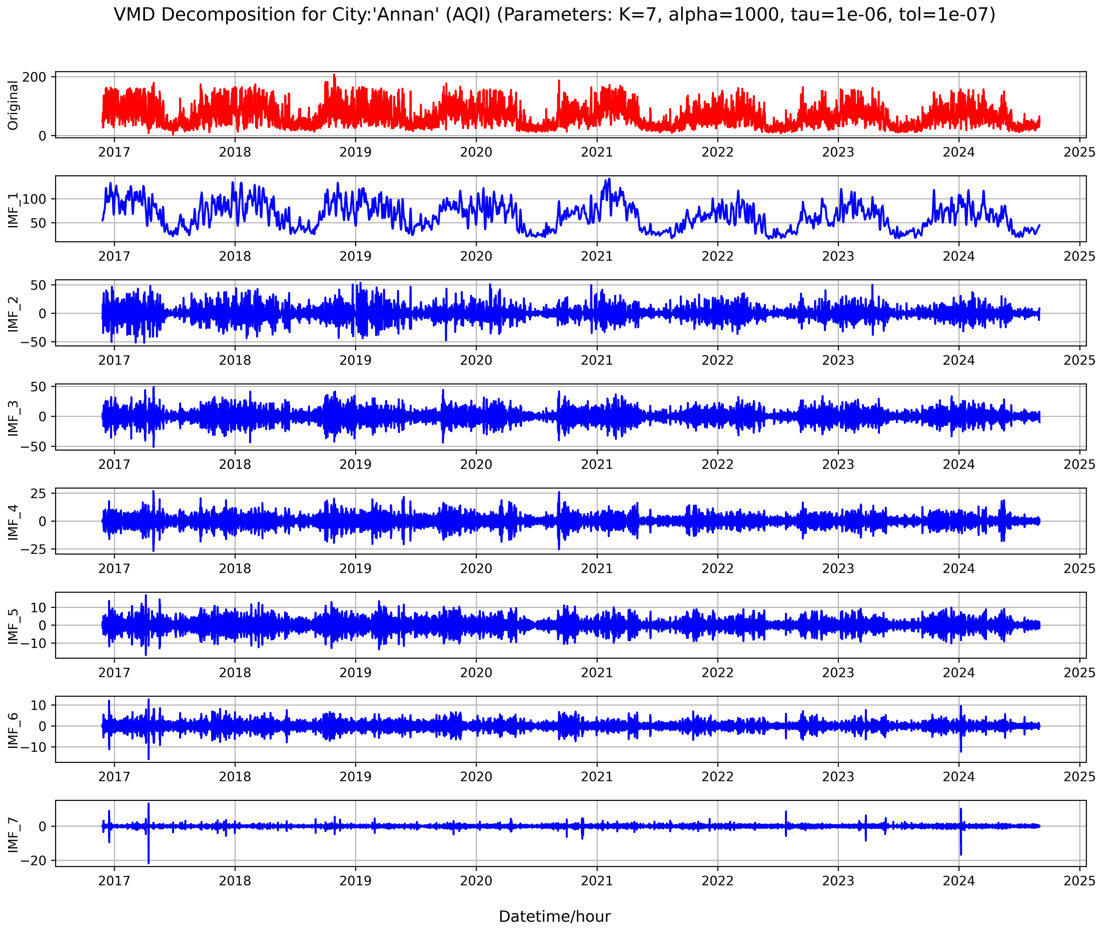


Figure 3: VMD Decomposition of AQI Time Series for Annan (Parameters: Number of Modes $K = 7$, Bandwidth Constraint $\alpha = 1000$, Noise-Tolerance $\tau = 1 \times 10^{-6}$, Convergence Tolerance $\text{tol} = 1 \times 10^{-7}$)

parameter combinations. Figure 3 illustrates the decomposed 7 IMFs of the AQI Time Series for Annan.

These IMFs captured the different underlying components of the AQI time series such as Long-Term Trends which include slow-changing patterns in air quality over time, Seasonal Variations which include periodic patterns due to meteorological and environmental factors, High-Frequency Noise representing short-term fluctuations. The IMFs generated for each city were saved as individual files, categorized by their parameter settings. This structured organization allowed for efficient experimentation with different forecasting models and parameters.

By applying VMD, the AQI time-series data was transformed into a sub time-series data, enabling the subsequent Fourier Analysis Network (FAN) on each of the components to focus on specific patterns and improve forecasting accuracy.

3.5 Model Development

The development of the forecasting model involved integrating the Variational Mode Decomposition (VMD) outputs with the Fourier Analysis Network (FAN). This hybrid approach leveraged the strengths of both techniques to enhance the accuracy of AQI predictions.

3.5.1 Fourier Analysis Network (FAN)

The Fourier Analysis Network (FAN) is used as the core forecasting model for its ability to effectively handle periodic and cyclical patterns in time-series data. The FAN model uses a combination of sinusoidal and linear transformations to capture both linear and non-linear relationships in the decomposed components of AQI data.

Based on Dong et al. (2024) FAN is designed with the FAN layer $\phi(x)$ defined as equation 2:

$$\phi(x) \triangleq [\cos(W_p x) \,\|\, \sin(W_p x) \,\|\, \sigma(B_p + W_{\bar{p}} x)], \tag{2}$$

where W_p, $W_{\bar{p}}$, and B_p are learnable parameters and σ denotes the activation function. The entire FAN is defined as the stacking of the FAN layer $\phi(x)$:

$$\mathrm{FAN}(x) = \phi_L \circ \phi_{L-1} \circ \cdots \circ \phi_1 \circ x,$$

where

$$\phi_l(x) = \begin{cases} [\cos(W_p x) \,\|\, \sin(W_p x) \,\|\, \sigma(B_p + W_{\bar{p}} x)], & \text{if } l < L, \\ B^L + W^L x, & \text{if } l = L. \end{cases}$$

The FAN consists of a custom Fourier layer designed to capture cyclic patterns. This layer uses sine and cosine transformations to extract features from the IMFs generated by VMD. An output layer maps these extracted features to AQI predictions. The model was implemented in a sequence-to-sequence format to predict future AQI values based on past data.

3.5.2 Sequence Creation

The IMFs generated by VMD were divided into overlapping sequences to create inputs for the FAN model. This was achieved by defining a window size, which specified the number of past observations used to predict the next AQI value. Each sequence captures the temporal dependencies in the IMFs.

3.5.3 Training Process

The FAN model was trained for each IMF separately, allowing it to focus on the specific frequency components represented by that IMF. The data was divided into training and testing subsets, with 80% used for training and 20% reserved for testing. The model optimize the Mean Squared Error (MSE) loss, which minimize the average squared difference between predicted and actual AQI values. The Adam optimizer is used for its ability to adapt learning rates during training, improving convergence speed and accuracy.

3.5.4 Aggregating Predictions

The predictions from all IMFs are aggregated to reconstruct the final AQI forecast represented by equation 3 as presented by Dragomiretskiy and Zosso (2014). This approach helps in capturing the unique information from each IMF to improve overall prediction accuracy.

$$\hat{f}(t) = \sum_{k=1}^{K} \hat{u}_k(t), \tag{3}$$

where $\hat{u}_k(t)$ are the individual IMF forecasts.

3.5.5 Parameter Tuning of FAN Model

The FAN model for each IMF was tunned to find the best parameters for the model based on the below parameter search grid Table 4

Table 4: Parameters for Fourier Analysis Network (FAN)

Parameter	**Description**	**Tested Values**
Window Size	Length of the input time-series window for processing.	12
Learning Rate	Step size for the optimizer during training.	0.0001,0.00001
P Ratio	Proportion of the series used for periodicity modeling.	0.15, 0.25
FAN Units	Number of units in the FAN.	64

3.5.6 Baseline Model

A simple feedforward neural network model BaseNN is included in the study as baseline model to benchmark the performance.The BaseNN consists of hidden layers with a ReLU activation function, followed by a linear output layer that maps the processed input to the target AQI prediction.The models architecture is simple thereby making it efficient computationally while also easy to interpret therefore enabling it to use as a baseline model. The selection of BaseNN can also be justified by the fact that in the time-series forecasting literature, a simple neural network is often used as a reference point.

3.5.7 Model Testing and Evaluation

The model is tested with the test data. The aggregated predictions are compared against actual AQI values using following evaluation metrics

Mean Absolute Error (MAE)

$$\text{MAE} = \frac{1}{N} \sum_{i=1}^{N} |y_i - \hat{y}_i|. \tag{4}$$

Mean Squared Error (MSE)

$$\text{MSE} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2, \tag{5}$$

Root Mean Squared Error (RMSE)

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (y_i - \hat{y}_i)^2}. \tag{6}$$

Mean Absolute Percentage Error (MAPE)

$$\text{MAPE} = \frac{100}{N} \sum_{i=1}^{N} \left| \frac{y_i - \hat{y}_i}{y_i} \right|. \quad (7)$$

By combining VMD and FAN, the model is able to decompose complex AQI patterns into different components and accurately forecast future values, demonstrating the effectiveness of the hybrid approach.

3.5.8 Experimental environment and configuration

The experiments in this research were conducted using Google Colab. The models were implemented using Python (version 3.10.12) and PyTorch (version 2.5.1+cu121) with GPU acceleration.

Table 5: Experimental Environment Configuration

Experimental Environment	**Specific Settings**
GPU	NVIDIA Tesla T4, Memory: 15,360 MB
CPU	Intel(R) Xeon(R) CPU @ 2.00GHz
Default Hard Disk	236 GB, 204 GB available
Additional Hard Disk	None
RAM	51 GB
Network	Cloud-based, dependent on Colab settings

The GPU used for the experiments was an NVIDIA Tesla T4 with 15,360 MB of dedicated memory, running CUDA version 12.2.and 51 GB of RAM, ensuring efficient handling of computational tasks. The specific environment configuration is summarized in Table 5

4 Evaluation

The detailed evaluation of the proposed VMD-FAN hybrid model and comparison with the baseline models are explained in this section.The analysis also shows the effect of the best parameter combinations on the performance metrics and also the importance of parameters in each model. The evaluation is based on the AQI dataset for Annan city to show that the model is capable of accurately predicting time series data.

4.1 Best Parameter Selection and Model Performance

To ensure a fair comparison, each model was tested with the best parameters for it, the best parameters are represented in Table 6. The BaseNN model was found to be optimal with a window size of 12, hidden dimension of 64, learning rate of 0.0001, and a forecast horizon of 1. Similarly, the FAN model was found to perform optimally with a window size of 12, learning rate of 0.0001, and a periodicity ratio (p_{ratio}) of 0.25, FAN units of 64, and a forecast horizon of 1. In the case of the hybrid models, VMD parameters were adjusted to ensure optimal decomposition of the signals. More specifically, the number of intrinsic mode functions (K) was fixed at 7, the bandwidth constraint (α) at 1000, the noise tolerance (τ) to 1×10^{-6}, and the convergence tolerance ($\texttt{tol}$) to 1×10^{-7}. For the forecasting, both hybrid models employed a window size of 12. The VMD+BaseNN

Table 6: Best Parameters for VMD and FAN

Component	Parameter	Best Value
VMD	K	7
	Alpha	1000
	Tau	1e-06
	DC	False
	Tol	1e-07
FAN	P_Ratio	0.15
	Fan Units	64
	Window Size	12
	Learning Rate	0.0001

model had a hidden dimension of 64 and a learning rate of 0.0001, while the VMD+FAN model used a periodicity ratio (p_{ratio}) of 0.15, FAN units of 64, and a learning rate of 0.0001.

The performance of each model is presented in Table 7 below. The BaseNN model, had higher error metrics, with a Mean Absolute Error (MAE) of 2.59 and a Root Mean Squared Error (RMSE) of 3.99. Likewise, the FAN model which is good at capturing periodicity was slightly better than BaseNN with an MAE of 2.53 and an RMSE of 3.86. However, the hybrid models that include VMD performed much better than the two baseline models. The VMD+BaseNN model had an MAE of 1.49 and an RMSE of 2.11, while the VMD+FAN model with the lowest overall MAE of 0.72 and an RMSE of 1.16. The Figure 4 shows the comparison of the final prediction results of each model

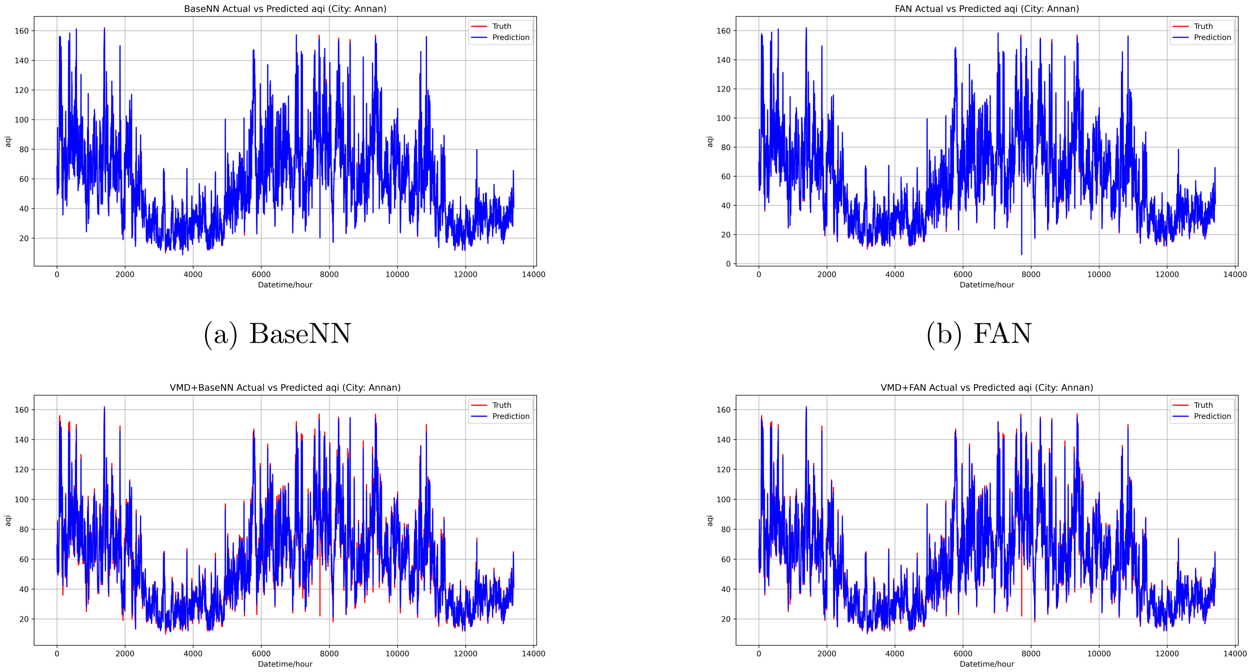
Table 7: Performance Metrics of the models.

Model	MAE	MSE	RMSE	MAPE
BaseNN	2.585101	15.953592	3.994195	5.052568%
FAN	2.534186	14.861638	3.855080	5.149411%
VMD+BaseNN	1.485718	4.450701	2.109669	2.924737%
VMD+FAN	0.717643	1.352704	1.163058	1.495354%

4.2 Analysis of Model Effectiveness

As it can be seen from the above results, the inclusion of VMD in the forecast approach has an effect on model accuracy. By decomposing the AQI time-series data into intrinsic mode functions (IMFs), VMD effectively successfully extracts features like long-term trends, periodicities and high frequency noise. This decomposition allows the models to focus in different aspects of the data, improving the accuracy. The MAE for the VMD+BaseNN model was reduced by over 77% compared to the BaseNN model without VMD. Similarly, the VMD+FAN model showed a 70% reduction in MAE compared to the FAN model, This highlights the effect of decomposition in addressing non-stationary and non-linear characteristics of AQI data.

The choice of parameters for VMD played a significant role in these outcomes. A higher number of IMFs (K) improved the granularity of decomposition but also increased the computational complexity. The selected value of $K = 7$ ensure balance by capturing



(a) BaseNN (b) FAN

(c) VMD+BaseNN (d) VMD+FAN

Figure 4: Comparison of the final prediction results of each model.

critical patterns without overfitting to noise. Similarly (α) of 1000 helped maintain trend stability, while the noise tolerance (τ) of 1×10^{-6} ensured the model focused on meaningful data features. These parameter choices emphasise the need to fine-tune the system to get the best out of the VMD methodology.

4.3 Comparison and Generalization

The results showcase the efficiency of the VMD+FAN model and proves the relevance of using the hybrid combination of Fourier Analysis Networks (FAN) with Variational Mode Decomposition (VMD). VMD is effective in reducing noise and isolating periodic components, while FAN models these periodicities. The VMD+FAN model has the lowest Mean Absolute Percentage Error (MAPE) of 1.36%, highlighting the combined strengths of two methods. The VMD+FAN model shows slightly better performance in terms of Mean Absolute Error (MAE) at 0.67 compared to 0.70 for VMD+BaseNN and Root Mean Squared Error (RMSE) at 1.08 compared to 1.09, also this highlights the precision in forecasting air quality data. The small differences between these metrics suggest that both hybrid model and the baseline approaches offers comparable accuracy, and VMD+FAN is particularly suitable for datasets with strong periodic behaviors.

The baseline models were acceptable but could not solve the issues related to complex AQI data. For example, the BaseNN model was not able to deal with periodicity which lead to higher error rate. While the FAN model partially deal with this shortcoming, but still did not attain the level of precision of the hybrid methods. This highlights the problem with using one model on its own and the need for efficient methods of combining decomposition with complex forecasting approaches.

4.3.1 Extension analysis

A model needs to be generalizable. In order to confirm that the resulting model performs with different scenarios, a extension analysis is conducted and the results are illustrated in Figure 5 and Table 8. For extension analysis 5 sites Banqiao, Cailiao, Dongshan, Zhongshan and Yilan in Taiwan other than Annan which was used to train and evaluate the method where chosen and the VMD-FAN model is applied. The results show that the hybrid model performs well with all the different sites confirming the models generalization.

Table 8: Results of Extension analysis using VMD+FAN.

Site	MAE	MSE	RMSE	MAPE
Annan	0.717643	1.352704	1.163058	1.495354%
Banqiao	0.978943	2.563178	1.600993	2.626042%
Cailiao	1.130901	3.208828	1.791320	2.914285%
Dongshan	0.645761	1.120890	1.058721	2.230362%
Yilan	0.613743	0.949443	0.974394	1.883478%
Zhongshan	1.001847	2.530185	1.590655	2.671513%

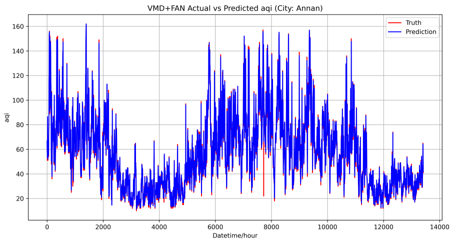
4.4 Summary

In conclusion, the results demonstrate the advantages of the proposed VMD-FAN hybrid model. By combining the strengths of signal decomposition and periodicity modeling, the hybrid approach achieved higher accuracy in forecasting AQI data. The analysis also highlights the importance of parameter optimization in enhancing model performance. The success of the VMD+FAN model, in particular, offers a promising direction for future research and applications in air quality forecasting.

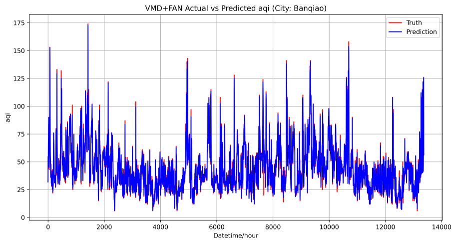
5 Conclusion and Future Work

5.1 Conclusion

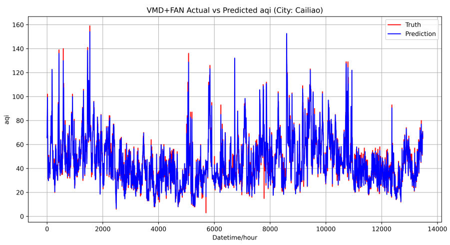
This research explored a novel hybrid model that combines Variational Mode Decomposition (VMD) and Fourier Analysis Network (FAN) to effectively forecast air quality index (AQI).The time series data of Air Quality Index (AQI) is very complex and nonstationary, therefore the forecasting and accurate prediction of AQI was challenging. The AQI timeseries extracted from the original dataset was decomposed into individual Intrinsic Mode Functions (IMFs) using VMD and each IMF is predicted using a FAN model subsequently aggregated to form a final forecast of AQI values.The proposed hybrid model predicts the AQI of Taiwan with a MAE,MSE,RMSE and MAPE as 0.717643, 1.352704, 1.163058, 1.495354% respectively and is better than the compared base model. The generalizability of the model is further validated with extension analysis on different city in Taiwan.



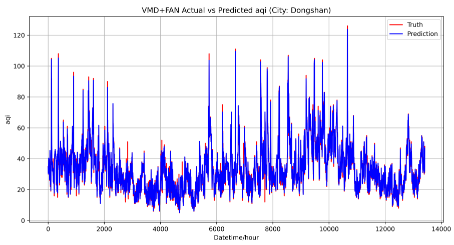
(a) Annan



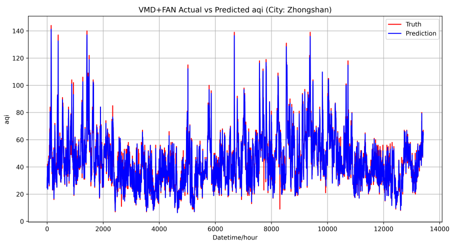
(b) Banqiao



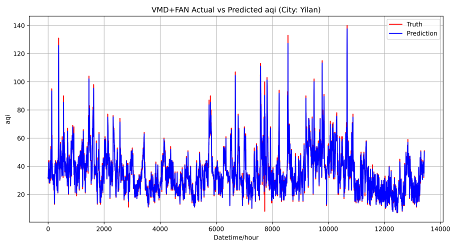
(c) Cailiao



(d) Dongshan



(e) Zhongshan



(f) Yilan

Figure 5: Results of Extension analysis.

5.2 Future Work

The study produced promising results, whereas there are few area which faced difficulties and there is a further scope for improvement.

This study considered only univariate time series forecasting focusing on AQI data, the dataset contained various other factors which contribute to air quality forecasting. This data was not able to be included because of the complexity it creates. Future studies can extend this aspect and conduct multivariate time series analysis to enhance the forecast and provide comprehensive insights. Also the parameter tunning done in this study was based on a grid search this added the computational complexity this can be improved with a optimization algorithm to find the best parameters. The FAN model used in this study was a basic implementation of the FAN layers by Dong et al. (2024) but this can be improved with incorporating FAN with Attention mechanisms or Transformers architectures.

Based on these aspects, it is possible to continue improving the hybrid VMD-FAN model to give more detailed and effective solution for the time series forecasting problems in air quality monitoring domain.

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