

<u>Influence of Cryptocurrencies on Portfolio Optimization:</u> <u>An Analysis of Diversification Opportunities</u>

MSc Research Project Master of Science in FinTech

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MSc Project Submission Sheet

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AI Acknowledgement Supplement

MSc Research Project Influence of Cryptocurrencies on Portfolio Optimization: An Analysis of Diversification Opportunities

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AI Acknowledgment

This section acknowledges the AI tools that were utilized in the process of completing this thesis.

Tool Name	Brief Description	Link to tool
ChatGPT	Understanding concepts more clearly. Paraphrasing raw content.	https://chatgpt.com/
Quillbot	Rewriting self-written content in a formal and academic manner.	https://quillbot.com/

Description of AI Usage

ChatGPT

ChatGPT was used to help in understanding financial and mathematical concepts. Additionally, it was used to help in rewriting Python code to be more readable and concise. It also was used to rectify any errors in the code.

Sample prompt: "Can you explain the mathematical concepts of Mean-Variance Optimization technique?"

Sample response from ChatGPT: "The expected return of a portfolio is the weighted average of the expected returns of the individual assets within the portfolio.

The risk (volatility) of the portfolio is captured by the variance of its return. Variance measures how much the returns of the portfolio deviate from the expected return.

The variance of a portfolio depends on the variances of the individual assets and the covariances between each pair of assets."

Quillbot

Quillbot was used to paraphrase and rewrite raw content to sound more formal and academic.

Sample text: "This research endeavours to examine the consequences of incorporating cryptocurrencies, specifically BTC and ETH, into conventional portfolios. A majority of previous research and related works focus on Sample text generated by Quillbot: "The purpose of this study is to look into what happens when you add cryptocurrencies like Bitcoin and Ethereum to regular investments. Most of the study that has been done on this topic so far has looked at how Bitcoin affects traditional

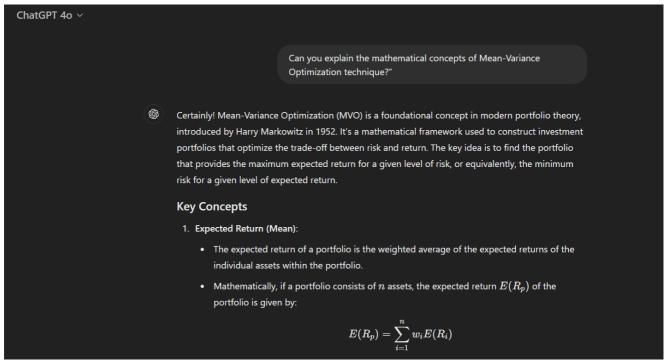
analysing the effects of Bitcoin in traditional portfolios. To resolve the knowledge gap, this study will investigate the implications of incorporating Ethereum into various combinations of portfolios consisting of traditional financial asserts and using different portfolio optimization strategies to generate optimal Sharpe Ratios."

portfolios. To fill in the gaps in our knowledge, this study will look into what happens when Ethereum is added to different types of portfolios that contain traditional financial assets and how to use different portfolio optimisation techniques to get the best Sharpe Ratios."

Evidence of AI Usage

ChatGPT was the main AI tool used for helping in this thesis. It was used to break down complex financial concepts for better understanding. It was also used to understand the mathematical frameworks of these models. It helped in summarizing similar research papers and understanding their limitations. It helped in refining ideas that would overcome these limitations. ChatGPT assisted in structuring the report and rewriting sections in a better way. Additionally, it was used to rectify any errors in the Python code. It also helped in making the code concise and more readable.

Additional Evidence A:



Additional significant prompts

- Can you explain the Critical Line Algorithm?
- How to I import the "CLA" class from PyportfolioOpt Library?
- Breakdown the Sharpe Ratio Formula
- Rectify this error in my code
- Give me the basic structure of a thesis. What sub sections should be included in the report?

<u>Influence of Cryptocurrencies on Portfolio Optimization:</u> <u>An Analysis of Diversification Opportunities</u>

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Abstract

The twenty-first century has been marked by extraordinary technological advancements that have had a profound impact on a variety of sectors, including financial services. Blockchain technology and cryptocurrencies have emerged as significant developments among these innovations. Cryptocurrencies, which were initially perceived as speculative assets, have gained the attention of both institutional and individual investors who are seeking to increase their returns in spite of their inherent volatility. This research examines the potential diversification advantages cryptocurrency, specifically Bitcoin (BTC) and Ethereum (ETH), provide when incorporated into conventional investment portfolios. This study conducts a thorough examination of the influence of BTC and ETH on portfolio performance, with a particular emphasis on the Sharpe ratios, returns, and risk of portfolios that incorporate these digital assets in addition to traditional assets. The objective of the research is to ascertain whether Ethereum offers superior diversification advantages in comparison to Bitcoin. This study utilises empirical data analysis and sophisticated portfolio optimisation techniques to provide useful insights, building on previous research that has emphasised the benefits of integrating cryptocurrencies into conventional portfolios.

Keywords: Portfolio Optimization, Cryptocurrencies, Bitcoin, Ethereum

1 Introduction

In recent years, there has been a substantial increase in study on cryptocurrencies and the underlying blockchain technology. People are progressively embracing digital currencies for several reasons, such as making payments, engaging in mining activities to get incentives, and directly investing by buying and selling. Despite the increasing interest, experts maintain a cautious stance about various elements of cryptocurrencies and their future direction. Blockchain technology, an innovative and groundbreaking advancement, has significantly disrupted the conventional finance industry and is leading the path towards a new era of financial technology. Blockchain, closely linked to Bitcoin, which was established in 2009 by the anonymous Satoshi Nakamoto, has given rise to various additional blockchain-driven innovations, including smart contracts, NFTs, and DeFi apps. This technology is gradually replacing traditional financial services because of its quick growth and numerous applications. Blockchain functions as a distributed, unchangeable system for transactions and data management, where records are spread across multiple interconnected nodes that form a chain. From the perspective of a distributed ledger, blockchain is decentralised and exists across a network of many locations, devices, and stakeholders. Digital currencies, blockchain technology, and cryptocurrencies are fundamentally changing and questioning conventional financial systems in many ways. Consumers frequently face the challenges of excessive expenses, delays in transactions, and intricate paperwork linked to financial intermediaries. Nevertheless, the technological advancements of digital currencies eliminate the necessity for intermediaries to verify transactions or identities, reduce transaction durations, and establish a transparent system for documenting data and information.

The cryptocurrency sector is growing at an extraordinary pace. As of 2024, there are over 22,000 cryptocurrencies listed on various platforms (CoinMarketCap, 2024). In recent years, the number of cryptocurrencies has increased exponentially. The global cryptocurrency market valuation currently stands at \$2.49 trillion, a 97.73% increase from the previous year (CoinGecko, n.d.). This rapid growth is fuelled by project developers who have expanded the application of cryptocurrencies to include decentralized finance (DeFi), gaming, non-fungible tokens (NFTs), and metaverse environments. The potential for profits from trading in these volatile markets has attracted numerous investors.

Due to the constant innovation of blockchain technologies and the rapid growth of numerous cryptocurrencies, investors now have a variety of options when it comes to diversification of their portfolios. Incorporation of cryptocurrencies for reducing risk and maximising returns is a definite possibility in today's age. However, the selection of cryptocurrencies for this task is of vital importance.

1.1 Research Question:

Is Ethereum a better diversifier than Bitcoin when added to traditional investing portfolios?

1.2 Research Objectives:

- Creating portfolios with and without cryptocurrencies
- Applying portfolio optimization techniques to these portfolios
- Calculating and comparing Sharpe Ratios

Therefore, this thesis aims to provide financial professionals and investors with practical insights pertaining to cryptocurrency investment, thereby contributing to the existing literature.

2 Related Work

In the midst of doubt, excitement, and financial transactions, researchers persist in examining and studying the possible consequences of digital currencies. For instance, according to (Yermack, 2014), cryptocurrencies do not completely fulfil all the requirements of conventional currencies, only partially meeting these parameters. Economists usually define money in terms of three functions it serves: as a medium of exchange, unit of account and standard for deferred payment (store of value). Bitcoin partially fulfils the initial requirement, as a growing number of merchants, especially in online markets, are open to accepting it as a means of payment. Nevertheless, the significant volatility of Bitcoin and the absence of arbitrage opportunities due to its varying prices across exchanges reduce its effectiveness as a unit of account. Bitcoin's role as a store of wealth is hindered by the considerable obstacles posed by widespread hacking assaults, thefts, and various security-related concerns. Additionally, as stated by (Grinberg, 2011), cryptocurrencies are not generated by any public entity, such as a governmental body or a financial institution. This indicates that they are decentralised and, to some extent, virtual. Their prices are determined exclusively by the forces of supply and demand, resulting in greater volatility when compared to traditional currencies. The absence of regulation, along with these variables, renders cryptocurrencies very vulnerable to speculation and the emergence of financial bubbles.

However, the majority of current research supports the idea that cryptocurrencies are slowly transforming into a new and separate category of assets. Furthermore, the cryptocurrency market has experienced significant growth in recent years, characterised by enhanced liquidity and a reduced bid-ask gap. The intrinsic worth of cryptocurrencies is evidenced by the enhanced security measures implemented in computers and trading platforms, as well as the stabilisation of volatility, resulting in a significant reduction in the danger of financial loss. While there is ongoing dispute about whether cryptocurrencies may be classified as a new type of asset, most current research indicates that they are increasingly being recognised as a distinct asset class. As per (Marie Brière, 2013), Bitcoin is an asset that is distinctive from the perspective of an investor due to its unique characteristics. The results of their research suggest that Bitcoin investments are highly volatile, but they also provide excellent returns. This elevated risk is mitigated by Bitcoin's low correlations with other assets for investors who maintain well-diversified portfolios. A small portion of Bitcoin into a well-diversified portfolio can dramatically improve the overall risk-return pattern. Another study conducted by (Jaywant, 2019) emphasises the distinctive characteristics of Bitcoin, identifying plenty of investment opportunities and a sufficient market capitalisation, thereby satisfying the criteria for investability. Financial markets have multiple access points into Bitcoin (direct investment, mining, etc), and that broad representation serves as an important source of confidence given the ongoing battle between risk-seeking inflation forces and those interested in price stability. The open-source and decentralised character of this asset class distinguishes its politicoeconomic profile from that of other asset classes. In addition, Bitcoin has no correlation or even a small positive one with other asset classes in most developed and developing economies, making it an excellent diversifier. Additionally, Bitcoin offers superior risk-adjusted returns when contrasted with numerous other asset classes. Similarly, (Shaen Corbet, 2019) conducted a study where the relationships of the three major cryptocurrencies (Bitcoin, Ethereum, and Ripple) with multiple financial assets were analysed in the time and frequency domain. Their results suggest that these cryptocurrencies are relatively isolated from conventional financial and economic assets. The findings indicate that cryptocurrencies may offer diversification advantages to investors with short-term investment horizons.

According to the research by (David Kuo Chuen Lee, 2017), the CRIX index and cryptocurrencies can be used to diversify portfolio risks as a result of the consistently low correlations between cryptocurrencies and traditional assets. Furthermore, traditional investments are outperformed by the average daily returns of the majority of cryptocurrencies. The efficient frontier analysis is further evidence that the efficient frontier of traditional asset classes is substantially expanded by the inclusion of the CRIX index. Another research conducted by (Krückeberg, 2019) indicates that cryptocurrencies satisfy Sharpe's 1992 criteria of mutual exclusivity, exhaustiveness, and differing returns by exhibiting high internal correlations and low correlations with traditional asset classes. In addition, traditional portfolios exhibit substantial and consistent outperformance with even a minimal 1% allocation. Although volatility is not reduced the inclusion of cryptocurrencies in conventional portfolios improves risk-adjusted returns (Sharpe ratios). In the same way, (Alexander Brauneis, 2018) conducted research that illustrated a substantial potential for risk reduction when traditional asset classes are combined with multiple cryptocurrencies, rather than a single cryptocurrency (frequently Bitcoin), in a portfolio. (Petukhina Alla, 2018) evaluated the effectiveness of conventional asset-allocation models by analysing the historical prices and trading volumes of 52 cryptocurrencies when combined with 16 traditional assets. Their research demonstrated that models that prioritise diversification and seek high returns generate greater anticipated returns for investors by integrating a broader selection of cryptocurrencies. The potential for profit is increased by the incorporation of cryptocurrencies in a portfolio of

diverse asset classes, as demonstrated by empirical evidence (Yechi Maa, 2020). The results, which are represented as efficient frontiers, suggest that diversified portfolios that include cryptocurrencies generate higher returns for the same level of risk.

This research endeavours to examine the consequences of incorporating cryptocurrencies, specifically BTC and ETH, into conventional portfolios. A majority of previous research and related works focus on analysing the effects of Bitcoin in traditional portfolios. To resolve the knowledge gap, this study will investigate the implications of incorporating Ethereum into various combinations of portfolios consisting of traditional financial assets and using different portfolio optimization strategies to generate optimal Sharpe Ratios.

3 Theoretical Background

3.1 Cryptocurrencies

Cryptocurrencies are digital or virtual currencies that make use of cryptography for security, making them difficult to counterfeit. Unlike fiat currencies, which are issued by governments, cryptocurrencies operate on decentralized networks based on blockchain technology. Blockchain is a distributed ledger enforced by a network of computers and nodes. Cryptocurrencies enable peer-to-peer transactions without intermediaries such as banks, resulting in benefits like quicker transfers, increased privacy, and reduced transaction fees.

The next section provides an overview of the two primary cryptocurrencies used in this research: Bitcoin and Ethereum.

1. Bitcoin:

In 2008, a paper was published by an individual or group operating under the pseudonym Satoshi Nakamoto, which defined the concept of digital currencies. This paper, (Nakamoto, 2008), introduced the Bitcoin network. Bitcoin is not supported by a central bank, in contrast to traditional government-issued currencies. Rather, it is based on a decentralised network that enables participants to create new Bitcoins through the process of mining.

According to (Rose, 2015), the process of mining Bitcoin is relatively simple, cost-effective, and accessible, as it is based on the principle of decentralised power, which states that each participant has equal influence. In the Bitcoin ecosystem, there are no institutions or intermediaries, and as a result, individuals are responsible for managing their own Bitcoin in virtual accounts known as wallets. Users are granted complete control over their funds and are capable of conducting transactions. Bitcoin's value is intrinsically volatile and is determined by market demand, fluctuating depending on the price that purchasers are willing to pay at any given moment. Many investors have experienced large financial gains and losses as a consequence of this volatility. Bitcoin's value has undergone significant fluctuations throughout history, rising from less than one dollar in 2011 to more than \$1,200 in 2013, and presently standing at more than \$66,000. The fluctuating market demand and the absence of a central authority that regulates its value are the driving forces behind these price fluctuations. This makes Bitcoin a distinctively innovative and adaptable asset in comparison to conventional currencies, providing investors with a lot of opportunities to profit from its fluctuations.

Conventional payment systems, including Visa, MasterCard, and PayPal, are different from Bitcoin, which functions as a peer-to-peer network. In traditional payment systems,

the network is supervised and secured by a central authority, which is typically a forprofit corporation, to guarantee its reliability. These companies serve as intermediaries, overseeing transactions and ensuring the network's integrity. In contrast, Bitcoin's security and operation are decentralised, as they are based on a cryptographic algorithm called the Secure Hash Algorithm (SHA-256) rather than being backed by any government or central institution. The integrity and security of transactions within the Bitcoin network are guaranteed by this cryptographic protocol. (Nakamoto, 2008) provides a comprehensive explanation of the intricate mechanisms that underlie Bitcoin.

1.1.Proof-of-Work:

The technical implementation of Bitcoin has its foundation on the fundamental concept of Proof of Work (PoW). A PoW system similar to Adam Back's Hashcash (Back, 2002) is employed to establish a distributed timestamp server on a peer-to-peer basis. The PoW procedure entails the identification of a value that, when hashed with an algorithm such as SHA-256, generates a hash that commences with a predetermined number of the zero bits. It is possible to verify the outcome with a single hash operation, despite the fact that the computational effort required to accomplish this is enormous in relation to the number of zero bits required.

PoW is implemented in the timestamp network of Bitcoin by incrementing a nonce within a block until a hash with the necessary number of zero bits is discovered. After this PoW is completed, it is computationally not feasible to modify the block without redoing the PoW. Securing the blockchain is dependent upon the reworking of the PoW for all subsequent blocks in the event that any block is modified, as each block is linked to the next.

The issue of representation in majority decision-making within the network is also addressed by PoW. Bitcoin employs a one-CPU-one-vote system, as opposed to a one-IP-address-one-vote system that could be compromised by allocating multiple IP addresses. The longest blockchain, which has the highest PoW invested in it, represents the majority decision.

In order for the attacker to modify a previous block, they would need to redo the PoW for that block and all subsequent blocks, and subsequently surpass the honest nodes. The complexity of this endeavour increases exponentially as additional blocks are incorporated. The difficulty of PoW is dynamically amended to accommodate the network's fluctuating participation and the growing capabilities of hardware. This adjustment, which is based on a moving average, guarantees a consistent average number of blocks per hour, thereby enhancing the difficulty as blocks are generated at a faster pace.

2. Ethereum:

In 2015, (Buterin, 2015) introduced Ethereum, which overcame numerous constraints of Bitcoin's scripting language. Its primary innovation is its full Turing-completeness, which allows it to perform all types of computations, including loops. Furthermore, Ethereum incorporates numerous enhancements to the blockchain structure and provides transaction status support.

Ethereum, which is a blockchain featuring its own Turing-complete programming language. It provides an abstraction layer where people can define their own definition of ownership, transaction types, and state transition functions. It does this using smart contracts - collections of cryptographic rules that only execute under certain conditions. What sets Ethereum blockchains apart from Bitcoin blocks is that in addition to the block number, nonce, and other data already familiar with — they include or point to not

only the most recent state but also a list of all transactions. We obtain the new state by looking into all transactions in the transaction list and applying previously generated states.

2.1.Proof-of-Stake (Ethereum, 2024):

Ethereum's blockchain operates using a consensus mechanism known as Proof of Stake (PoS), which offers a more secure, energy-efficient, and scalable alternative to the previous Proof of Work (PoW) system. To enhance network security, reduce energy consumption, and implement scaling solutions more effectively, Ethereum transitioned to PoS in 2022.

Proof of Stake ensures validators have a common interest in maintaining the network's integrity by requiring them to stake their own assets. In Ethereum's PoS, validators stake Ether (ETH) in a smart contract. They are responsible for verifying new blocks and, occasionally, for creating and propagating new blocks. Validators who act dishonestly, such as by proposing multiple blocks or sending conflicting attestations, risk forfeiting part or all of their staked ETH. To become a validator on Ethereum, a user must deposit 32 ETH into a deposit contract and run three types of software: an execution client, a consensus client, and a validator client. After depositing their ETH, validators enter an activation queue to manage the rate of new validators joining the network. Once active, validators receive and verify new blocks from their peers on the Ethereum network, ensuring the proposed changes to the blockchain's state are legitimate. They then disseminate their attestation of the block throughout the network.

In PoS, time is divided into 32-slot epochs and 12-second slots. A validator is randomly chosen to propose a new block in each slot, and a committee of validators is selected to validate the block. This system helps manage network traffic and ensures consistent validator participation. Transactions are initiated by users signing them with their private key, typically through a wallet or a library like web3.js or ether.js. The transaction is then submitted to an Ethereum execution client, which verifies its validity. Once validated, the transaction is broadcasted to other nodes and added to the local mempool. The validator selected as the block proposer for the current slot aggregates transactions from the mempool into an execution payload, executes them locally, and updates the global state. This payload is included in a beacon block, which contains additional information, such as rewards and penalties, to facilitate network consensus. Other nodes then reexecute the transactions locally to validate the block and verify the proposed state change. A transaction is considered finalized when it is included in a chain with a supermajority link between two checkpoints, occurring at the beginning of each epoch. This finality mechanism ensures that reverting a finalized block would be economically unfeasible due to the substantial loss of staked ETH an attacker would incur.

Validators are incentivized to maintain reliable network connectivity and hardware. They receive ETH rewards for their participation but face penalties for dishonest behavior, such as proposing multiple blocks in a single slot or submitting contradictory attestations. The severity of penalties depends on the number of validators involved in dishonest actions, with correlation penalties applied during mass slashing events. This system ensures that a coordinated attack would be prohibitively expensive.

Overall, PoS offers numerous benefits over PoW, including improved energy efficiency, reduced entry barriers, and enhanced security. The transition to PoS has made Ethereum more adaptable and resilient, boosting economic security and decentralization within the network.

3.2 Traditional Assets

The next section provides an overview of the traditional financial assets that have been incorporated in the portfolios.

1. SPY (SPDR, n.d.):

The 500 most famous U.S. firms listed for trading on the NYSE and NASDAQ systems make up the Standard & Poor's 500 Index, a common stock market-capitalization-weighted index. It shows how the country's biggest corporations did during that session and is often used as a measure of the US stock market.

2. GLD (SPDR, n.d.):

The goal of investing in the SPDR Gold Trust (GLD) is for the value of GLD to track the price of gold less expenses.

3. BND (Vanguard, n.d.):

Investing in the Vanguard Total Bond Market Exchange-Traded Fund (BND) is done with the intention of imitating the outcomes of a market-weighted bond index that covers a wide range of bonds. The fund offers a diverse range of exposure to the market for investment-grade U.S. bonds, with the exception of bonds that are insured against inflation and bonds that are exempt from taxation.

4. USDU (WisdomTree, n.d.):

In terms of total returns, the objective of the U.S. Dollar Bullish Fund (USDU) offered by WisdomTree is to achieve a higher level of performance than the Bloomberg Dollar Total Return Index (before fees). A dynamic, efficient, and all-encompassing approach to gaining exposure to the United States dollar in relation to a set of foreign currencies inside the framework of an exchange-traded fund (ETF) is provided by this strategy.

3.3 Portfolio Optimization Methods

This section presents a theoretical framework for analysing how cryptocurrencies affect portfolio diversification and optimisation. The key models investigated in this study are Mean-Variance Optimisation, Critical Line Algorithm, and Monte Carlo Simulation. These models are critical for analysing the possible benefits and dangers of adding cryptocurrencies, since they provide insights into their impact on achieving optimal portfolio performance. Sharpe Ratio is used to evaluate the performance of each model and each portfolio. By thoroughly exploring these models, this part establishes the framework for the empirical investigation that follows.

1. Mean-Variance Optimization:

(Milhomem, 2020), states that in order to optimise a portfolio, one must first identify a set of assets and the weights assigned to each so that the investor is satisfied with the risk-return binomial combination. The Modern Portfolio Theory (MPT), put forth by economist Harry Markowitz in 1952 (Markowitz, 1952), aimed to address this issue. Portfolio construction using MPT aims to maximise projected return with minimal risk. The fundamental concept is that an investor can get diversification benefits by holding a variety of assets, lowering the total risk of the portfolio. Mean Variance Optimisation (MVO) is a quantitative method utilised within the MPT framework to meet the theory's goals. MVO makes MPT's theoretical notions more practical by offering a mathematical

approach to portfolio development. To summarise, MVO is a practical implementation of MPT's theoretical foundation.

The model calculates the predicted return by averaging the stock's past returns and calculating the risk based on their variance. The Mean Variance model considers individual asset returns as random variables and calculates anticipated return and variance to assess investment risk (Zhang Yuanyuan, 2018).

E(R) represents the return on a portfolio of m assets, as shown below:

$$E(R) = \sum_{j=1}^{m} w_j \mu_j$$

Where μ_j is the return on the asset j, and w_j is the weight of the asset j in the portfolio. The variance of the portfolio is calculate using the formula,

Variance = $w^T \Sigma w$,

where $\mathbf{w} = (w_1, w_2, ..., w_n)$ is the vector of weights and Σ denotes the variance-covariance matrix of the assets of the portfolio.

Therefore, in order to maximize the returns while minimizing variance (volatility),

Max E(R)subject to $w_i \ge 0$, for all i,

The variance measures the level of risk, denoted as α . The second requirement limits the sum of weights to one, and the final condition demands long positions in all assets, with no short selling permitted. The testing process consisted of two independent phases: one without the optimisation function and one with the optimisation function providing optimal weights to the portfolio's assets. The goal was to monitor and compare the differences in portfolio performance between the two scenarios. In the initial phase, the portfolio was built without using the optimisation algorithm. This suggests that the assets were given equal weight without regard for optimising the risk-return trade-off. In the second phase, the MVO model's optimisation function was used to assign optimal weights to each asset in the portfolio. The optimisation sought to maximise the portfolio's predicted return for a given level of risk.

2. Critical Line Algorithm:

Financial professionals frequently face the difficulty of optimising their portfolios. According to (Markowitz, 1952) this approach consists of selecting the asset combination that maximises the expected return within a certain risk budget, resulting in the Efficient Frontier idea for a variety of risk levels. While this problem may be solved analytically without limitations on portfolio weights, it becomes more complicated when inequality constraints, such as upper and lower weight bounds, are introduced. Harry Markowitz created the Critical Line Algorithm (CLA) to optimise general quadratic functions under linear inequality constraints (Markowitz, 1956). Additionally, according to (David H. Bailey, 2013), the Critical Line Algorithm (CLA) is the first technique created expressly for quadratic portfolio optimisation problems with linear inequality constraints that

ensures an exact solution after a finite number of iterations. Unlike other methods, CLA computes the full efficient frontier, putting it ahead of general-purpose quadratic programming algorithms for certain problems. However, it is important to highlight that CLA cannot tackle portfolio optimisation problems that cannot be expressed in quadratic terms (David H. Bailey, 2013).

Markowitz's CLA involves identifying the turning point with the highest expected return first, followed by a sequence of turning points with smaller expected returns.

In-depth explanation of the CLA optimization method: (Markowitz, 2019).

1. Minimize portfolio variance subject to linear constraints:

Portfolio variance $(\sigma_p^2) = w^T \sum w$,

w: vector of portfolio weights

 Σ : covariance matrix of asset returns

 w^T : transpose of vector of portfolio weights

Expected return $(\mu_p) = w^T \mu$,

 μ : vector of expected returns of each asset

Constraints:

 $\sum_{i=1}^{n} \omega_i = 1$ (weights sum to 1)

 $l_i \le \omega_i \le u_i$ (bounds on weights)

2. Lagrange Function:

 $L = \frac{1}{2} w^T \sum w - \lambda (w^T \mu - \mu_p) - \gamma (\mathbf{1}^T w - \mathbf{1}),$

L: Lagrange function

 $\frac{1}{2}w^T \sum w$: Portfolio variance (to be minimized)

 $\lambda(w^T\mu - \mu_p)$: The constraint term ensuring the portfolio's expected return matches the target expected return (μ_p)

λ: Lagrange multiplier associated with expected return constraint

 $w^T\mu$: expected return of the portfolio

 μ_p : target expected return of the portfolio

 $\gamma(1^T w - 1)$: The constraint term ensuring the portfolio weights sum to 1

 γ : The Lagrange multiplier associated with the sum of weights constraint.

 $1^T w$: sum of portfolio weights.

3. First-Order Conditions:

Solve the system of equations resulting from the first-order requirements for weights, expected return, and risk.

4. Turning Points:

CLA highlights turning points where the status of constraints changes (for example, when a weight moves from lower bound to free). Between turning points, the efficient frontier is a convex combination of the portfolios at the adjacent turning points.

5. Iterative Process:

- Begin with the portfolio with the highest return, with weights set to their top bounds.
- Calculate turning points iteratively by altering weights within their limits to shift to a portfolio with a lower return.
- Continue until the global minimum variance portfolio is achieved.

6. Efficient Frontier:

Created by solving a succession of quadratic programming problems between turning points.

3. Monte Carlo Simulation:

A brief background and summary of the Monte Carlo Simulation method is given in (RL., 2010). This simulation was carefully developed as part of the Manhattan Project, the United States' attempt to produce nuclear weapons during World War II. It was proposed by John von Neumann and Stanislaw Ulam to examine neutron transit via radiation shielding, and was named after Monaco's Monte Carlo Casino. Monte Carlo simulation is now commonly used to solve problems that can't be solved analytically and to avoid doing expensive or impractical tests. It enables researchers to investigate complex systems, uncover hidden values, and simply repeat or alter studies. When dealing with a large amount of uncertainty in forecasting or estimates, several strategies replace the variable that is uncertain with an average value. Instead, the Monte Carlo simulation employs a series of values and then calculates the average. The purpose of MC is to generate a significant amount of "plausible realities" from the input space through repeat random sampling, which are then used to conduct risk analyses, metrics, and additional assessments.

Processes involved in Monte Carlo Simulation:

1. Random Sampling:

For a normal distribution with mean (μ) and standard deviation (σ) , random samples (X) are created as follows:

 $x_i \sim N(\mu, \sigma^2)$,

- x_i : A random sample from the normal distribution.
- *u*: Mean of normal distribution
- σ : Standard deviation of normal deviation
- $N(\mu, \sigma^2)$: Normal distribution with mean and variance

2. Iteration Process:

Let f(X1, X2, ..., Xn) f(X1, X2, ..., Xn) be the model function with n n input variables X1, X2, ..., Xn. For each iteration, k:

 $Y_k = f(x_{1k}, x_{2k, \dots}, x_{nk})$

- $f(X_1, X_2, ..., X_n)$: Model function that describes the system or process
- n: Number of input variables
- $x_{1k}k, x_{2k...}, x_{nk}$: Random sample for the i-th variable in the k-th iteration
- Y_k : Output of the model for the k-th iteration

3. Statistical Analysis:

a. Mean of the output distribution

$$\overline{Y} = \frac{1}{N} \Sigma_{k=1}^N Y_k,$$

- *N*: Total number of iterations
- Y_k : Output of the model for the k-th iteration
- $\Sigma_{k=1}^{N} Y_{k}$: Sum of all outputs from each iteration

b. Variance of the output distribution:

$$VAR(Y) = \frac{1}{N-1} \sum_{k=1}^{N} (Y_k - \overline{Y})^2 ,$$

- \bar{Y} : Mean of the normal distribution
- $(Y_k \bar{Y})^2$: Squared difference between k-th output and mean

4. Confidence intervals:

Used to find the range within which the true mean of the output falls with a given level of confidence. Calculated using the output distribution's mean, variance, and standard error.

3.4 Portfolio Performance Metric

For the purposes of this investigation, the Sharpe Ratio was chosen as the portfolio performance statistic.

1. Sharpe Ratio:

The Sharpe Ratio, initially known as the "reward-to-variability" ratio, was invented by William F. Sharpe in 1966 with the objective of simplifying the evaluation of an investment's performance by taking into account both risk and return (Sharpe, 1966). The ratio rapidly established itself as a critical instrument in the field of finance, providing a simple method for comparing various investments. The formula is employed to determine the performance of a portfolio and an individual asset. Additionally, it facilitates the comparison of the performance of two or more portfolios or investments. A Sharpe ratio that exceeds one is deemed acceptable. Furthermore, the risk-free rate, the portfolio, or the assets generate more excessive returns as the Sharpe ratio increases. The formula is given below:

Sharpe Ratio =
$$\frac{R_p - R_f}{\sigma_P}$$
,

Excess return (R_n) : Expected return of the portfolio

 R_f : Risk free rate of return

Standard deviation of the portfolios excess return (σ_P) : $\sqrt{\frac{1}{N-1}\Sigma_{i=1}^N(R_{p_i}-\bar{R}_p)^2}$,

13

N: Number of observations

 R_{p_i} : Return of portfolio in the i-th period

 \bar{R}_p : Average return of the portfolio over N periods

The optimisation problem of maximising the Sharpe Ratio, subject to the constraint on the weights in a portfolio, is solved in addition to supplying an efficient frontier for various portfolios. The information is presented as:

$$\max_{\frac{R_p - R_f}{\sigma_p}} \frac{R_p - R_f}{\sigma_p}$$
subject to
$$\sum_{i=1}^m w_i = 1$$

where w_i are the weights of the assets in the portfolio.

The second condition, $\sum_{i=1}^{m} w_i = 1$, indicates that the sum of weights allocated to all the assets in the portfolio should equal to 1. This is done by only allowing long positions in the assets such that $w_i \ge 0 \forall i$.

4 Research Methodology

4.1 Data Selection

The selection of assets for this study is based on 4 reasons

- Diversification: By selecting assets from various asset classes, the unsystematic risk can be reduced.
- Risk Management: Assets like BND and GLD help mitigate the risk of high volatility assets like BTC, ETH and even SPY.
- Growth: Assets such as BTC and ETH, even though exhibit high volatility, also help in obtaining high returns.
- Hedging: GLD and USDU can act as hedges against inflation, currency risk and economic uncertainty.

The portfolios seek to establish a harmonious combination of risk and return by including different assets, using the advantages of each asset class while minimising their respective drawbacks.

4.2 Portfolio creation

To evaluate and compare the outcomes of this study, four distinct portfolios were developed:

Portfolio 1 (P1): Contains all assets used in the study.

Portfolio 2 (P2): Consists solely of traditional financial assets, not cryptocurrency.

Portfolio 3 (P3): Traditional assets and Bitcoin (BTC-USD).

Portfolio 4 (P4): Traditional assets combined with Ethereum (ETH-USD).

Portfolio 1 (P1)	SPY, BND, GLD, USDU, BTC-USD, ETH-
	USD
Portfolio 2 (P2)	SPY, BND, GLD, USDU
Portfolio 3 (P3)	SPY, BND, GLD, USDU, BTC-USD
Portfolio 4 (P4)	SPY, BND, GLD, USDU, ETH-USD

4.3 Optimisation techniques

Three portfolio optimization techniques have been used on the 4 created portfolios.

The first optimization technique is the Mean-Variance Optimization. The optimal weight's function is added to the MVO function to generate portfolios with the highest Sharpe Ratios.

The second optimization technique is the Critical Line Algorithm. By continuously repeating the allocations of asset weights in the portfolio along the critical lines, CLA offers a precise way to generate portfolios with high Sharpe Ratios and finds the optimal efficient frontier.

The third technique used is the Monte Carlo Simulation that repeats the process of asset allocation numerous times in order to find the best possible combination that provides the highest Sharpe Ratio.

4.4 Sharpe Ratio Calculation

The Sharpe Ratio is used to evaluate the performance of each portfolio when applied with different optimization techniques. The higher the Sharpe Ratio the better the portfolio performance.

5 Implementation

5.1 Data Collection, Preparation and Exploratory Data Analysis

Python language has been implemented to code the entire study.

The Python library 'yfinance' was employed to gather data from Yahoo Finance. The data collection period spans from May 23, 2020, to May 23, 2024. Also, it is important to note that 'Adjusted Close' is being used instead of 'Close' for the following reasons:

- The adjusted closing price is a modification of the closing price of a stock to reflect its value after accounting for any corporate actions, such as stock splits, dividends, and rights offerings, whereas the closing price merely refers to the cost of shares at the end of the day.
- The adjusted closing price assists investors in determining the amount of profit they would have generated by investing in a specific asset.
- The adjusted closing price enables investors to compare the performance of two or more assets.

Initially, the collected data had 1461 rows. After cleaning the data, i.e. dropping the null values the data frame consisted of 1006 rows and 6 columns. This is due to the fact that cryptocurrencies such as Bitcoin and Ethereum can be traded 365 days a year since they run on a decentralised blockchain which means that they are not owned by any country, institution or central exchange. On the other hand, traditional assets such as SPY, GLD, BND and USDU are operated through centralised exchanges that which are bound by the working hours of the financial institutions of their respective countries. The number of trading days in a year is 252.

Ticker	BND	BTC-USD	ETH-USD	GLD	SPY	USDU
Date						
2020-05-26	78.185394	8835.052734	201.902313	160.889999	280.711945	24.137737
2020-05-27	78.221100	9181.017578	208.863434	161.179993	284.888763	24.163599
2020-05-28	78.247864	9525.750977	219.840424	161.720001	284.363098	23.999807
2020-05-29	78.470993	9439.124023	220.675125	162.910004	285.630219	23.965324
2020-06-01	78.407501	10167.268555	246.991760	163.660004	286.784668	23.792910

Figure 1. First 5 rows of the dataset

Ticker	BND	BTC-USD	ETH-USD	GLD	SPY	USDU
count	1006.000000	1006.000000	1006.000000	1006.000000	1006.000000	1006.000000
mean	73.773169	33818.215485	1998.615672	175.574572	403.711913	23.858855
std	4.547752	16043.207767	1056.042830	12.571953	52.617092	1.449606
min	65.384163	8835.052734	201.902313	151.229996	280.711945	21.621813
25%	69.884865	20694.235352	1333.456726	167.092499	372.133278	22.510293
50%	71.695095	30394.599609	1843.859192	174.014999	403.909592	23.956904
75%	78.576565	44346.247070	2770.175659	182.037495	434.905617	24.980879
max	80.306908	73083.500000	4812.087402	224.559998	529.651245	26.750000

Figure 2. Descriptive Statistics of the dataset

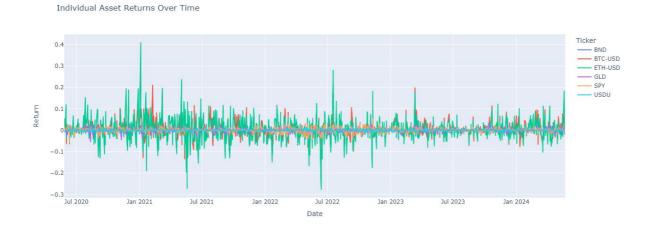


Figure 3. Individual Asset Returns

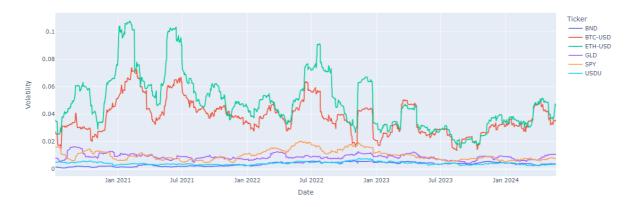


Figure 4. Individual Asset Volatility

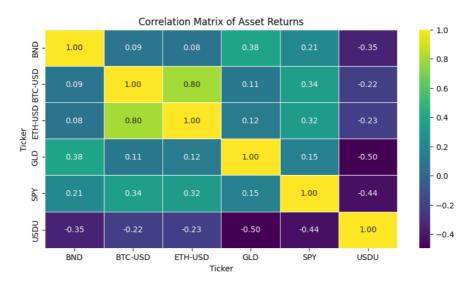


Figure 5. Correlation Matrix

5.2 PyPortfolioOpt

<u>PyPortfolioOpt</u> is a python library that includes various portfolio optimization models that can be implemented. This study utilizes this library for implementing optimization techniques on the 4 portfolios created and generating portfolios with optimal Sharpe Ratios.

5.3 Models using PyPortfolioOpt

1. Mean-Variance Optimization:

The "Efficient Frontier" class from PyPortfolioOpt initialises the Mean-Variance optimizer along with calculating the expected returns and covariance matrix required.

2. Critical Line Algorithm:

The "CLA" class from PyPortfolioOpt is called with the expected returns and covariance matrix already calculated in the step above. Then the "cla.max_sharpe()" optimization function is used to calculate the Sharpe Ratios.

3. Monte Carlo Simulation:

Since the PyportfolioOpt has no class that implements Monte Carlo Simulation this code was done manually by generating random weights for each asset in each portfolio using loops in Python. Then the Sharpe Ratios were calculated and the portfolios with the highest Sharpe Ratios were displayed.

6 Evaluation, Results and Discussion

6.1 Evaluation and Results

Python functions were employed to determine the annual returns and standard deviations (volatility) of each asset from May 23, 2020, to May 23, 2024. They are displayed below:

→		Annual Return	Annual Volatility
	Ticker		
	BND	-0.021820	0.060782
	BTC-USD	0.714966	0.630619
	ETH-USD	1.080003	0.837332
	GLD	0.089280	0.146076
	SPY	0.173383	0.172282
	USDU	0.026094	0.069281

Figure 6. Annual Returns and Volatilities

As seen above, the cryptocurrencies had the highest annual returns among the assets. But at the same time, they also had the highest volatilities. This makes cryptocurrencies a good choice for investors who are willing to take risks in order to maximise gains.

Equally Weighted Portfolio Analysis:

The first analysis done is on portfolios where each asset is given equal weightage. The Sharpe Ratio is calculated to compare them. Additionally, the expected return and standard deviation (volatility) is also calculated. The results indicate that P4 which consists of traditional assets and Ethereum had the highest Sharpe Ratio of 1.44



Figure 7. Equal Weighted Portfolio Performance

1. Mean-Variance Optimization:

Portfolio 1 (P1) and Portfolio 4 (P4) obtained the highest Sharpe Ratios at 1.6021, which indicates superior risk-adjusted returns among the four portfolios, as evidenced by the results. The negligible weight assigned to Bitcoin in P1 by the Mean Variance Optimisation method and the absence of Bitcoin in P4 are the reasons for the similarity in Sharpe Ratios. The highest expected returns (11.84%) and standard deviation (6.14%) were also demonstrated by both portfolios.

In contrast, Portfolio 3 (P3), which comprises Bitcoin, exhibited a Sharpe Ratio of 1.4398, which indicates that Ethereum offers superior diversification advantages over Bitcoin. Finally, Portfolio 2 (P2), which contains exclusively traditional assets, exhibited the lowest Sharpe Ratio at 1.1782, which underscores the performance improvement that cryptocurrencies provide when incorporated into a diversified portfolio.



Figure 8. Mean-Variance Optimization

An additional analysis was conducted by increasing the weight of Bitcoin (BTC-USD) in the portfolio and observing its performance. The results suggest that the Sharpe Ratio is steadily decreasing as the Bitcoin weight increases from 20% to 50%. This is accompanied by an increase in both standard deviation (volatility) and expected returns.

At a 50% Bitcoin allocation, the Sharpe Ratio was the lowest, at 1.14. This allocation had an expected return of 42.3% and a corresponding volatility of 35.4%. These findings demonstrate that the proportion of Bitcoin increases returns considerably, but it also substantially increases risk, which may not be optimal for portfolio diversification.



Figure 9. BTC Weight Analysis

Similar analysis was done for Ethereum. The Sharpe Ratio experiences a consistent decline as the weight of Ethereum (ETH-USD) in the portfolio increases from 20% to 50%, while both the standard deviation and returns increase. This is demonstrated by the analysis.

The substantial increase in Sharpe Ratios and returns for portfolios with higher Ethereum weights in comparison to Bitcoin is a noteworthy observation. For example, a portfolio that contains 40% Ethereum (36.9% standard deviation) generates a Sharpe Ratio of 1.35 and a return of 51.9%, while a portfolio that contains 50% Bitcoin (35.4% standard deviation) generates a Sharpe Ratio of 1.14 and a return of 42.3%. This again suggests that Ethereum provides a superior risk-return trade-off at greater allocation levels than Bitcoin.



Figure 10. ETH Weight Analysis

2. Critical Line Algorithm:

The strongest risk-adjusted performance is demonstrated by P1 and P4, which have the highest returns (0.0973) and Sharpe ratio (1.5665). P2 exhibits the lowest returns (0.0686) and Sharpe ratio (1.1566), indicating that the exclusion of cryptocurrencies leads to a decrease in overall performance in terms of both returns and risk-adjusted metrics.

The least volatile portfolio is P2, which has the lowest standard deviation (0.0421). This is consistent with the exclusion of high-volatility assets, such as cryptocurrencies. The high standard deviation of both P1 and P4 is 0.0494, which suggests that the volatility of the portfolio is increased by the inclusion of both Bitcoin and Ethereum.

In comparison to P2, the expected returns of traditional portfolios are increased by the incorporation of Bitcoin (P3) and Ethereum (P4). However, the Sharpe Ratio (1.57) was greater for P4 with Ethereum than for P3 with Bitcoin (1.41). This goes back to the original goal of the study, which was to show that Ethereum is a superior diversification choice than Bitcoin.

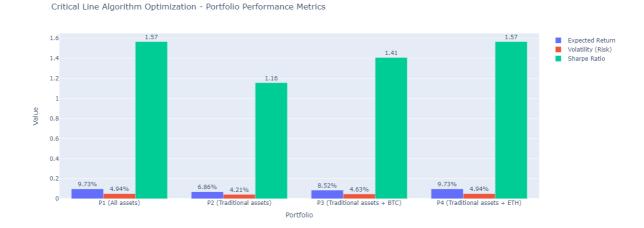


Figure 11. Critical Line Algorithm

3. Monte Carlo Simulation:

Out of the three optimisation techniques employed, Monte Carlo Simulation yielded the highest Sharpe ratios. This is because this procedure repeatedly assigns random weights to each asset in the portfolios in order to achieve the maximum Sharpe Ratio (10,000 times in this instance).

P3 had the highest Sharpe Ratio, 1.87, and was composed of Ethereum and traditional assets. The Sharpe Ratio of 1.86 was achieved by P1, which was comprised of all the assets utilised in the study, which was a close second. However, this was accompanied by a 6.16% increase in risk and a 11.47% increase in returns when contrasted with P3. P2, which contained Bitcoin, underperformed P3, which contained Ethereum, at approximately equivalent levels of risk. Their Sharpe Ratios were 1.76 and 1.87, respectively. Finally, the Sharpe Ratio of P2, which did not contain any cryptocurrency, was the lowest of the four portfolios.

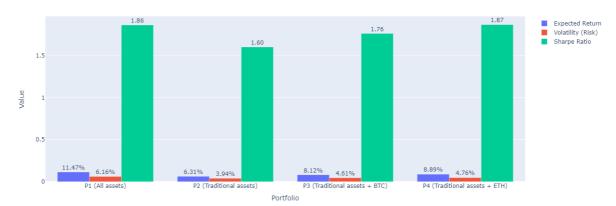


Figure 12. Monte Carlo Simulation

The following image displays the percentage of assets in each portfolio at the time the highest Sharpe Ratio was achieved. Ethereum was allocated the highest weightage in both portfolios P1 and P2, as evidenced by the table. In the absence of cryptocurrencies in P2, USDU, BND, and GLD were considered to be less risky assets than SPY and were therefore given a higher weighting. In order to achieve the highest Sharpe Ratio, the cryptocurrency Bitcoin was once again assigned the greatest weightage in P3. These findings demonstrate that cryptocurrencies are highly advantageous in the development of diverse portfolios that optimise returns. And lastly, from the acquired data we can conclude that Ethereum does have an advantage in terms of returns over Bitcoin when it comes to diversification.

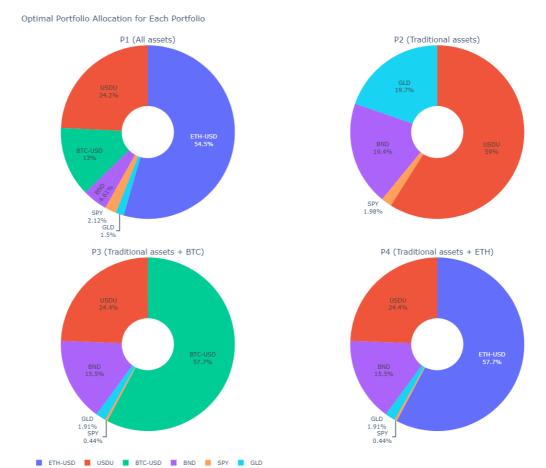


Figure 13. Monte Carlo Simulation - Asset Allocation

6.2 Discussion

From the above conducted analysis, we can observe that the Monte Carlo Simulation (MCS) optimization technique provided the highest Sharpe Ratios. This is due to the fact that this particular method keeps reallocating asset weights until it finds the best combination that generates the highest Sharpe Ratio.

Mean Variance Optimisation (MVO) ranks second, which is based on the time-tested principle of reducing the overall risk of an investment portfolio while maintaining an initial target rate of return.

With a maximum Sharpe Ratio of 1.57 for P1 and P4, the Critical Line Algorithm (CLA) comes in at number three. In order to find efficient frontier portfolios, CLA employs an advanced approach to quadratic optimisation. On the other hand, unlike MCS, it is deterministic and may not investigate as many possible outcomes. Consequently, CLA's more strict structure in portfolio construction may explain why it underperforms MCS and MVO in terms of Sharpe Ratios, even though it can offer optimised solutions. Also, all the optimization techniques implemented show that Ethereum allocation takes priority over Bitcoin allocation.

It is important to note that the risk-free rate which plays an important role in the calculation of Sharpe Ratio has been intentionally set to 0. This has been done because the core goal

of this study is to verify if Ethereum proves better diversification opportunities than Bitcoin. However, future research can include calculating the average risk-free rate for the period of study.

Various studies done in the past such (Borri, 2019), (Emmanouil Platanakis, 2018), (Emmanouil Platanakis, 2019), (Liu, 2019) and (Alexander Brauneis, 2019) have analysed the effects of incorporating cryptocurrencies in traditional portfolio for diversification and maximizing gains. But most of the previous studies such as (Symitsi Efthymia, 2018), (Khaled Gusemi, 2019), (Anton Kajtazi, 2019), etc. focus on only incorporating Bitcoin in order to diversify portfolios.

However, the aim of this research was to demonstrate evidence that Ethereum proves to be a better diversification option than Bitcoin. The results acquired show conclusive evidence that the inclusion of cryptocurrencies in traditional portfolios help in maximising returns and prove to be good diversification options. Additionally, the analysis done on 4 portfolios using 3 different optimization techniques provide conclusive evidence that Ethereum proves to be a better diversification option than Bitcoin by generating higher Sharpe Ratios. Similarly, this research aims to add to previous literature such as (Petukhina Alla, 2018), (Simon Trimborn, 2019) and (Yechi Maa, 2020) which have made use of cryptocurrencies other than Bitcoin for portfolio diversification.

7 Conclusion and Future Work

This study explored the effects of including cryptocurrencies in traditional portfolios. Decentralisation which is the core underlying foundation of blockchain technologies is challenging traditional financial systems. Additionally, with the constant innovation of cryptocurrencies, investors will have a new way to diversify and manage their portfolios while targeting maximum returns. Through this study, the advantages of incorporating cryptocurrencies into traditional portfolios have been observed. The rapid growth of cryptocurrencies due to the phenomenal use cases of their underlying blockchains can be seen very evidently and investors can take advantage of these cryptocurrencies to maximise their returns and also diversify risk.

Future work on this study can include incorporation of stable coins such as USDT, USDC, etc. along with cryptocurrencies. Additionally, various other portfolio optimization techniques such as Hierarchical Risk Parity and Black-Litterman Model can also be implemented to find the most optimal portfolios. Similarly, short selling can be allowed while developing these portfolio optimization models.

To conclude, the incorporation of cryptocurrencies into traditional portfolios indicates a substantial shift in modern investment methods. This shift is motivated by the possibility of blockchain technology to be decentralised and inventive. Including cryptocurrencies in investment portfolios has been shown to have significant advantages, both in terms of increased diversification and the possibility of higher returns, as proved by this study. By accepting new asset classes such as cryptocurrencies, investors are required to adapt in order to keep up with the constantly shifting environment of the financial industry. Their ability to manage the intricacies of today's marketplaces and capitalise on emerging opportunities is improved as a result of their actions. The pursuit of truly optimal and resilient portfolios will become increasingly attainable as future research expands on this

foundation. This will provide investors with the tools they require to thrive in a financial environment that is constantly changing.

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