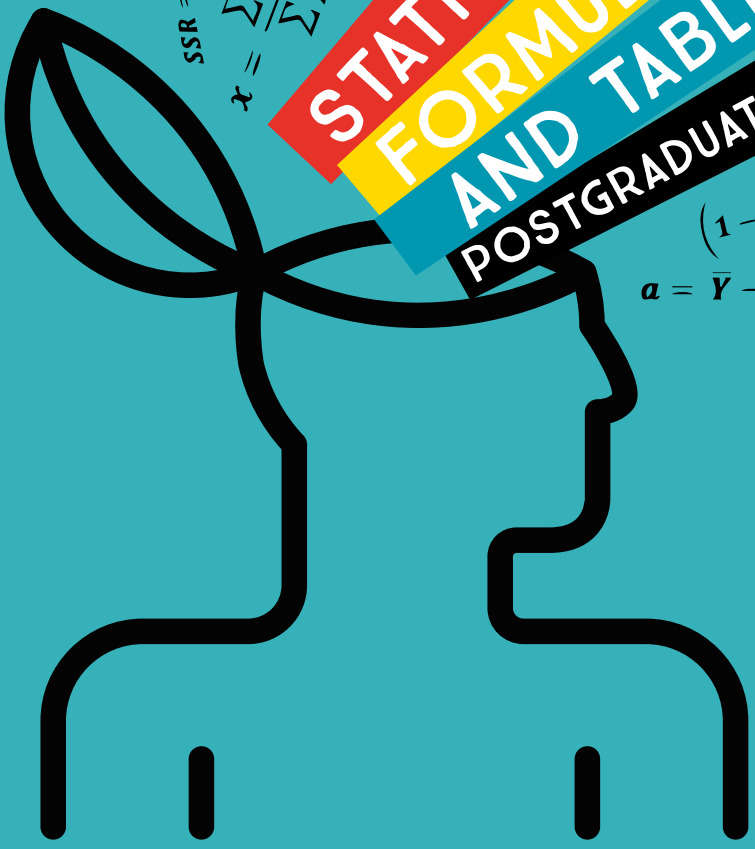
National College of Ireland

STATISTICAL FORMULAE AND TABLES

POSTGRADUATE



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Contents

STATISTICAL SYMBOLS

Population Statistical Symbols

Symbol	Meaning
N	Size of the Population
μ	Mean of the Population
σ^2	Variance of the Population
σ	Standard Deviation of the Population
f	Frequency of Occurrence
F	Cumulative Frequency

Sample Statistical Symbols

Symbol	Meaning
n	Size of the Sample
$\bar{x}$	Mean of the Sample
s^2	Variance of the Sample
s	Standard Deviation of the Sample
f	Frequency of Occurrence
F	Cumulative Frequency

DESCRIPTIVE STATISTICS FORMULAE

Population Parameter and Sample Statistic Formulae when Dealing with Raw Data.

	Population	Sample
Mean	$\mu = \frac{\sum x}{N}$	$\bar{x} = \frac{\sum x}{n}$
Variance	$\sigma^2 = \frac{\sum (x - \mu)^2}{N}$	$s^2 = \frac{\sum (x - \bar{x})^2}{n - 1}$
Standard Deviation	$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{N}}$	$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$

Population Parameter and Sample Statistic Formulae when Dealing with Frequency Distributions.

	Population	Sample
Mean	$\mu = \frac{\sum fx}{\sum f}$	$\bar{x} = \frac{\sum fx}{\sum f}$
Variance	$\sigma^2 = \frac{\sum f(x - \mu)^2}{\sum f}$	$s^2 = \frac{\sum f(x - \bar{x})^2}{\sum f - 1}$
Standard Deviation	$\sigma = \sqrt{\frac{\sum f(x - \mu)^2}{\sum f}}$	$s = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f - 1}}$

DESCRIPTIVE STATISTICS FORMULAE CONTINUED

Median and Mode given a Frequency Distribution.

Median	$$Median = L_M + \left(\frac{\frac{\sum f}{2} - F_{M-1}}{f_M}\right).C_M$$
Mode	$$Mode = L + \left(\frac{D_1}{D_1 + D_2}\right).C_M$$

First and Third Quartiles given Frequency Distribution.

1st Quartile	$$Q_1 = L_{Q1} + \left(\frac{\frac{\sum f}{4} - F_{Q1-1}}{f_{Q1}}\right).C_{Q1}$$
3rd Quartile	$$Q_3 = L_{Q3} + \left(\frac{\frac{3\sum f}{4} - F_{Q3-1}}{f_{Q3}}\right).C_{Q3}$$

DESCRIPTIVE STATISTICS FORMULAE CONTINUED

Skewness and Kurtosis

Sample Skewness	$S_K = \left[\frac{n}{(n-1)(n-2)}\right]\frac{\sum(x-\bar{x})^3}{s^3}$
Sample Excess Kurtosis	$K = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)}\right]\frac{\sum(x-\bar{x})^4}{s^4} - \frac{3(n-1)^2}{(n-2)(n-3)}$

Coefficient of Variation and Chebyshev's Rule

Coefficient of Variation	$C_V = \frac{s}{\bar{x}} \times 100\%$
Percentage of values found within k standard deviations of the mean	$\left(1 - \frac{1}{k^2}\right) \times 100\%$

COUNTING RULES

Counting Rule 1:

If any one of k mutually exclusive and exhaustive events can occur on each of n trials, the number of possible outcomes is equal to:	k^n

Counting Rule 2:

If there are k_1 events on the first trial, k_2 events on the second trial, ..., and k_n events on the nth trial, then the number of possible outcomes is:	$(k_1)(k_2) \dots (k_n)$

Counting Rule 3:

The number of ways that all n items can be arranged in order is:	$n! = (n)(n-1) \dots (1)$ where $n!$ is called n factorial and $0!$ is defined as 1.

Permutations:

The number of ways of arranging x objects selected from n objects in order is:	$^nP_x = \frac{n!}{(n-x)!}$

Combinations:

The number of combinations from selecting x objects from n objects, irrespective of order, is equal to:	$^nC_x \equiv \binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

RULES OF PROBABILITY

Simple Probability:

Probability of occurrence of a simple event A.	$P(A) = \frac{Number\ of\ Favourable\ Outcomes}{Number\ of\ Possible\ Outcomes}$

Addition Rule of Probability:

When A and B **are not** mutually exclusive	$P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
When A and B **are** mutually exclusive	$P(A\ or\ B) = P(A) + P(B)$

Multiplication Rule of Probability:

When A and B **are not** independent	$P(A\ and\ B) = P(A)P(B\|A)$
When A and B **are** independent	$P(A\ and\ B) = P(A)P(B)$

Bayes Theorem:

To find the conditional probability of **B** given **A**	$P(B\|A) = \frac{P(A\|B)P(B)}{P(A)}$ **alternatively** $P(B\|A) = \frac{P(A\|B)P(B)}{P(A\|B)P(B) + P(A\|B^c)P(B^C)}$

RULES OF PROBABILITY CONTINUED

Binomial Distribution:

The probability of **x** successes given the number of observations **n** and the probability of success **p**	$P(X = x) = {}^{n}_{x}C\, p^x\, (1-p)^{n-x}$
Expected value of the Binomial Distribution	$\mu = E(X) = np$
Variance of the Binomial Distribution	$\sigma^2 = Var(X) = np(1-p)$
Standard Deviation of the Binomial Distribution	$\sigma = \sqrt{Var(X)} = \sqrt{np(1-p)}$

Poisson Distribution:

The probability of **X** events occurring given the expected number of events $\boldsymbol{\lambda}$	$P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

CORRELATION

Spearman Rank Correlation Coefficient (r_s)	$$r_s = 1 - \frac{6\sum_{i=1}^{n} d_i^2}{n(n^2-1)}$$
Pearson Product Moment Correlation Coefficient (r)	$$r = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$$ **alternatively** $$r = \frac{\sum((X-\bar{X})(Y-\bar{Y}))}{\sqrt{\sum(X-\bar{X})^2\sum(Y-\bar{Y})^2}}$$

REGRESSION (SINGLE DEPENDENT VARIABLE AND SINGLE INDEPENDENT VARIABLE)

Linear Regression Function	$y = a + bx$
Slope of Regression Line	$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2}$ alternatively $b = r\frac{s_y}{s_x}$
Y-Intercept of Regression Line	$a = \frac{\sum Y}{n} - b\frac{\sum X}{n}$ alternatively $a = \bar{Y} - b\bar{X}$

REGRESSION (SINGLE DEPENDENT VARIABLE AND TWO INDEPENDENT VARIABLES)

Multiple Regression Function:

Linear Regression Function	$$Y' = a + b_1X'_1 + b_2X'_2$$
Regression Equation Slope Parameters	$$b_1 = \frac{(\sum x_2^2)(\sum x_1 y) - (\sum x_1 x_2)(\sum x_2 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$ $$b_2 = \frac{(\sum x_1^2)(\sum x_2 y) - (\sum x_1 x_2)(\sum x_1 y)}{(\sum x_1^2)(\sum x_2^2) - (\sum x_1 x_2)^2}$$
Y-Intercept of Regression Plane	$$a = \overline{Y} - b_1\overline{X}_1 - b_2\overline{X}_2$$
Regression Equation Slope Coefficients	$$\sum y^2 = \sum Y^2 - \frac{(\sum Y)^2}{N}$$ $$\sum x_i^2 = \sum X_i^2 - \frac{(\sum X_i)^2}{N}$$ $$\sum x_i y = \sum X_i Y - \frac{(\sum X_i)(\sum Y)}{N}$$ $$\sum x_1 x_2 = \sum X_1 X_2 - \frac{(\sum X_1)(\sum X_2)}{N}$$

REGRESSION (SINGLE DEPENDENT VARIABLE AND TWO INDEPENDENT VARIABLES) CONTINUED

Regression Sum of Squares	$$SSR = \sum (\hat{Y}_i - \overline{Y})^2$$
Error Sum of Squares	$$SSE = \sum (Y_i - \hat{Y})^2$$
Total Sum of Squares	$$SST = SSR + SSE$$ $$SST = \sum (Y_i - \overline{Y})^2$$
Coefficient of Determination R^2	$$R^2 = \frac{SSR}{SST}$$ alternatively $$R^2 = 1 - \frac{SSE}{SST}$$
Adjusted coefficient of Determination $\bar{R}^2$	$$\bar{R}^2 = 1 - \left(\frac{n-1}{n-k-1}\right)(1 - R^2)$$

INFERENTIAL STATISTICS

Standard Normal Transformation

Standardised Normal Transformation	$Z = \frac{X - \mu}{\sigma}$
Standardised Normal Transformation for a Sampling Distribution (σ known)	$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$
Standardised Normal Transformation for a Sampling Distribution (σ unknown)	$Z = \frac{\bar{X} - \mu}{\frac{s}{\sqrt{n}}}$

INFERENTIAL STATISTICS

Single Sample Hypothesis Testing (Parametric)

Test Statistic for Hypothesis Tests of the Population Mean (σ known)	$z = \dfrac{\bar{X} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}}$
Test Statistic for Hypothesis Tests of the Population Mean (σ unknown)	$z = \dfrac{\bar{X} - \mu_0}{\dfrac{s}{\sqrt{n}}}$
Test Statistic for Hypothesis Tests of the Population Mean (σ known)	$t_{n-1} = \dfrac{\bar{X} - \mu_0}{\dfrac{\sigma}{\sqrt{n}}} \qquad df = n - 1$
Test Statistic for Hypothesis Tests of the Population Mean (σ unknown)	$t_{n-1} = \dfrac{\bar{X} - \mu_0}{\dfrac{s}{\sqrt{n}}} \qquad df = n - 1$
Test Statistic for Hypothesis Tests of the Population Proportion	$z = \dfrac{p - \pi}{\sqrt{\dfrac{\pi(1-\pi)}{n}}}$
Test Statistic for Tests Concerning the Value of a Population Variance	$\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2} \qquad df = n - 1$

Two-Sample Hypothesis Testing (Parametric)

Test Statistic for a Test of the Difference between Two Population Means (σ known)	$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$
Test Statistic for a Test of the Difference between Two Population Means (σ unknown; assumed equal)	$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}$$ $$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$ $$df = n_1 + n_2 - 2$$
Test Statistic for a Test of the Difference between Two Population Means (σ unknown; assumed unequal)	$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$ $$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(s_1^2/n_1\right)^2}{n_1 - 1} + \frac{\left(s_2^2/n_2\right)^2}{n_2 - 1}}$$

Test Statistic for a Test of the Difference between Two Population Proportions	$$t = \frac{(p_1 - p_2) - (\pi_1 - \pi_2)}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$ $$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} \quad p_1 = \frac{X_1}{n_1} \quad p_2 = \frac{X_2}{n_2}$$
Test Statistic for a Test of Mean Differences (σ known)	$$z = \frac{\bar{d} - \mu_d}{\frac{\sigma_{\bar{d}}}{\sqrt{n}}}$$
Test Statistic for a Test of Mean Differences (σ unknown)	$$t = \frac{\bar{d} - \mu_{d0}}{s_{\bar{d}}} \qquad df = n - 1$$
Test Statistic for Tests of Independence between two Variables (Chi-square)	$$\chi^2 = \sum\left(\frac{(f_o - f_e)^2}{f_e}\right) \qquad df = (r-1)(c-1)$$ $$f_e = \frac{Row\ total \times Column\ Total}{n}$$ **alternatively** $$\chi^2 = \sum\left(\frac{(O-E)^2}{E}\right)$$

Two-Sample Hypothesis Testing (Continued)

Test Statistic for the Difference between Two Population Variances	$$F = \frac{s_1^2}{s_2^2}$$ $$df_{numerator} = n_1 - 1$$ $$df_{denominator} = n_2 - 1$$

Inferential Statistics: Hypothesis Testing (Nonparametric)

Test Statistic for the comparison of two independent (unrelated) samples **(Mann-Whitney U-Test)**	$$U = min(U_1, U_2)$$ **where** $$U_i = n_1 n_2 + \frac{n_i(n_i+1)}{2} - \sum R_i$$
Test Statistic for the comparison of more than two independent (unrelated) samples **(Kruskal-Wallis H-Test)**	$$H = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1)$$ **where** $$df = k - 1$$ **correction for ties** $$C_H = 1 - \frac{\sum(T^3 - T)}{N^3 - N}$$
Test Statistics for the comparison of two dependent (related) samples **(Wilcoxon Signed Ranks Test)**	$$T = min\left(\sum R_+, \sum R_-\right)$$
Test Statistics for the comparison of more than two dependent (related) samples **(Friedman Test)**	$$F_r = \left[\frac{12}{nk(k+1)} \sum_{i=1}^{k} R_i^2\right] - 3n(k+1)$$

INFERENTIAL STATISTICS CONTINUED

Hypothesis Testing Significance of Correlation and Regression Coefficients

Test Statistic for the Spearman Rank Correlation Coefficient (r_s)	$t_{n-2} = \frac{r_s\sqrt{(n-2)}}{\sqrt{(1-r_s^2)}} \qquad df = n-2$
Test Statistic for the Pearson Product Moment Correlation Coefficient (r)	$t_{n-2} = \frac{r\sqrt{(n-2)}}{\sqrt{(1-r^2)}} \qquad df = n-2$
Test Statistic for the Population Slope in a Regression	$t_{n-2} = \frac{b_1 - \beta_1}{SE} \qquad df = n-2$

CONFIDENCE INTERVAL ESTIMATION

Confidence Interval for a Mean (σ known)	$$\bar{X} \pm Z\frac{\sigma}{\sqrt{n}}$$ **alternatively** $$\bar{X} - Z\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z\frac{\sigma}{\sqrt{n}}$$
Confidence Interval for a Mean (σ unknown)	$$\bar{X} \pm t_{n-1}\frac{s}{\sqrt{n}}$$ **alternatively** $$\bar{X} - t_{n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{n-1}\frac{s}{\sqrt{n}}$$
Confidence Interval for a single Proportion	$$p \pm z\sqrt{\frac{p(1-p)}{n}}$$ **alternatively** $$p - z\sqrt{\frac{p(1-p)}{n}} < \pi < p + z\sqrt{\frac{p(1-p)}{n}}$$
Confidence Interval for the Difference between Two Independent Population Means	$$(\bar{X}_1 - \bar{X}_2) \pm t_{n_1+n_2-2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$$
Confidence Interval for the Difference between Two Dependent Population Means	$$\bar{d} \pm t_{n-1}\frac{s_{\bar{d}}}{\sqrt{n}}$$

TESTS OF NORMALITY

Shapiro Wilks Test of Normality	$$W = \frac{b^2}{SS}$$ where $$SS = \sum (x - \bar{x})^2$$ $$b = \sum_{i=1}^{m} a_i \left(x_{(n+1-i)} - x_i \right)$$ and $$\begin{cases} n \text{ even}: & m = \frac{n}{2} \\ n \text{ odd}: & m = \frac{(n-1)}{2} \end{cases}$$

ANALYSIS OF VARIANCE (ANOVA)

Within Groups Mean Sum of Squares	$$MSS_W = \frac{\sum_{g \in G} (X - \bar{X}_g)^2}{n - k}$$
Between Groups Mean Sum of Squares	$$MSS_B = \frac{\sum_{g \in G} n_g(\bar{X}_g - \bar{X}_G)^2}{k - 1}$$
Test Statistic for Tests Concerning the Differences between the Variances of Two Populations (Normally Distributed Populations)	$$F = \frac{MSS_B}{MSS_W}$$ $$df_B = k - 1$$ $$df_W = n - k$$

SCALE RELIABILITY

Cronbach's Alpha Reliability Estimate	$$\alpha = \frac{N\bar{\rho}}{[1 + \bar{\rho}(N-1)]}$$

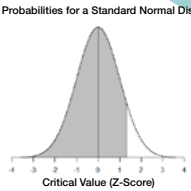
PORTFOLIO CONSTRUCTION

Two Asset Case:

Expected Return	$E(R_P) = w_1E(R_1) + w_2E(R_2)$
Variance	$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$

Three Asset Case:

Expected Return	$E(R_P) = w_1E(R_1) + w_2E(R_2) + w_3E(R_3)$
Variance	$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 + 2w_1w_3\rho_{1,3}\sigma_1\sigma_3 + 2w_2w_3\rho_{2,3}\sigma_2\sigma_3$



Cumulative Probabilities for a Standard Normal Distribution

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.60	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.70	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.80	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.90	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.00	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.10	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.20	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.30	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.40	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.50	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.60	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.70	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.80	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.90	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.00	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.10	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.20	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.30	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.40	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.50	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.60	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.70	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.80	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.90	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.00	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.10	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.20	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.30	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.40	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.50	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.60	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.70	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.80	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.90	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.00	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Critical Values for Right-Hand tail areas: Chi-Square Distribution

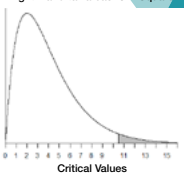


Table of χ_2 Distribution

	Probability in Right Tail								
df	**0.99**	**0.975**	**0.95**	**0.9**	**0.1**	**0.05**	**0.025**	**0.01**	**0.005**
1	0.000	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

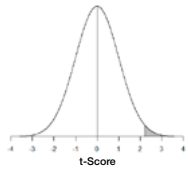


Table of the Student's t-Distribution
Critical values for right-hand tail areas

df	p = 0.1	p = 0.05	p = 0.025	p = 0.01	p = 0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756

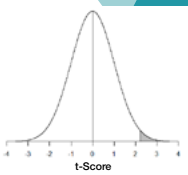


Table of the Student's t-Distribution

Critical values for right-hand tail areas

df	p = 0.1	p = 0.05	p = 0.025	p = 0.01	p = 0.005
30	1.310	1.697	2.042	2.457	2.750
31	1.309	1.696	2.040	2.453	2.744
32	1.309	1.694	2.037	2.449	2.738
33	1.308	1.692	2.035	2.445	2.733
34	1.307	1.691	2.032	2.441	2.728
35	1.306	1.690	2.030	2.438	2.724
36	1.306	1.688	2.028	2.434	2.719
37	1.305	1.687	2.026	2.431	2.715
38	1.304	1.686	2.024	2.429	2.712
39	1.304	1.685	2.023	2.426	2.708
40	1.303	1.684	2.021	2.423	2.704
41	1.303	1.683	2.020	2.421	2.701
42	1.302	1.682	2.018	2.418	2.698
43	1.302	1.681	2.017	2.416	2.695
44	1.301	1.680	2.015	2.414	2.692
45	1.301	1.679	2.014	2.412	2.690
46	1.300	1.679	2.013	2.410	2.687
47	1.300	1.678	2.012	2.408	2.685
48	1.299	1.677	2.011	2.407	2.682
49	1.299	1.677	2.010	2.405	2.680
50	1.299	1.676	2.009	2.403	2.678
60	1.296	1.671	2.000	2.390	2.660
70	1.294	1.667	1.994	2.381	2.648
80	1.292	1.664	1.990	2.374	2.639
90	1.291	1.662	1.987	2.368	2.632
100	1.290	1.660	1.984	2.364	2.626
110	1.289	1.659	1.982	2.361	2.621
120	1.289	1.658	1.980	2.358	2.617
200	1.286	1.653	1.972	2.345	2.601

Table of the F-Distribution

Critical values for right-hand tail area equal to **0.10**

df1:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
df2: 1	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	60.2	60.5	60.7	60.9	61.1	61.2
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.40	9.41	9.41	9.42	9.42
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.22	5.21	5.20	5.20
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.91	3.90	3.89	3.88	3.87
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.28	3.27	3.26	3.25	3.24
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.92	2.90	2.89	2.88	2.87
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.68	2.67	2.65	2.64	2.63
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.52	2.50	2.49	2.48	2.46
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.40	2.38	2.36	2.35	2.34
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.30	2.28	2.27	2.26	2.24
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.23	2.21	2.19	2.18	2.17
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.17	2.15	2.13	2.12	2.10
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.12	2.10	2.08	2.07	2.05
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.07	2.05	2.04	2.02	2.01
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.04	2.02	2.00	1.99	1.97
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	2.01	1.99	1.97	1.95	1.94
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.98	1.96	1.94	1.93	1.91
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.95	1.93	1.92	1.90	1.89
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.93	1.91	1.89	1.88	1.86
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.91	1.89	1.87	1.86	1.84
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.90	1.87	1.86	1.84	1.83
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.88	1.86	1.84	1.83	1.81
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.87	1.84	1.83	1.81	1.80
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.85	1.83	1.81	1.80	1.78
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.84	1.82	1.80	1.79	1.77
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.79	1.77	1.75	1.74	1.72
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.74	1.71	1.70	1.68	1.66
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.68	1.66	1.64	1.62	1.60
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.63	1.60	1.58	1.56	1.55

Critical Values for Right-Hand Tail Areas: F-Distribution

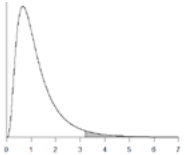
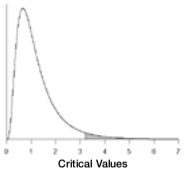


Table of the F-Distribution

Critical values for right-hand tail area equal to **0.10**

Critical Values for Right-Hand Tail Areas: F-Distribution



df1:	16	17	18	19	20	21	22	23	24	25	30	40	50	60	120
df2: 1	61.3	61.5	61.6	61.7	61.7	61.8	61.9	61.9	62.0	62.1	62.3	62.5	62.7	62.8	63.1
2	9.43	9.43	9.44	9.44	9.44	9.44	9.45	9.45	9.45	9.45	9.46	9.47	9.47	9.47	9.48
3	5.20	5.19	5.19	5.19	5.18	5.18	5.18	5.18	5.18	5.17	5.17	5.16	5.15	5.15	5.14
4	3.86	3.86	3.85	3.85	3.84	3.84	3.84	3.83	3.83	3.83	3.82	3.80	3.80	3.79	3.78
5	3.23	3.22	3.22	3.21	3.21	3.20	3.20	3.19	3.19	3.19	3.17	3.16	3.15	3.14	3.12
6	2.86	2.85	2.85	2.84	2.84	2.83	2.83	2.82	2.82	2.81	2.80	2.78	2.77	2.76	2.74
7	2.62	2.61	2.61	2.60	2.59	2.59	2.58	2.58	2.58	2.57	2.56	2.54	2.52	2.51	2.49
8	2.45	2.45	2.44	2.43	2.42	2.42	2.41	2.41	2.40	2.40	2.38	2.36	2.35	2.34	2.32
9	2.33	2.32	2.31	2.30	2.30	2.29	2.29	2.28	2.28	2.27	2.25	2.23	2.22	2.21	2.18
10	2.23	2.22	2.22	2.21	2.20	2.19	2.19	2.18	2.18	2.17	2.16	2.13	2.12	2.11	2.08
11	2.16	2.15	2.14	2.13	2.12	2.12	2.11	2.11	2.10	2.10	2.08	2.05	2.04	2.03	2.00
12	2.09	2.08	2.08	2.07	2.06	2.05	2.05	2.04	2.04	2.03	2.01	1.99	1.97	1.96	1.93
13	2.04	2.03	2.02	2.01	2.01	2.00	1.99	1.99	1.98	1.98	1.96	1.93	1.92	1.90	1.88
14	2.00	1.99	1.98	1.97	1.96	1.96	1.95	1.94	1.94	1.93	1.91	1.89	1.87	1.86	1.83
15	1.96	1.95	1.94	1.93	1.92	1.92	1.91	1.90	1.90	1.89	1.87	1.85	1.83	1.82	1.79
16	1.93	1.92	1.91	1.90	1.89	1.88	1.88	1.87	1.87	1.86	1.84	1.81	1.79	1.78	1.75
17	1.90	1.89	1.88	1.87	1.86	1.86	1.85	1.84	1.84	1.83	1.81	1.78	1.76	1.75	1.72
18	1.87	1.86	1.85	1.84	1.84	1.83	1.82	1.82	1.81	1.80	1.78	1.75	1.74	1.72	1.69
19	1.85	1.84	1.83	1.82	1.81	1.81	1.80	1.79	1.79	1.78	1.76	1.73	1.71	1.70	1.67
20	1.83	1.82	1.81	1.80	1.79	1.79	1.78	1.77	1.77	1.76	1.74	1.71	1.69	1.68	1.64
21	1.81	1.80	1.79	1.78	1.78	1.77	1.76	1.75	1.75	1.74	1.72	1.69	1.67	1.66	1.62
22	1.80	1.79	1.78	1.77	1.76	1.75	1.74	1.74	1.73	1.73	1.70	1.67	1.65	1.64	1.60
23	1.78	1.77	1.76	1.75	1.74	1.74	1.73	1.72	1.72	1.71	1.69	1.66	1.64	1.62	1.59
24	1.77	1.76	1.75	1.74	1.73	1.72	1.71	1.71	1.70	1.70	1.67	1.64	1.62	1.61	1.57
25	1.76	1.75	1.74	1.73	1.72	1.71	1.70	1.70	1.69	1.68	1.66	1.63	1.61	1.59	1.56
30	1.71	1.70	1.69	1.68	1.67	1.66	1.65	1.64	1.64	1.63	1.61	1.57	1.55	1.54	1.50
40	1.65	1.64	1.62	1.61	1.61	1.60	1.59	1.58	1.57	1.57	1.54	1.51	1.48	1.47	1.42
60	1.59	1.58	1.56	1.55	1.54	1.53	1.53	1.52	1.51	1.50	1.48	1.44	1.41	1.40	1.35
120	1.53	1.52	1.50	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.41	1.37	1.34	1.32	1.26

Table of the F-Distribution

Critical values for right-hand tail area equal to **0.05**

df1:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
df2: 1	161	200	216	225	230	234	237	239	241	242	243	244	245	245	246
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.4
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.76	8.74	8.73	8.71	8.70
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.94	5.91	5.89	5.87	5.86
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.70	4.68	4.66	4.64	4.62
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.03	4.00	3.98	3.96	3.94
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.60	3.57	3.55	3.53	3.51
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.31	3.28	3.26	3.24	3.22
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.10	3.07	3.05	3.03	3.01
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.94	2.91	2.89	2.86	2.85
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.82	2.79	2.76	2.74	2.72
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.72	2.69	2.66	2.64	2.62
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.63	2.60	2.58	2.55	2.53
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.57	2.53	2.51	2.48	2.46
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.51	2.48	2.45	2.42	2.40
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.46	2.42	2.40	2.37	2.35
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.41	2.38	2.35	2.33	2.31
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.37	2.34	2.31	2.29	2.27
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.34	2.31	2.28	2.26	2.23
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.31	2.28	2.25	2.22	2.20
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.28	2.25	2.22	2.20	2.18
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.26	2.23	2.20	2.17	2.15
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.24	2.20	2.18	2.15	2.13
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.22	2.18	2.15	2.13	2.11
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.20	2.16	2.14	2.11	2.09
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.13	2.09	2.06	2.04	2.01
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.04	2.00	1.97	1.95	1.92
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.95	1.92	1.89	1.86	1.84
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.87	1.83	1.80	1.78	1.75

Critical Values for Right-Hand Tail Areas: F-Distribution

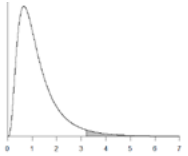
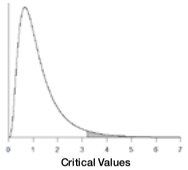


Table of the F-Distribution

Critical values for right-hand tail area equal to **0.05**

Critical Values for Right-Hand Tail Areas: F-Distribution



df1:	16	17	18	19	20	21	22	23	24	25	30	40	50	60	120
df2: 1	246	247	247	248	248	248	249	249	249	249	250	251	252	252	253
2	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5	19.5
3	8.69	8.68	8.67	8.67	8.66	8.65	8.65	8.64	8.64	8.63	8.62	8.59	8.58	8.57	8.55
4	5.84	5.83	5.82	5.81	5.80	5.79	5.79	5.78	5.77	5.77	5.75	5.72	5.70	5.69	5.66
5	4.60	4.59	4.58	4.57	4.56	4.55	4.54	4.53	4.53	4.52	4.50	4.46	4.44	4.43	4.40
6	3.92	3.91	3.90	3.88	3.87	3.86	3.86	3.85	3.84	3.83	3.81	3.77	3.75	3.74	3.70
7	3.49	3.48	3.47	3.46	3.44	3.43	3.43	3.42	3.41	3.40	3.38	3.34	3.32	3.30	3.27
8	3.20	3.19	3.17	3.16	3.15	3.14	3.13	3.12	3.12	3.11	3.08	3.04	3.02	3.01	2.97
9	2.99	2.97	2.96	2.95	2.94	2.93	2.92	2.91	2.90	2.89	2.86	2.83	2.80	2.79	2.75
10	2.83	2.81	2.80	2.79	2.77	2.76	2.75	2.75	2.74	2.73	2.70	2.66	2.64	2.62	2.58
11	2.70	2.69	2.67	2.66	2.65	2.64	2.63	2.62	2.61	2.60	2.57	2.53	2.51	2.49	2.45
12	2.60	2.58	2.57	2.56	2.54	2.53	2.52	2.51	2.51	2.50	2.47	2.43	2.40	2.38	2.34
13	2.51	2.50	2.48	2.47	2.46	2.45	2.44	2.43	2.42	2.41	2.38	2.34	2.31	2.30	2.25
14	2.44	2.43	2.41	2.40	2.39	2.38	2.37	2.36	2.35	2.34	2.31	2.27	2.24	2.22	2.18
15	2.38	2.37	2.35	2.34	2.33	2.32	2.31	2.30	2.29	2.28	2.25	2.20	2.18	2.16	2.11
16	2.33	2.32	2.30	2.29	2.28	2.26	2.25	2.24	2.24	2.23	2.19	2.15	2.12	2.11	2.06
17	2.29	2.27	2.26	2.24	2.23	2.22	2.21	2.20	2.19	2.18	2.15	2.10	2.08	2.06	2.01
18	2.25	2.23	2.22	2.20	2.19	2.18	2.17	2.16	2.15	2.14	2.11	2.06	2.04	2.02	1.97
19	2.21	2.20	2.18	2.17	2.16	2.14	2.13	2.12	2.11	2.11	2.07	2.03	2.00	1.98	1.93
20	2.18	2.17	2.15	2.14	2.12	2.11	2.10	2.09	2.08	2.07	2.04	1.99	1.97	1.95	1.90
21	2.16	2.14	2.12	2.11	2.10	2.08	2.07	2.06	2.05	2.05	2.01	1.96	1.94	1.92	1.87
22	2.13	2.11	2.10	2.08	2.07	2.06	2.05	2.04	2.03	2.02	1.98	1.94	1.91	1.89	1.84
23	2.11	2.09	2.08	2.06	2.05	2.04	2.02	2.01	2.01	2.00	1.96	1.91	1.88	1.86	1.81
24	2.09	2.07	2.05	2.04	2.03	2.01	2.00	1.99	1.98	1.97	1.94	1.89	1.86	1.84	1.79
25	2.07	2.05	2.04	2.02	2.01	2.00	1.98	1.97	1.96	1.96	1.92	1.87	1.84	1.82	1.77
30	1.99	1.98	1.96	1.95	1.93	1.92	1.91	1.90	1.89	1.88	1.84	1.79	1.76	1.74	1.68
40	1.90	1.89	1.87	1.85	1.84	1.83	1.81	1.80	1.79	1.78	1.74	1.69	1.66	1.64	1.58
60	1.82	1.80	1.78	1.76	1.75	1.73	1.72	1.71	1.70	1.69	1.65	1.59	1.56	1.53	1.47
120	1.73	1.71	1.69	1.67	1.66	1.64	1.63	1.62	1.61	1.60	1.55	1.50	1.46	1.43	1.35

Table of the F-Distribution

Critical values for right-hand tail area equal to **0.025**

df1:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
df2: 1	648	800	864	900	922	937	948	957	963	969	973	977	980	983	985
2	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4	39.4
3	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	14.4	14.4	14.3	14.3	14.3	14.3
4	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.79	8.75	8.71	8.68	8.66
5	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.57	6.52	6.49	6.46	6.43
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.41	5.37	5.33	5.30	5.27
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.71	4.67	4.63	4.60	4.57
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.24	4.20	4.16	4.13	4.10
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.91	3.87	3.83	3.80	3.77
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.66	3.62	3.58	3.55	3.52
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.47	3.43	3.39	3.36	3.33
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.32	3.28	3.24	3.21	3.18
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.20	3.15	3.12	3.08	3.05
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.09	3.05	3.01	2.98	2.95
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	3.01	2.96	2.92	2.89	2.86
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.93	2.89	2.85	2.82	2.79
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.87	2.82	2.79	2.75	2.72
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.81	2.77	2.73	2.70	2.67
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.76	2.72	2.68	2.65	2.62
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.72	2.68	2.64	2.60	2.57
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.68	2.64	2.60	2.56	2.53
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.65	2.60	2.56	2.53	2.50
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.62	2.57	2.53	2.50	2.47
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.59	2.54	2.50	2.47	2.44
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.56	2.51	2.48	2.44	2.41
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.46	2.41	2.37	2.34	2.31
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.33	2.29	2.25	2.21	2.18
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.22	2.17	2.13	2.09	2.06
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.10	2.05	2.01	1.98	1.94

Critical Values for Right-Hand Tail Areas: F-Distribution

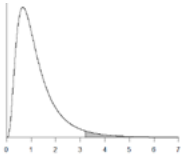
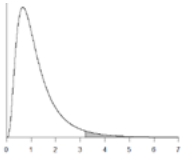


Table of the F-Distribution

Critical values for right-hand tail area equal to **0.025**

Critical Values for Right-Hand Tail Areas: F-Distribution



df1:	16	17	18	19	20	21	22	23	24	25	30	40	50	60	120
df2: 1	987	989	990	992	993	994	995	996	997	998	1001	1006	1008	1010	1014
2	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5	39.5
3	14.2	14.2	14.2	14.2	14.2	14.2	14.1	14.1	14.1	14.1	14.1	14.0	14.0	14.0	13.9
4	8.63	8.61	8.59	8.58	8.56	8.55	8.53	8.52	8.51	8.50	8.46	8.41	8.38	8.36	8.31
5	6.40	6.38	6.36	6.34	6.33	6.31	6.30	6.29	6.28	6.27	6.23	6.18	6.14	6.12	6.07
6	5.24	5.22	5.20	5.18	5.17	5.15	5.14	5.13	5.12	5.11	5.07	5.01	4.98	4.96	4.90
7	4.54	4.52	4.50	4.48	4.47	4.45	4.44	4.43	4.41	4.40	4.36	4.31	4.28	4.25	4.20
8	4.08	4.05	4.03	4.02	4.00	3.98	3.97	3.96	3.95	3.94	3.89	3.84	3.81	3.78	3.73
9	3.74	3.72	3.70	3.68	3.67	3.65	3.64	3.63	3.61	3.60	3.56	3.51	3.47	3.45	3.39
10	3.50	3.47	3.45	3.44	3.42	3.40	3.39	3.38	3.37	3.35	3.31	3.26	3.22	3.20	3.14
11	3.30	3.28	3.26	3.24	3.23	3.21	3.20	3.18	3.17	3.16	3.12	3.06	3.03	3.00	2.94
12	3.15	3.13	3.11	3.09	3.07	3.06	3.04	3.03	3.02	3.01	2.96	2.91	2.87	2.85	2.79
13	3.03	3.00	2.98	2.96	2.95	2.93	2.92	2.91	2.89	2.88	2.84	2.78	2.74	2.72	2.66
14	2.92	2.90	2.88	2.86	2.84	2.83	2.81	2.80	2.79	2.78	2.73	2.67	2.64	2.61	2.55
15	2.84	2.81	2.79	2.77	2.76	2.74	2.73	2.71	2.70	2.69	2.64	2.59	2.55	2.52	2.46
16	2.76	2.74	2.72	2.70	2.68	2.67	2.65	2.64	2.63	2.61	2.57	2.51	2.47	2.45	2.38
17	2.70	2.67	2.65	2.63	2.62	2.60	2.59	2.57	2.56	2.55	2.50	2.44	2.41	2.38	2.32
18	2.64	2.62	2.60	2.58	2.56	2.54	2.53	2.52	2.50	2.49	2.44	2.38	2.35	2.32	2.26
19	2.59	2.57	2.55	2.53	2.51	2.49	2.48	2.46	2.45	2.44	2.39	2.33	2.30	2.27	2.20
20	2.55	2.52	2.50	2.48	2.46	2.45	2.43	2.42	2.41	2.40	2.35	2.29	2.25	2.22	2.16
21	2.51	2.48	2.46	2.44	2.42	2.41	2.39	2.38	2.37	2.36	2.31	2.25	2.21	2.18	2.11
22	2.47	2.45	2.43	2.41	2.39	2.37	2.36	2.34	2.33	2.32	2.27	2.21	2.17	2.14	2.08
23	2.44	2.42	2.39	2.37	2.36	2.34	2.33	2.31	2.30	2.29	2.24	2.18	2.14	2.11	2.04
24	2.41	2.39	2.36	2.35	2.33	2.31	2.30	2.28	2.27	2.26	2.21	2.15	2.11	2.08	2.01
25	2.38	2.36	2.34	2.32	2.30	2.28	2.27	2.26	2.24	2.23	2.18	2.12	2.08	2.05	1.98
30	2.28	2.26	2.23	2.21	2.20	2.18	2.16	2.15	2.14	2.12	2.07	2.01	1.97	1.94	1.87
40	2.15	2.13	2.11	2.09	2.07	2.05	2.03	2.02	2.01	1.99	1.94	1.88	1.83	1.80	1.72
60	2.03	2.01	1.98	1.96	1.94	1.93	1.91	1.90	1.88	1.87	1.82	1.74	1.70	1.67	1.58
120	1.92	1.89	1.87	1.84	1.82	1.81	1.79	1.77	1.76	1.75	1.69	1.61	1.56	1.53	1.43

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