The Global Minimum Variance Portfolio and Efficient Frontier Approach in Developed and Developing Economies: A Case Study of International Stock Markets

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Abstract

Global minimum variance portfolio (GMVP) is the portfolio with lowest variance among all other feasible portfolios. In addition, efficient frontier is the combination of all other feasible portfolios which have higher standard deviations and higher expected return. The aim of achieve GMVP and efficient frontier is to enable investors considered risk averse to define the risk they are willing to take according to the expected return. The study had the objective to find the GMVP and the efficient frontier of five developed economies and five developing economies, using five stocks of each country. Furthermore, the study aimed to gather the developed economies in one group, the developing economies in another and finally, created one group of all of them. The research was conducted using the Solver tool, from Excel, and graphs were plotted to illustrate and facilitate the understanding. The time frame used for the study was from 01/01/2010 to 31/12/2020, and the adjusted closing price was collected from Yahoo Finance over the period. From the results it was possible to conclude that diversification of the portfolio brings lowest risk and highest return to the investment. Even though the number of assets included in the study is considerably high if invested by an individual, the distribution of weights over developed and developing economies result in achieve a higher expected return assuming less risk to the portfolio.

Keywords: Global Minimum Variance Portfolio, Efficient Frontier, Portfolio Diversification, Portfolio Theory.

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1. Introduction

Portfolio diversification has become a trend in recent times. Economists, brokers, and investment companies all agree that the best way to mitigate risk is by spreading the wealth across different assets. Investment platforms worldwide allow investors to both buy a share in Japan and sell a bond in the USA with just one click. Thus, access to information is key, and people are becoming interested in the financial market more than ever. The frequent changes in the world cause concern in investors regarding the growth of their wealth, and as far as they know diversification means decreasing the risks and increasing the return.

The presented study aims to make a comparison of the global minimum variance portfolio in stocks from developed and developing countries individually, as well as in groups. In addition, the research has the objective to find the efficient frontier of the portfolio.

The developed economies that will be used in this study are from the Group of 7 (G7). The G7 is the organisation with the seven advanced economies in the world, represented by: UK, Canada, France, Germany, Italy, Japan and USA (BBC, 2021). However, to facilitate the comparison, only 5 countries of the G7 will be cited in the study. They will be Canada, Germany, Japan, the UK and the USA. The G7 is responsible for 45% of the world GDP, in contrast to 70% thirty years ago (CFR.org Editors, 2021).

In the group of developing economies, the study will use the BRICS group, which consists of Brazil, Russia, India, China and South Africa. BRICS is considered an informal group of developing economies that was created to promote the dialogue and cooperation between these countries (Infobrics, 2020). The BRICS countries represents 27% of the global GDP and has 42% of the world population.

Every country has its own particularity regarding the economy. The investors must assume the risk when making the decision to invest their wealth in countries which are not fully developed. Notwithstanding, it is plausible to say that when investors decide to invest their money in abroad, the expected relationship is one with both sides winning, as they expect economy to be strengthened in unison with the growth of the investments, Bhutto (2020).

Lekovic (2018) developed a study saying that investment professionals and investors in general are looking for the best way to achieve higher results while taking less risk. He also mentioned that a good portfolio must contain safe assets and risk assets, in order to succeed independent

of the market's current activity. Lekovic (2018) quoted that the study of not investing the wealth in a single asset originated many years ago and has since improved consistently.

In this study, only the stock market will be analysed, and will be based upon the primary index of each country. The data collected will support the results in relation to which portfolio has the lowest standard deviation, while also taking into consideration the expected return. The purpose of this study is to provide the global minimum variance among approaches made by country and by group.

Harry Markovitz has created a formula of portfolio variance, which combines the assets that provide the best results for each risk taken. His formula will guide the data collected and developed in this study in order to elaborate on the conclusion of each result identified by the formulas.

The global minimum variance portfolio is suitable for investors that are averse to risk and is defined as the lowest variance of the efficient frontier of the portfolio. The GMPV takes into consideration the minimization of the variance of the efficient frontier and any slight addition of risk will result in considerable increase of expected return (Wortche, 2010).

According to Michaud and Michaud (2008), efficient frontier is the combination of all portfolios returns with maximum alphas for many levels of portfolio risk. The efficient frontier in the graph begins soon after the global minimum variance portfolio, with an increase in expected return and standard deviation.

2. Literature Review

2.1 Utility Theory

The act of creating a portfolio is a daily basis task based in decisions taken by investors, institutions, asset managers, risk managers and every person that in some way invest their wealth in the financial market (Alexander, 2008). The main objective is to allocate the capital in an optimal manner to satisfy the needs previously determined. Capital allocation means the distribution of the weights of wealth in different products with the purpose to achieve a performance, portfolio management or even a compensation (Zaks and Tsanakas, 2014). Xu and Mao (2013) mentioned that the allocation of investments has to consider the risk that the investor or the manager are willing to assume when defining the expected outcome.

In order to define the risk someone is susceptible to take an utility theory was developed to try to evaluate the outcomes of a decision taken. This theory aims to evaluate the aversion or preference for risk the investor is and how biased they are to achieve the expected return (Lambert and McCarl, 1985). The expected utility formula is given by:

$$E[U(P)] = \sum_{i=1}^{n} p_i U(W_i)$$

Equation 1: Expected Utility

Where W_i are the possible outcomes multiplied by the sum of all the probabilities p_i.

According to Livanas (2011), the researchers Friedman and Savage developed an hypothesis in 1952 about how the utility function drives the investor behaviours due to risk and uncertainty. Many experiments were made at that time to analyse the behaviour of the investors in moments of uncertainty. The function was created after the experiments and guides investors through investment decisions.

The utility function gives three different outcomes and determines the preference of risk by decision makers (Pennings and Smidts, 2003). These can be:

- Risk loving: when the investor tends to assume more risk;
- Risk averse: when the investor is averse to the risk and does not want to take risk;
- Risk neutral: when the investor presents neutral preference for risk.

2.2 Portfolio Theory

In the investment business portfolio means the assets that compound the wealth invested by companies or individuals. The construction of a portfolio aims at two main objectives, ie: to increase returns and to reduce risks, Branke (2009). Investment decisions are widely used based on the Mean-variance portfolio theory, which was developed by Harry Markovitz (Fahmy, 2020). Markovitz stated that investors should spread their investments across a variety of sectors and not keep the wealth in assets considerably correlated, Pinheiro (2019).

The diversification of portfolio became notorious after the Markowitz theory in 1950s. However, researches found that the act of diversify the wealth in different assets was adopted in the nineteenth century (Rutterford and Sotiropoulos, 2016). Other authors at the beginning of the twentieth century approached the benefits of spreading the investments. Nevertheless, at that time an optimisation model was not well developed. Furthermore, Rutterford and Sotiropoulos, also added that the lack of technology hampered the practice and testing of the models until 1970.

The Mean-Variance approached by Markowitz in his theory consists of quantifying the risk of assets by their variance, and quantifying the returns of the same assets by their mean. With the results of both variables, investors are able to trade-off the portfolio maximising the results (Chen, 2021). The MV theory aims to minimize the standard deviation of a portfolio considering a target rate of return previous defined, spreading the weights of the assets (Fahmy, 2020). The Markowitz model supposes a portfolio is created by different assets, and according to the returns and the risk of those assets, a vary number of proportions produce an efficient envelope (Karandikar and Sinha, 2012). The theory created by Markowitz was enhanced over the years by many researchers and new elements such as income, dynamic formulation and market incompleteness, were included to bring more trustworthiness and reliance to the results (Pfiffelmann *et al.*, 2016).

Baker and Filbeck (2013) described in their book the basic framework for the Mean variance theory created by Markowitz. The approach considers a period of time of assets with normally distributed returns and an arbitrary value for the risk-free asset. From the calculation is possible to set different feasible portfolios, with different distribution of weights that can be considered optimal portfolios according to the investor's risk aversion.

On the other hand, Markowitz recognized that the mean-variance model failed to differentiate whether the performance of the mean was over or under valuated. Hence, in this situation, the deviations from the mean must be analysed in each scenario so as to avoid mistakes, Elton (2003). Different approaches were created after Markowitz Portfolio Theory in order to try to replace variance as the main measure of risk (Vo, 2021). Value-at-risk VaR and Conditional Value-at-Risk (CVaR), according to Vo (2021), are the best candidates trying to propose a new overview of the optimal portfolio achievement.

2.3 Global Minimum Variance Portfolio

The global minimum variance portfolio (GMVP) is considered an specific optimal portfolio which has the lowest variance among the efficient frontier (Bodnar *et al.*, 2017). This portfolio is set for an investor considered risk averse that is interested in decrease the volatility of the investments, even though it means a lower expected return. Furthermore, there are constraints that can be added to the GMVP in order to achieve better results (Chiu and Jiang, 2016). The authors stated that portfolios where short sales are allowed presents a lower volatility than portfolio that constraints are not included. Constraints, according to Chiu and Jiang (2016), are included in the portfolio to improve the results, trying to find a GMVP with highest return assuming less risk.

When constructing a portfolio is necessary to infer if short sales are allowed or not. According to Benninga (2014), short sales allow negative proportions in the portfolio. This means that assets can represent a negative participation in the portfolio. On the other hand, Benninga cited that short sales in the real world is far more complicated since restrictions that can be applied to investors. Some brokerages companies, assets and even countries do not permit this kind of investment since the high level of leverage and speculation intrinsic in the products.

The global minimum variance has the objective to determine the portions of a wealth to be allocated in different assets according to the option previously given. This approach aims to minimize the variance of the portfolio (Clemente *et al.,* 2021).

Benninga (2014) stated in his book that the two main objectives of the variance covariance matrix are to find the GMVP and efficient portfolios. He assured that when portfolios are feasible, that one with the lowest variance is considered the global minimum variance portfolio. The definition of the GMVP, according to Benninga, is illustrated by:

$$x_{GMVP} = \begin{Bmatrix} x_{GMVP,1} \\ x_{GMVP,2} \\ \vdots \\ x_{GMVP,N} \end{Bmatrix} = \frac{S^{-1}1_{column}}{1_{column}^{T} \cdot S^{-1} \cdot 1_{column}}, \text{ where } 1_{column} = \begin{Bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{Bmatrix}$$

Equation 2: Benninga GMVP Definition

Merton (1972) is the person responsible for this formula and also for discuss and analyse the evolution of the Modern Portfolio Theory, created by Markowitz. Merton (1972), approached many derivations from the efficient portfolio frontier and he has proved mathematically the efficacy of it.

2.4 Efficient Portfolio

Investors are affected by uncertainty. According to Elton (2003), if individuals and institutions were not influenced by uncertainty, they would invest the money in assets that bring higher returns. Regardless the risk intrinsic in investment products, good balance of risk and return is the best path to acquire consistent gains.

Markowitz developed the Modern Portfolio Theory in 1950s which has been enhanced over the years. The theory is based on maximising the mean (expected return) and minimising the variance (risk). Even though the theory has presented some limitations, the constant development of technology and new research has added feasibility to efficient portfolio.

2.5 Developed Countries

The USA has the largest economy in the world, with nominal GDP of 21,433 billions of dollars in 2019, seven billions more than the second country, which is China with 14,341 (Lee, 2021). In addition, the USA stock market is highly developed and attract investors all over the world. The U.S. stock market has 43% of the market share considering the world stock market, and in 2017, the USA stock market capitalization was $34 trillion, against $43 trillion from the rest of the world (Surz, 2018). In contrast, due to the Covid-19 outbreak, investors tend to send their money to countries considered safe, and in 2021 the USA achieved almost 56% of the stock market share worldwide (Statista, 2021).

Stock markets are considered well-developed in relation to their market size and liquidity. Soomro (2021) stated that the size is in relation to the volume of capital available. Liquidity means the trades executed by the listed companies, or in other words, the ease of buying and selling stock, turning paper into cash. The liquidity of the stock market is highly connected with the investor sentiment (Liu, 2015). There are three types of investors: one insider, market

maker and noise traders. In order to prove the connection between stock market and investor sentiment, Liu (2015) cited the Kyle's model:

$$P = P0 + \lambda * y$$

Equation 3: Kyle's Model

Where P is the pricing rule, P0 the intrinsic value, and y the flow of the aggregate net order from insider and noise traders. The model consider λ as the measure in the price impact by the order flow.

2.6 Developing Countries

The BRICS group of developing countries (Brazil, Russia, India, China and South Africa), is considered an important opportunity for investments to highlight the comprehension of increasing return. According to Bhutto (2020), the BRICS represents 40% of the worldwide population and 15% of the global GDP, and its stock exchange market is expected to achieve 40% of the world stock exchange market by 2030. The expectation is that the BRICS GDP will surpass the G7 very soon in the future, due to the opportunities in these countries (Zhang *et al.,* 2021). The information above helps to understand the importance of this group to the world economy development and even though they are considered developing, a considerably role is performed.

Cloutier (2018) approached the idea that developing countries have much more opportunities to increase the return in investments due to the possible absence of growth in the past. Developed countries usually have a flat curve of growth while emerging countries have an exponential curve. Although, the higher the return, the higher the risk.

Nonetheless, Pinheiro (2018) stated that the financial market in developing countries must be enhanced in order to add more products to the portfolio. The author cited the Brazilian market in specific that when compared to the US stock market, it has a far less products available to the investors. Due to the constant development of the capital markets, if the investors choose to diversify their portfolios abroad, they will find a range of products and must select that which better suits their needs. Regarding the products presented, there is no doubt emerging countries must occupy a percentage of investor's portfolio. In order to be attractive, those countries, and especially their central banks, need to be aware of the currency, politics, interest rates and other economic issues (Pinheiro, 2018).

2.7 International Diversification

One of the main objectives to invest internationally is to achieve better results. A national market, by itself, has limits on its products. Lekovic (2018) concluded in his research that the economies conditions are fundamental in order to define the number of products a portfolio needs to increase return and decrease risk. The foreign investment in the BRICS countries has grown from $81 billion in 2000 to $221 in 2012 (Al-Mohamad, 2020). Over the years, according to Al-Mohamad *et al.* (2020), investors in general discovered that securities usually are less correlated internationally than if concentrated in one country. In addition, they complemented that the Modern Portfolio Theory states that international diversification is beneficial if the markets are not perfectly correlated, otherwise the diversification is not considered an opportunity. Cloutier (2018) added another opportunity of investing in emerging countries. Generally, the currency in emerging markets is worth less if compared to the currency in developed economies, which means that the power of buying shares is higher and it costs less than investing in bigger economies, i.e.: 1 dollar = 5.23 Brazilian real on August 8th, according to Morningstar.

International diversification is a practice used since many years ago. That said at the beginning only a small number of investors had access to those options. According to Cloutier (2018) after the term *diversification* became well-known, people found that the yield of their investments was limited and it was possible to achieve better results by reducing the risks. The globalization of the capital market allows people to invest their wealth worldwide. Bhutto (2020) quoted that developing countries improved their financial market to attract foreign investments. This movement allows economies to capture money and invest internally, collaborating to deliver a better result.

Ashfaq (2020) mentioned when the investor decides to put their wealth in different economies, the market risk is considerably decreased. Nevertheless, the author also noted that the capital market is highly integrated nowadays, and the information released in one continent might immediately affect activity on the other side of the world.

Ozcelebi (2020) said that the first action that investors take when there is any signal of crisis is take the money out of high-risk assets and developing economies, and invest in something considered to be safer. When this type of action happens suddenly, the situation tends to deteriorate and the crisis accelerates. The confidence of the investor also tends to decrease,

which does not help the developing country to get through the crisis easily. Due to the correlation between global financial markets, when a crisis begins in one place, all may be affected. Jaggia (2000) says investors are optimistic while the economy is going up. However, emerging markets are not considered, yet, as safe markets. During a crisis, once the developing economies are not considered safe, the outflow of money is inevitable.

A study mentioned by Cloutier (2018) compared similar bonds worldwide over 20 years. He concluded that investors who diversified internationally, assuming the same risk, made two times the profits when compared to investors who had maintained their portfolio only in the USA. In addition, the author considered that investing in assets around the world requires a long term horizon by the investor. Firstly, due to the costs intrinsic to operations which are usually higher than the domestic market. Secondly, because of the taxation imposed by the country in which they are investing. Thirdly, although the investor could be buying a riskless asset, he is frequently buying in another currency, which needs to incorporate that fluctuation. Last but not least, we must consider the volatility of the market by itself. In this case, the investor must consider keeping the wealth invested in order not to suffer any loss. Lekovic (2018) also affirmed the horizon as being one of the most important criteria to be considered when investing internationally.

The authors are unanimous in agreeing that investing part of the portfolio internationally helps the investors to increase profits and reduce risks. Mukherji (2020) analysed the data from 21 countries over 118 years and concluded that investing abroad brought higher results than maintaining the wealth in just one economy. The result of the study showed that a diversification with equal proportion over the 21 countries has increased the Sharpe ratio in 28%.

3. Research Question

The literature presented a wide range of studies that have been made over the years. The studies encouraged to create further analysis in diversifying internationally, adding more variables to the research.

Moreover, some researches were created to evaluate the global minimum variance portfolio and the efficient frontier from one country specifically and have suggested further studies internationally.

The aim of this study is achieved by the following research question:

Does the Global Minimum Variance Portfolio present less risk and higher expected return when diversifying internationally? Is the efficient frontier improved when more stocks are added to the portfolio?

The main objective of this study is to create a comparison between the stock market of the five developed countries, from G7, and the five developing countries, from BRICS.

The study aims to present the global minimum variance portfolio and the efficient frontier per country, per group and together. Furthermore, the research aims to show what are the weights of each stock in the composition of the GMVP and the efficient frontier.

4. Methodology

4.1 Research Type:

Quantitative research is the method of analysing and describing information from numerical data applying different techniques of statistic, according to Edwards (2020). Using historical data is the best path to gather information from the indexes over the years, and then summarise in a statistical analysis to obtain the results of the research question.

The longitudinal approach uses the same sample over different period of times and it allows one to create a relationship between the data analysed. Moreover, the data collected will be used in a longitudinal research includes long periods and facilitates the analysis of how it changed over the period (Creswell and Creswell, 2009).

A similar approach has been made by Faustino and Neto (2016) and Rubesam and Beltrame (2013), where they have used the global minimum variance portfolio to achieve the variance of portfolios in the Brazilian market.

4.2 Sample

The sample considered for this study consists in ten different countries. Five countries considered developed and are part of the Group of 7 (G7): The United States of America, the UK, Canada, Germany and Japan. Five countries considered developing and are part of the BRICS: Brazil, Russia, India, China and South Africa. From each country five stocks from the main stock index will be considered in order to develop the calculations and findings of the study.

The stocks will be considered by weight in the main index of the country, based on market capitalization in 19/07/2020. The weights of the index change constantly due to the ups and downs of the stock market. Nevertheless, since the stocks provide data information for the period studied, the calculations will not be compromised.

4.3 Developed Countries

United States of America – S&P500

Index	Company	Yahoo Finance Symbol	Sector
S&P500	Apple Inc.	AAPL	Technology Consumer Electronics

S&P500	Microsoft Corporation	MSFT	Technology Software/Infra
S&P500	Amazon.com, Inc.	AMZN	Consumer Cyclical Internet Retail
S&P500	Alphabet Inc.	GOOGL	Communication Services
S&P500	Berkshire Hathaway Inc.	BRK.B	Financial Services Diversified

Table 1: The USA Stocks Description

United Kingdom – FTSE100

Index	Company	Yahoo Finance Symbol	Sector
FTSE100	AstraZeneca Plc	AZN.L	Healthcare Drug Manufacturers
FTSE100	HSBC Holdings Plc	HSBA.L	Financial Services Diversified
FTSE100	Diageo Plc	DGE.L	Consumer Defensive Beverages
FTSE100	Rio Tinto Group	RIO.L	Basic Materials Metals and Mining
FTSE100	GlaxoSmithKline Plc	GSK.L	Healthcare Drug Manufacturers

Table 2: UK Stocks Description

Canada – S&P/TSX

Index	Company	Yahoo Finance Symbol	Sector
S&P/TSX	Royal Bank of Canada	RY.TO	Financial Services Diversified
S&P/TSX	The Toronto - Dominion Bank	TD.TO	Financial Services Diversified
S&P/TSX	Enbridge Inc.	ENB.TO	Energy Oil and Gas
S&P/TSX	The Bank of Nova Scotia	BNS.TO	Financial Services Diversified
S&P/TSX	Canadian National Railway Company	CNR.TO	Industrials Railroads

Table 3: Canada Stocks Description

Germany – DAX

Index	Company	Yahoo Finance Symbol	Sector
DAX	SAP SE	SAP.DE	Technology Software
DAX	Linde Plc	LIN.DE	Basic Materials Chemicals

DAX	Siemens Aktiengesellschaft	SIE.DE	Industrials Machinery
DAX	Volkswagen AG	VOW.DE	Consumer Cyclical Auto Manufacturers
DAX	Deutsche Telekom AG	DTE.DE	Communication Services

Table 4: Germany Stocks Description

Japan – Nikkei 225

Index	Company	Yahoo Finance Symbol	Sector
Nikkei 225	Fast Retailing Co., Ltd.	9983.T	Consumer Cyclical Apparel Retail
Nikkei 225	Tokyo Electron Limited	8035.T	Technology Equipments/Materials
Nikkei 225	SoftBank Group Corp.	9984.T	Communication Services
Nikkei 225	Fanuc Corporation	6954.T	Industrials Machinery
Nikkei 225	KDDI Corporation	9433.T	Communication Services

Table 5: Japan Stocks Description

4.4 Developing Countries

Brazil – Ibovespa

Index	Company	Yahoo Finance Symbol	Sector
Ibovespa	Vale S.A.	VALE3.SA	Basic Materials Metals and Mining
Ibovespa	Petróleo Brasileiro S.A. - Petrobras	PETR4.SA	Energy Oil and Gas
Ibovespa	Banco Bradesco S.A.	BBDC4.SA	Financial Services Banks
Ibovespa	B3 S.A. - Brasil, Bolsa, Balcão	B3SA3.SA	Financial Services Stock Exchange
Ibovespa	Ambev S.A.	ABEV3.SA	Consumer Defensive Beverages

Table 6: Brazil Stocks Description

Russia – MOEX

Index	Company	Yahoo Finance Symbol	Sector
MOEX	PJSC Gazprom	GAZP.ME	Energy Oil and Gas
MOEX	PJSC LUKOIL	LKOH.ME	Energy Oil and Gas

MOEX	PJSC Mining and Metallurgical Company Norilsk Nickel	GMKN.ME	Basic Materials Metals and Mining
MOEX	PJSC Rosneft Oil Company	ROSN.ME	Energy Oil and Gas
MOEX	PJSC Polyus	PLZL.ME	Basic Materials Gold

Table 7: Russia Stocks Description

India – BSE Sensex

Index	Company	Yahoo Finance Symbol	Sector
BSE Sensex	Reliance Industries Limited	RELIANCE.NS	Energy Oil and Gas
BSE Sensex	HDFC Bank Limited	HDFCBANK.NS	Financial Services Banks
BSE Sensex	Infosys Limited	INFY.NS	Technology Information Services
BSE Sensex	Housing Development Finance Corporation Limited	HDFC.NS	Financial Services Mortgage Finance
BSE Sensex	ICICI Bank Limited	ICICIBANK.NS	Financial Services Banks

Table 8: India Stocks Description

China – SSE

Index	Company	Yahoo Finance Symbol	Sector
SSE	Kweichow Moutai Co., Ltd.	600519.SS	Consumer Defensive Beverages
SSE	Industrial and Commercial Bank of China Limited	601398.SS	Financial Services Diversified
SSE	China Merchants Bank Co., Ltd.	600036.SS	Financial Services Banks
SSE	Ping An Insurance (Group) Company of China, Ltd.	601318.SS	Financial Services Insurance
SSE	China Life Insurance Company Limited	601628.SS	Financial Services Insurance

Table 9: China Stocks Description

South Africa – JSE40

Index	Company	Yahoo Finance Symbol	Sector
JSE40	British American Tobacco p.l.c.	BTI.JO	Consumer Defensive Tobacco
JSE40	Naspers Limited	NPN.JO	Communication Services Internet Content
JSE40	BHP Group	BHP.JO	Basic Materials Metals and Mining

JSE40	Compagnie Financière Richemont SA	CFR.JO	Consumer Cyclical Luxury Goods
JSE40	Anglo American Plc	AGL.JO	Basic Materials Metals and Mining

Table 10: South Africa Stocks Description

4.5 Data Collection

The data will be collected for the period of 01/01/2010 to 31/12/2020, from Yahoo Finance website. The author Simon Benninga wrote many books explaining financial modelling and he uses the Yahoo Finance website to access the data. Benninga (2011) says clearly in his recorded YouTube classes that Yahoo Finance is a reliable website that provides accurate information.

The data from Russian stocks are available in the Yahoo Finance website only from 01/03/2010. Specifically from this country, the range of the data will vary from 01/03/2010 to 31/12/2020. However, the number of observations are still high and the results will not be compromised.

The data will be collected from the index to which each stock belongs. Hence, it will be disclaimed in their own currency. However, as the calculations will be done considering the returns, the currency will not affect the result.

4.6 Construction of Calculations

4.6.1 Return Calculations

The data that will be collected presents the daily adjusted closing price for the stocks over the period. Firstly it will be necessary to calculate the returns of the stocks, daily. According to Benninga (2014), the return of each stock is defined by the formula on excel:

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

Equation 4: Returns Formula on Excel

Where, P_t is the adjusted closing price in day t, and P_{t-1} is the adjusted closing price the day before.

Furthermore, it will be assumed that the way stocks behaved in the past represents the distribution of the returns in the future. Consequently, the average return of the stocks over the period also means the expected daily return of each one (Benninga, 2014).

The average daily return and the standard deviation formula is given by:

$$\text{Average} = \frac{1}{N}\sum_{i=1}^{N} r_i$$

Equation 5: Average Daily Return

$$\text{Stdev.p} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(r_i - \bar{r})^2}$$

Equation 6: Standard Deviation

The calculation on excel is given by the formula:

=AVERAGE(range of the returns daily);

=STDEVP(range of the returns daily).

4.6.2 Correlation of the Stocks

The correlation matrix will be built to illustrate how the stocks are related and how their price behaves to each other. The correlation vary from -1 to 1, where:

-1 presents a strong negative correlation;

0 presents no correlation;

1 presents a strong positive correlation.

The formula to achieve the correlation between two stocks is (Alexander, 2008):

$$\text{Corr}(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{V(X)V(Y)}}$$

Equation 7: Correlation Formula

The correlation matrix will be drawn to illustrate the correlation of all the stocks that are being considered in this study. The calculations will be done on excel using the Data Analysis tool, Correlation and selecting the daily returns of all the stocks (Naughton, 2021).

4.6.3 Volatility Vector

The volatility vector is achieved by the multiplication of the stock standard deviation by the stock weight in the portfolio (Naughton, 2021).

4.6.4 Portfolio Standard Deviation

The risk of the portfolio is measured by the standard deviation (Alexander, 2008). The standard deviation is achieved by the square root of the variance. The formula of variance according to portfolio returns is:

$$V(R) = \sum_{i=1}^{k} w_i^2 V(R_i) + \sum_{i=1}^{k} \sum_{j=1}^{k} w_i w_j \operatorname{Cov}(R_i, R_j)$$

Equation 8: Portfolio Variance

In addition, the multiplication of each stock weight by each standard deviation achieves the volatility vector for the stock itself.

According to Joe (2021), to calculate the portfolio standard deviation on excel, the formula below will be used:

=SQRT(MMULT(volatility_row_vector,MMULT(stocks_correl,TRANSPOSE(volatility_row_vector)))).

4.6.5 Portfolio Expected Return

The portfolio expected return is given by the sum of the multiplications of the assets weights by the returns of each asset (Alexander, 2008). The formula is:

$$R = \sum_{i=1}^{k} w_i R_i.$$

Equation 9: Portfolio Expected Return

The result will present the expected return of the portfolio according to the weights attributed to each stock that is inserted in the portfolio. Considering that the stocks have different mean returns, the weight of each will be different in creating the portfolio, bringing different results, regarding the risk assumed.

4.6.6 Constraints

When creating a portfolio, it must be defined whether short sales are allowed or not (Benninga, 2014). In this study specifically short sales are not allowed, which means that the weight of any stock cannot be less than 0%. Moreover, the sum of the weights must be equal to 100%.

The constraints that will be included in this study are:

- Stock weights >= 0;
- Stock weights <= 1;
- Sum of stock weights = 1.

4.6.7 Global Minimum Variance Portfolio

The Global Minimum Variance Portfolio is the result of the highest expected return of the portfolio with the lowest possible standard deviation. The formula to achieve the global minimum variance portfolio, assuming no short sales are allowed, is (Alexander, 2008):

$$\min_{\mathbf{w}} \mathbf{w}'\mathbf{V}\mathbf{w} \text{ such that } \sum_{i=1}^{n} w_i = 1,\ \mathbf{w}'E(\mathbf{r}) = \bar{R},\ w_i \geq 0 \text{ for all } i.$$

Equation 10: Global Minimum Variance Portfolio

In his book, Alexander (2008) presents in his book the method to find the GMVP using Solver in Excel. In order to include the constraints explained before, the Solver tool assists in the calculation.

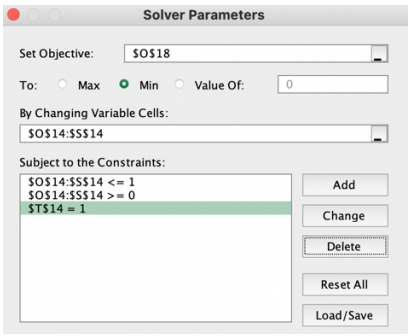

Figure 1: Solver GMVP Calculation

- Set Objective: The result will be allocated in the Portfolio Standard Deviation cell;
- To: minimizing the objective;
- By Changing Variable Cells: the cells of the weights of each stock;
- Constraints: Stock weights <= 1

  Stock weights >=0

  Sum of stock weights = 1

The result solved by the tool will present the lowest standard deviation with the highest expected return.

4.6.8 Efficient Frontier

The efficient frontier is the increasing of the return assuming the lowest standard deviation. This study will assume that no short sales are allowed. In this case, the efficient frontier is the envelope of all portfolios starting in the global minimum variance portfolio and finishing at the maximum expected return portfolio (Alexander, 2008).

According to Joe (2021), it is feasible to achieve the efficient frontier on excel adding a constraint to the Solver tool, previously used to find the GMVP. The constraint includes an arbitrary value for the expected return, in order to achieve the lowest standard deviation for that return. A slight increasing value for the expected return will be added regularly until it reaches the maximum value.

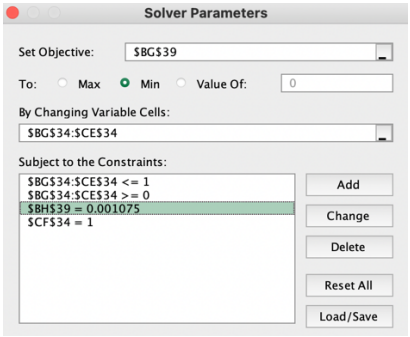

Figure 2: Solver Efficient Frontier Calculation

- Set Objective: The result will be allocated in the Portfolio Standard Deviation cell;
- To: minimizing the objective;
- By Changing Variable Cells: the cells of the weights of each stock;
- Constraints: Stock weights <= 1

  Stock weights >=0

  Expected Return = arbitrary value

  Sum of stock weights = 1

Each time the value of expected return is changed, the Solver will find the lowest standard deviation for the portfolio.

5. Data Analysis and Interpretation

The analysis will be carried out considering each country individually, the results of the global minimum variance portfolio, the efficient frontier and the weight that each stock has in the composition of the portfolio. Moreover, the analysis will also approach the portfolio of the group of 5 developed countries, the 5 developing countries and all the stocks of all 10 countries.

5.1 Developed Countries

The countries analysed below are the United States of America, the United Kingdom, Canada, Germany and Japan. An inference will be made for each one separately and a portfolio will be created at the end with the 5 stocks of each country evaluated together.

5.1.1 United States of America – S&P 500

	AAPL	MSFT	AMZN	GOOGL	BRK-B
Daily Average Returns	0.1088%	0.0802%	0.1157%	0.0618%	0.0449%
Standard Deviation	1.7847%	1.5997%	1.9926%	1.6285%	1.2774%
Number of Observations	2767				

Figure 3: The USA Returns / Std. Deviation

The table presents the daily mean returns and the standard deviation of data collected from 01/01/2010 to 31/12/2020. 2767 observations were considered during the period. From the 5 stocks, Amazon had the highest average daily return, in addition to the highest standard deviation as well. Berkshire Hathaway presented the less than a half of average return, however, with only 20% less standard deviation.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	AAPL	MSFT	AMZN	GOOGL	BRK-B
GMVP	1.1899%	0.0611%	9%	11%	6%	13%	61%
Efficient Frontier	1.2034%	0.0700%	17%	13%	12%	7%	52%
	1.2224%	0.0750%	21%	14%	16%	3%	46%
	1.2493%	0.0800%	25%	15%	20%	0%	41%
	1.2842%	0.0850%	29%	15%	23%	0%	33%
	1.3271%	0.0900%	33%	15%	26%	0%	26%
	1.3772%	0.0950%	38%	15%	29%	0%	18%
	1.4338%	0.1000%	42%	15%	33%	0%	11%
	1.4962%	0.1050%	46%	15%	36%	0%	3%
	1.5673%	0.1100%	52%	6%	42%	0%	0%

Figure 4: The USA GMVP - Efficient Frontier

Regarding a portfolio created with the 5 stocks above, the global minimum variance showed a standard deviation of 1.19% and expected return of 0.06% daily. The weight spread presented

a concentration of 61% of the portfolio in Berkshire Hathaway and less than 10% each in Apple and Amazon.

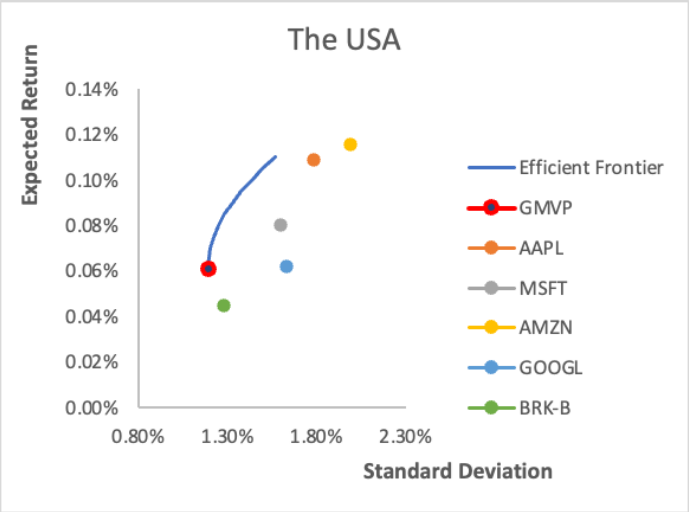

Figure 5: The USA GMVP Chart

Furthermore, as long as the expected returns increases, the standard deviation also increases, bringing more volatility to the portfolio. The efficient frontier shows the expected returns above the minimum variance.

5.1.2 UK – FTSE100

	AZN.L	HSBA.L	DGE.L	RIO.L	GSK.L
Daily Average Returns	0.0515%	-0.0030%	0.0476%	0.0342%	0.0206%
Standard Deviation	1.4348%	1.4559%	1.2510%	2.0715%	1.1946%
Number of Observations	2772				

Figure 6: UK Returns / Std. Deviation

2772 observations were considered from 01/01/2010 to 31/12/2020 according to the data collected. Astrazeneca presented the highest return, even though it does not have the highest volatility. Rio Tinto has a standard deviation above 2%, however it does not delivery a considerable average return when compared to Astrazeneca or even Diageo Plc. HSBA has a negative average daily return, as so as the second highest standard deviation.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	AZN.L	HSBA.L	DGE.L	RIO.L	GSK.L
GMVP	1.0002%	0.0290%	16%	19%	29%	2%	34%
Efficient Frontier	1.0004%	0.0300%	16%	18%	30%	2%	33%
	1.0030%	0.0325%	19%	14%	33%	3%	30%
	1.0089%	0.0351%	21%	11%	37%	4%	27%
	1.0175%	0.0376%	24%	7%	40%	5%	24%
	1.0283%	0.0400%	26%	4%	43%	6%	22%
	1.0424%	0.0425%	28%	1%	46%	7%	19%
	1.0603%	0.0450%	33%	0%	50%	6%	11%
	1.0853%	0.0476%	38%	0%	54%	6%	3%

Figure 7: UK GMVP - Efficient Frontier

Creating a portfolio with the 5 stocks from FTSE100, the global minimum variance presents a standard deviation of 1% and an expected return of 0.03%. The weights in each stock are well spread, except those in Rio Tinto which concentrates only 2%.

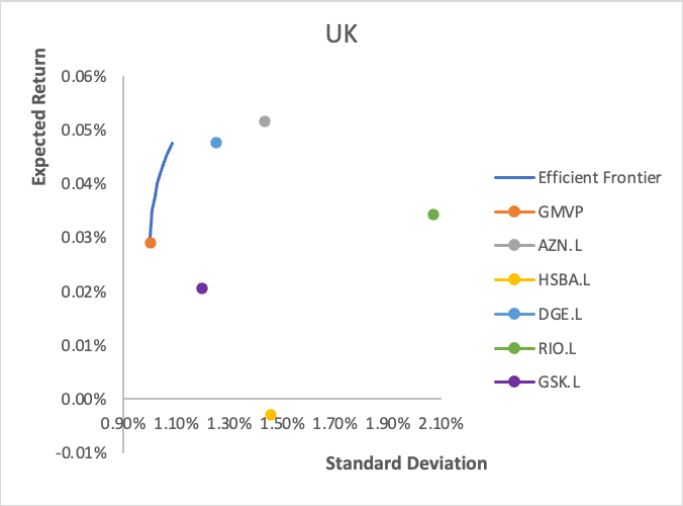

Figure 8: UK GMVP Chart

The graph perfectly illustrates the achievement of higher return with less standard deviation by the global minimum variance portfolio. It is also possible to infer that each stock has its participation to create a portfolio with better results while taking less risk.

5.1.3 CANADA – S&P/TSX

	RY.TO	TD.TO	ENB.TO	BNS.TO	CNR.TO
Daily Average Returns	0.0380%	0.0431%	0.0360%	0.0292%	0.0644%
Standard Deviation	1.1415%	1.1520%	1.5008%	1.1751%	1.2542%
Number of Observations	2758				

Figure 9: Canada Returns / Std. Deviation

The five stocks analysed from Canada S&P/TSX index presented 2758 observations, with a well-balanced standard deviation among all of them and slight difference in the average returns.

Canadian National Railways delivers the highest return with the second highest standard deviation.

	Port Standard Deviation	Port Expected Return	Portfolio Weights				
			RY.TO	TD.TO	ENB.TO	BNS.TO	CNR.TO
GMVP	1.0203%	0.0453%	29%	16%	10%	15%	30%
Efficient Frontier	1.0225%	0.0475%	28%	21%	9%	7%	35%
	1.0308%	0.0501%	26%	25%	8%	0%	42%
	1.0387%	0.0515%	22%	26%	6%	0%	47%
	1.0469%	0.0526%	18%	27%	4%	0%	51%
	1.0708%	0.0550%	11%	28%	1%	0%	59%
	1.1058%	0.0576%	2%	29%	0%	0%	69%
	1.1203%	0.0585%	0%	28%	0%	0%	72%
	1.1481%	0.0600%	0%	21%	0%	0%	79%
	1.2046%	0.0625%	0%	9%	0%	0%	91%

Figure 10: Canada GMVP - Efficient Frontier

In order to find out the minimum variance portfolio for the five stocks, the highest return is 0.045% with the volatility of 1.02%. It is possible to infer that none of the stocks have the lowest standard deviation found in the portfolio, and the well-distributed weight of each stock allows one to create an efficient portfolio. However, as long as the expected return increases, stocks such as Enbridge and Bank of Nova Scotia Halifax do not participate in the portfolio anymore.

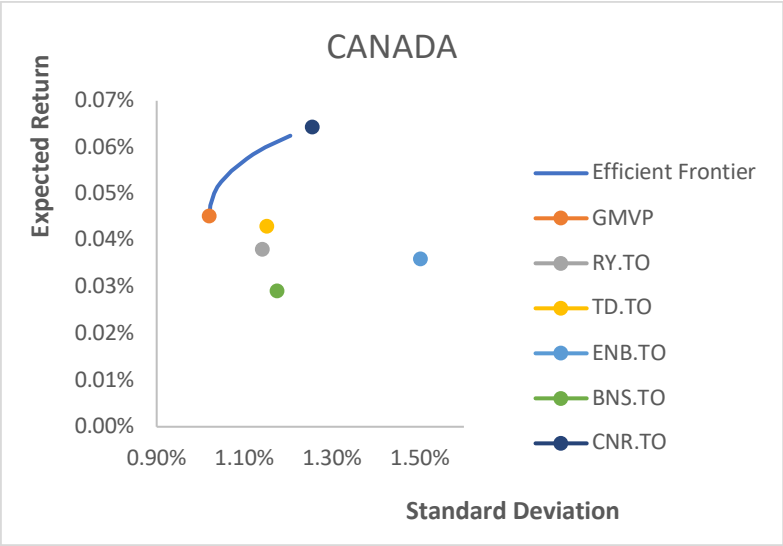

Figure 11: Canada GMVP Chart

The chart demonstrates how low the global minimum variance is for the 5 Canadian stocks. Nevertheless, the expected return is considerably high if compared to each stock individually.

5.1.4 GERMANY – DAX

	SAP.DE	LIN.DE	SIE.DE	VOW.DE	DTE.DE
Daily Average Returns	0.0485%	0.0600%	0.0358%	0.0384%	0.0359%
Standard Deviation	1.4855%	1.5386%	1.5316%	2.0051%	1.3851%
Number of Observations	2789				

Figure 12: Germany Returns / Std. Deviation

The five stocks from the Germany's DAX index presented 2789 observations during the period. The Linde plc has the highest return over the period, even though it has 20% less volatility than Volkswagen, which presented more than 2% of standard deviation.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	SAP.DE	LIN.DE	SIE.DE	VOW.DE	DTE.DE
GMVP	1.0404%	0.0473%	22%	36%	8%	1%	34%
Efficient Frontier	1.0463%	0.0490%	26%	40%	3%	0%	30%
	1.0552%	0.0500%	29%	43%	0%	0%	27%
	1.0684%	0.0510%	31%	46%	0%	0%	23%
	1.0976%	0.0525%	34%	51%	0%	0%	15%
	1.1097%	0.0530%	35%	53%	0%	0%	13%
	1.1689%	0.0550%	38%	59%	0%	0%	3%
	1.2066%	0.0560%	35%	65%	0%	0%	0%
	1.2632%	0.0570%	26%	74%	0%	0%	0%
	1.4295%	0.0590%	9%	91%	0%	0%	0%

Figure 13: Germany GMVP - Efficient Frontier

Considering that a portfolio has been created with the five stocks, the GMVP found a daily expected return of 0.047% and a standard deviation of 1.04%. Volkswagen, which has the highest standard deviation does not have an average return above the other stocks. Thus, it represents only 1% allocation in the portfolio global minimum variance portfolio. Whether the expected return increases only 0.001%, the company is no longer part of the portfolio anymore. SAP, Linde and Deutsche Telekom are well-distributed in weights.

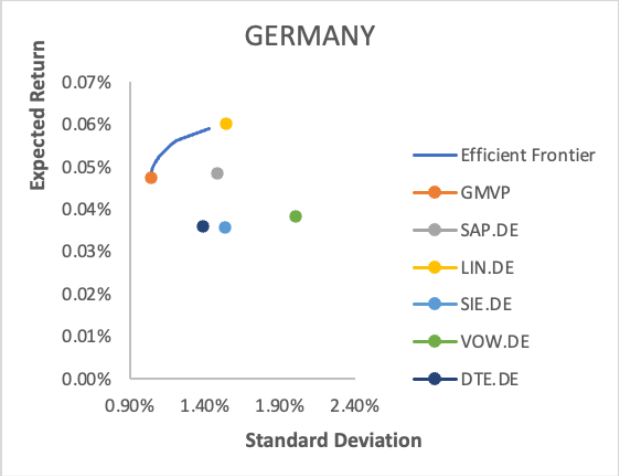

Figure 14: Germany GMVP Chart

The chart allows to see clearly where each stock is, according to the standard deviation and expected return. While 4 of the stocks have similar standard deviations, VOW.DE is farther to the right, demonstrating the high volatility. The GMVP has a higher expected return than DTE.DE, VOW.DE, and SIE.DE, with a markedly lower standard deviation.

5.1.5 Japan – Nikkei225

	9983.T	8035.T	9984.T	6954.T	9433.T
Daily Average Returns	0.0659%	0.0783%	0.0768%	0.0465%	0.0601%
Standard Deviation	2.2134%	2.2318%	2.3870%	2.0374%	1.7209%
Number of Observations	2710				

Figure 15: Japan Returns / Std. Deviation

The Japan's Nikkei225 index has the lowest number of observations over the period analysed, with 2710. Furthermore, the standard deviation of the stocks is the highest when compared to the other five developed countries cited above. The average returns of the stocks presented almost the same numbers, with only a slight difference among them. Tokyo Electron (8035.T) presented the highest average return and the second highest standard deviation.

	Port Standard Deviation	Port Expected Return	Portfolio Weights				
			9983.T	8035.T	9984.T	6954.T	9433.T
GMVP	1.4319%	0.0613%	13%	12%	7%	21%	47%
Efficient Frontier	1.4354%	0.0630%	13%	16%	9%	16%	46%
	1.4406%	0.0640%	14%	18%	10%	13%	45%
	1.4481%	0.0650%	14%	20%	11%	10%	45%
	1.4764%	0.0675%	15%	25%	14%	3%	43%
	1.5008%	0.0690%	15%	29%	16%	0%	40%
	1.5252%	0.0700%	14%	32%	19%	0%	35%
	1.5965%	0.0720%	11%	39%	25%	0%	25%
	1.7517%	0.0750%	7%	49%	33%	0%	10%
	1.8813%	0.0770%	5%	56%	38%	0%	1%

Figure 16: Japan GMVP - Efficient Frontier

The GMVP presented an expected return of 0.061% with a standard deviation of 1.43%. Moreover, with the weight distributed among the five stocks, in the GMVP analysis, one can see a concentration of 47% in the KDDI Corp. (9433.T), which has the lowest standard deviation itself. The other companies are well-distributed and presents an even weight. Nonetheless, when the expected return increases, the weight in KDDI Corp. (9433.T) and Fanuc Copr. (6954.T) decrease, and the opposite happens with the other 3 stocks.

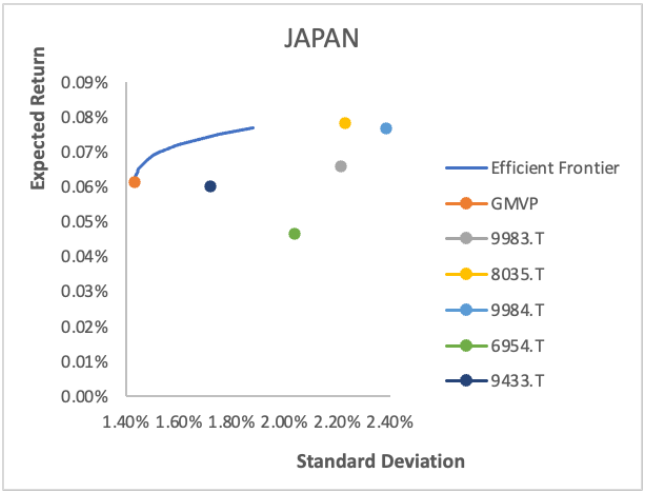

Figure 17: Japan GMVP Chart

The chart shows the sharp contrast between the GMVP standard deviation and expected return, and the results of the stocks. Even though, the Japanese index presents the highest volatility among the countries, the expected return is also high.

5.1.6 G5

	AAPL	MSFT	AMZN	GOOGL	BRK-B	AZN.L	HSBA.L	DGE.L	RIO.L	GSK.L	RY.TO	TD.TO	ENB.TO	BNS.TO	CNR.TO	SAP.DE	LIN.DE	SIE.DE	VOW.DE	DTE.DE	9983.T	8035.T	9984.T	6954.T	9433.T
Daily Average Returns	0.1054%	0.0799%	0.1170%	0.0573%	0.0435%	0.0577%	-0.0115%	0.0445%	0.0285%	0.0229%	0.0353%	0.0366%	0.0333%	0.0219%	0.0649%	0.0580%	0.0607%	0.0361%	0.0357%	0.0370%	0.0659%	0.0783%	0.0768%	0.0465%	0.0601%
Standard Deviation	1.7763%	1.5991%	1.9913%	1.6254%	1.2766%	1.4211%	1.4243%	1.2335%	2.0810%	1.1871%	1.1436%	1.1491%	1.4954%	1.1713%	1.2554%	1.4047%	1.5363%	1.5246%	2.0028%	1.3892%	2.2134%	2.2318%	2.3870%	2.0374%	1.7209%
Number of Observations	2710																								

Figure 18: G5 Returns / Std. Deviation

The period analysed considering the 25 stocks is the same, from 01/01/2010 to 31/12/2020. However, in order to achieve the same number of observations, 4 countries (USA, UK, Canada and Germany) had their last days of the period omitted from the analysis. There is still a high number of observations, with 2710 in total, which can clearly demonstrates the results without any loss of information. Amazon has the highest daily average return with 0.117%, followed by Apple with 0.015%, both from S&P500 index. On the other hand, HSBC Holding, from the FTSE100 index presented a negative daily average return, with -0.011%.

	AAPL	MSFT	AMZN	GOOGL	BRK-B	AZN.L	HSBA.L	DGE.L	RIO.L	GSK.L	RY.TO	TD.TO	ENB.TO	BNS.TO	CNR.TO	SAP.DE	LIN.DE	SIE.DE	VOW.DE	DTE.DE	9983.T	8035.T	9984.T	6954.T	9433.T
AAPL	1	0.5566	0.4423	0.5327	0.4791	0.0853	0.0325	0.0305	0.0472	0.0593	0.1174	0.1108	0.0867	0.1084	0.0735	-0.0070	-0.0189	-0.0027	-0.0117	-0.0387	-0.0261	-0.0144	-0.0316	-0.0117	-0.0136
MSFT	0.5566	1	0.5224	0.6119	0.5612	0.0691	0.0414	0.0450	0.0347	0.0729	0.1185	0.1055	0.1098	0.1046	0.1020	-0.0164	-0.0346	-0.0074	-0.0326	-0.0320	-0.0128	-0.0064	-0.0077	-0.0070	0.0083
AMZN	0.4423	0.5224	1	0.5804	0.4042	0.0480	0.0475	0.0648	0.0528	0.0372	0.0881	0.0783	0.0700	0.0814	0.0812	-0.0176	-0.0111	0.0009	-0.0016	-0.0208	0.0361	0.0204	0.0212	0.0378	0.0390
GOOGL	0.5327	0.6119	0.5804	1	0.5171	0.0469	0.0324	0.0413	0.0594	0.0561	0.1110	0.1058	0.0905	0.0981	0.0956	-0.0095	-0.0318	0.0045	0.0004	-0.0125	-0.0047	-0.0175	-0.0222	0.0053	0.0100
BRK-B	0.4791	0.5612	0.4042	0.5171	1	0.0129	0.0071	-0.0126	0.0142	0.0290	0.1573	0.1482	0.1466	0.1449	0.1045	-0.0477	-0.0154	-0.0225	-0.0481	-0.0350	-0.0077	-0.0096	0.0044	-0.0166	0.0068
AZN.L	0.0853	0.0691	0.0480	0.0469	0.0129	1	0.3382	0.3977	0.2551	0.5690	-0.0232	-0.0278	-0.0041	-0.0305	-0.0213	-0.0061	0.0034	-0.0088	0.0015	-0.0115	-0.0013	0.0170	0.0055	-0.0071	0.0188
HSBA.L	0.0325	0.0414	0.0475	0.0324	0.0071	0.3382	1	0.4417	0.5246	0.3842	-0.0371	-0.0308	-0.0314	-0.0358	-0.0448	-0.0388	0.0298	-0.0348	-0.0329	-0.0248	-0.0057	0.0000	-0.0146	0.0175	0.0078
DGE.L	0.0305	0.0450	0.0648	0.0413	-0.0126	0.3977	0.4417	1	0.3387	0.5062	-0.0156	-0.0323	-0.0119	-0.0291	-0.0273	-0.0308	0.0317	-0.0130	-0.0140	-0.0137	0.0020	-0.0021	0.0007	-0.0241	0.0410
RIO.L	0.0472	0.0347	0.0528	0.0594	0.0142	0.2551	0.5246	0.3387	1	0.2984	-0.0333	-0.0365	-0.0406	-0.0289	-0.0271	-0.0420	0.0054	-0.0321	-0.0500	-0.0349	0.0389	0.0310	0.0274	0.0293	0.0052
GSK.L	0.0593	0.0729	0.0372	0.0561	0.0290	0.5690	0.3842	0.5062	0.2984	1	-0.0331	-0.0377	-0.0084	-0.0400	-0.0027	-0.0209	0.0059	-0.0065	-0.0001	-0.0219	-0.0053	0.0223	0.0229	0.0084	0.0311
RY.TO	0.1174	0.1185	0.0881	0.1110	0.1573	-0.0232	-0.0371	-0.0156	-0.0333	-0.0331	1	0.8423	0.5194	0.8081	0.5524	0.0317	-0.0165	0.0165	0.0128	0.0177	-0.0136	0.0089	0.0090	-0.0196	-0.0215
TD.TO	0.1108	0.1055	0.0783	0.1058	0.1482	-0.0278	-0.0308	-0.0323	-0.0365	-0.0377	0.8423	1	0.5308	0.8318	0.5701	0.0274	-0.0120	0.0125	0.0183	0.0141	-0.0125	0.0211	0.0092	-0.0114	-0.0263
ENB.TO	0.0867	0.1098	0.0700	0.0905	0.1466	-0.0041	-0.0314	-0.0119	-0.0406	-0.0084	0.5194	0.5308	1	0.5546	0.4689	0.0160	-0.0037	-0.0007	-0.0098	-0.0110	-0.0120	-0.0061	-0.0002	0.0120	-0.0185
BNS.TO	0.1084	0.1046	0.0814	0.0981	0.1449	-0.0305	-0.0358	-0.0291	-0.0289	-0.0400	0.8081	0.8318	0.5546	1	0.5515	0.0190	-0.0010	-0.0004	0.0076	0.0167	-0.0070	0.0001	0.0027	-0.0090	-0.0276
CNR.TO	0.0735	0.1020	0.0812	0.0956	0.1045	-0.0213	-0.0448	-0.0273	-0.0271	-0.0027	0.5524	0.5701	0.4689	0.5515	1	0.0301	0.0256	0.0453	0.0310	0.0377	-0.0161	-0.0183	0.0140	-0.0075	-0.0255
SAP.DE	-0.0070	-0.0164	-0.0176	-0.0095	-0.0477	-0.0061	-0.0388	-0.0308	-0.0420	-0.0209	0.0317	0.0274	0.0160	0.0190	0.0301	1	0.1608	0.6181	0.4893	0.5010	0.0203	0.0088	0.0317	0.0144	0.0085
LIN.DE	-0.0189	-0.0346	-0.0111	-0.0318	-0.0154	0.0034	0.0298	0.0317	0.0054	0.0059	-0.0165	-0.0120	-0.0037	-0.0010	0.0256	0.1608	1	0.1987	0.1603	0.1331	0.0164	0.0058	0.0191	0.0125	-0.0070
SIE.DE	-0.0027	-0.0074	0.0009	0.0045	-0.0225	-0.0088	-0.0348	-0.0130	-0.0321	-0.0065	0.0165	0.0125	-0.0007	-0.0004	0.0453	0.6181	0.1987	1	0.5973	0.5693	-0.0038	-0.0104	0.0081	0.0204	-0.0003
VOW.DE	-0.0117	-0.0326	-0.0016	0.0004	-0.0481	0.0015	-0.0329	-0.0140	-0.0500	-0.0001	0.0128	0.0183	-0.0098	0.0076	0.0310	0.4893	0.1603	0.5973	1	0.4322	0.0144	-0.0004	0.0195	0.0230	0.0145
DTE.DE	-0.0387	-0.0320	-0.0208	-0.0125	-0.0350	-0.0115	-0.0248	-0.0137	-0.0349	-0.0219	0.0177	0.0141	-0.0110	0.0167	0.0377	0.5010	0.1331	0.5693	0.4322	1	0.0200	-0.0161	0.0242	0.0157	0.0165
9983.T	-0.0261	-0.0128	0.0361	-0.0047	-0.0077	-0.0013	-0.0057	0.0020	0.0389	-0.0053	-0.0136	-0.0125	-0.0120	-0.0070	-0.0161	0.0203	0.0164	-0.0038	0.0144	0.0200	1	0.3740	0.3970	0.4210	0.3647
8035.T	-0.0144	-0.0064	0.0204	-0.0175	-0.0096	0.0170	0.0000	-0.0021	0.0310	0.0223	0.0089	0.0211	-0.0061	0.0001	-0.0183	0.0088	0.0058	-0.0104	-0.0004	-0.0161	0.3740	1	0.4055	0.5366	0.3051
9984.T	-0.0316	-0.0077	0.0212	-0.0222	0.0044	0.0055	-0.0146	0.0007	0.0274	0.0229	0.0090	0.0092	-0.0002	0.0027	0.0140	0.0317	0.0191	0.0081	0.0195	0.0242	0.3970	0.4055	1	0.3957	0.3786
6954.T	-0.0117	-0.0070	0.0378	0.0053	-0.0166	-0.0071	0.0175	-0.0241	0.0293	0.0084	-0.0196	-0.0114	0.0120	-0.0090	-0.0075	0.0144	0.0125	0.0204	0.0230	0.0157	0.4210	0.5366	0.3957	1	0.3051
9433.T	-0.0136	0.0083	0.0390	0.0100	0.0068	0.0188	0.0078	0.0410	0.0052	0.0311	-0.0215	-0.0263	-0.0185	-0.0276	-0.0255	0.0085	-0.0070	-0.0003	0.0145	0.0165	0.3647	0.3051	0.3786	0.3051	1

Figure 19: G5 Correlation Matrix

The correlation matrix above presents the correlation of the 25 stocks. The majority of the stocks present a positive correlation, but not a strong positive. Stocks from the same index are susceptible to present the strongest correlation, as is possible to see in the matrix.

			Portfolio Weights																							
	Port Standard Deviation	Port Expected Return	AAPL	MSFT	AMZN	GOOGL	BRK-B	AZN.L	HSBA.L	DGE.L	RIO.L	GSK.L	RY.TO	TD.TO	ENB.TO	BNS.TO	CNR.TO	SAP.DE	LIN.DE	SIE.DE	VOW.DE	DTE.DE	9983.T	8035.T	9984.T	6954.T	9433.T
GMVP	0.5044%	0.0439%	1.26%	0.88%	0.00%	1.71%	10.87%	3.77%	5.86%	8.15%	0.41%	8.12%	4.97%	4.69%	1.74%	4.51%	6.36%	7.01%	8.26%	1.26%	0.21%	7.84%	1.77%	1.62%	0.22%	2.80%	5.70%
Efficient Frontier	0.5046%	0.0450%	1.68%	1.01%	0.04%	1.57%	10.60%	4.11%	5.30%	8.44%	0.45%	7.69%	5.01%	4.93%	1.68%	3.73%	6.84%	7.23%	8.40%	1.07%	0.20%	7.77%	1.81%	1.71%	0.22%	2.74%	5.76%
	0.5062%	0.0475%	2.44%	1.63%	0.05%	1.21%	9.98%	4.81%	3.97%	9.08%	0.68%	6.72%	5.01%	5.38%	1.54%	2.22%	7.86%	7.79%	8.70%	0.54%	0.16%	7.68%	1.85%	1.99%	0.29%	2.57%	5.85%
	0.5094%	0.0500%	3.26%	1.93%	0.16%	0.92%	9.43%	5.52%	2.61%	9.67%	1.03%	5.74%	4.95%	5.93%	1.38%	0.72%	8.84%	8.29%	8.98%	0.26%	0.00%	7.53%	1.91%	2.16%	0.54%	2.34%	5.89%
	0.5137%	0.0525%	3.90%	2.10%	1.19%	0.05%	8.86%	6.11%	1.53%	10.10%	1.16%	5.03%	4.88%	5.96%	1.21%	0.00%	9.64%	8.66%	9.20%	0.00%	0.00%	7.37%	1.94%	2.38%	0.65%	2.13%	5.94%
	0.5196%	0.0550%	4.27%	1.98%	2.42%	0.05%	7.99%	6.78%	0.29%	10.59%	1.31%	4.19%	4.23%	5.84%	0.97%	0.00%	10.45%	8.95%	9.42%	0.00%	0.00%	7.04%	1.97%	2.62%	0.73%	1.93%	5.98%
	0.5276%	0.0575%	5.14%	2.16%	3.14%	0.04%	6.86%	7.63%	0.00%	10.95%	1.15%	2.64%	3.50%	5.40%	0.59%	0.00%	11.71%	9.35%	9.66%	0.00%	0.00%	6.52%	2.05%	2.98%	0.93%	1.56%	6.03%
	0.5377%	0.0600%	6.09%	2.35%	3.90%	0.00%	5.67%	8.47%	0.00%	11.23%	0.92%	0.98%	2.77%	4.89%	0.13%	0.00%	13.07%	9.77%	9.91%	0.00%	0.00%	5.98%	2.12%	3.36%	1.16%	1.17%	6.08%
	0.5500%	0.0625%	7.14%	2.50%	4.79%	0.00%	4.21%	9.08%	0.00%	11.25%	0.60%	0.00%	1.83%	4.17%	0.12%	0.00%	14.39%	10.21%	10.16%	0.00%	0.00%	5.35%	2.21%	3.80%	1.42%	0.67%	6.11%
	0.5647%	0.0650%	8.32%	2.64%	5.79%	0.00%	2.52%	9.36%	0.00%	10.90%	0.15%	0.00%	0.85%	3.29%	0.10%	0.00%	15.82%	10.67%	10.43%	0.00%	0.00%	4.63%	2.34%	4.27%	1.69%	0.11%	6.11%
	0.5819%	0.0675%	9.60%	2.78%	6.87%	0.00%	0.64%	9.57%	0.00%	10.36%	0.00%	0.00%	0.00%	2.28%	0.00%	0.00%	17.28%	11.17%	10.68%	0.00%	0.00%	3.81%	2.35%	4.58%	1.93%	0.01%	6.08%
	0.6017%	0.0700%	10.91%	2.38%	8.15%	0.00%	0.00%	9.77%	0.00%	9.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	18.82%	11.82%	10.90%	0.00%	0.00%	2.53%	2.32%	4.92%	2.18%	0.01%	5.95%
	0.6259%	0.0725%	12.57%	1.17%	10.08%	0.00%	0.00%	9.92%	0.00%	7.52%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	18.92%	12.59%	11.08%	0.00%	0.00%	0.42%	2.20%	5.34%	2.58%	0.01%	5.61%
	0.6551%	0.0750%	14.45%	0.00%	12.29%	0.00%	0.00%	10.01%	0.00%	5.12%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	18.74%	12.31%	11.08%	0.00%	0.00%	0.00%	2.05%	5.81%	3.03%	0.01%	5.08%
	0.6899%	0.0775%	16.04%	0.00%	14.37%	0.00%	0.00%	10.02%	0.00%	2.35%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	18.33%	11.70%	11.00%	0.00%	0.00%	0.00%	1.85%	6.36%	3.53%	0.01%	4.44%
	0.7297%	0.0800%	17.69%	0.00%	16.51%	0.00%	0.00%	9.78%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	17.82%	11.00%	10.86%	0.00%	0.00%	0.00%	1.67%	6.90%	4.05%	0.00%	3.72%

Figure 20: G5 GMVP - Efficient Frontier

The global minimum variance portfolio reached an expected return of 0.044% and a standard deviation of 0.504%. When compared to the GMPV from each country individually, the standard deviation of the G5 presented very small number. This means that the diversification of the portfolio among the 25 stocks decreased the volatility considerable. Berkshire Hathaway, from the S&P500 has the highest weight in the minimum variance portfolio, with 10.9%, followed by Linde plc, from DAX, with 8.26%. Amazon is not a part of the portfolio, due to

its high volatility. Volkswagen, from DAX, and Softbank Group, from Nikkei225, have only 0.22% each also considering the high standard deviation.

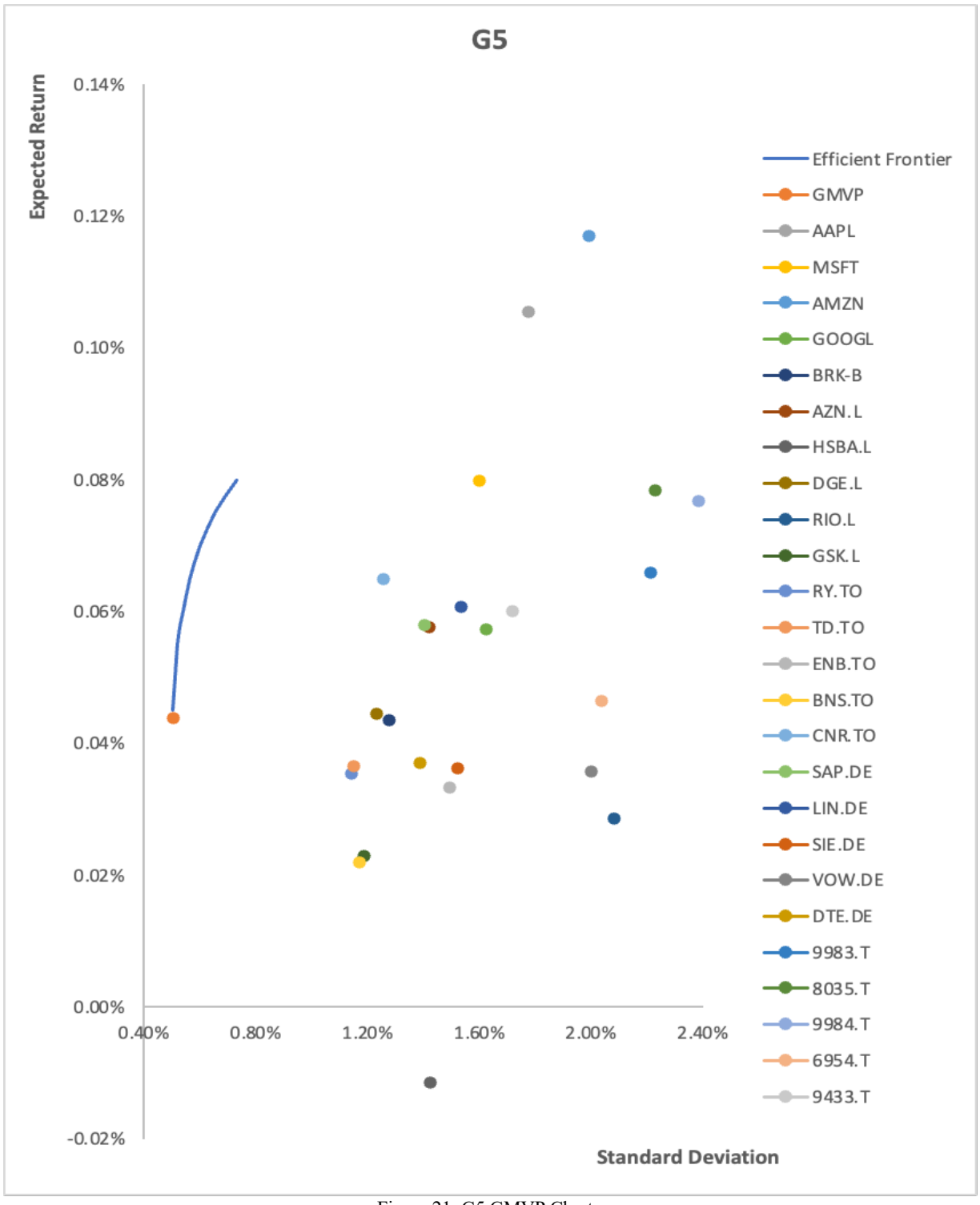

Figure 21: G5 GMVP Chart

The chart clearly illustrates how low the standard deviation of the global minimum variance portfolio is, and why it is used often by investment managers. In this study, the diversification

of the portfolio of, 5 countries and 25 stocks brought a result of a better expected return taking less risk if investing in one market.

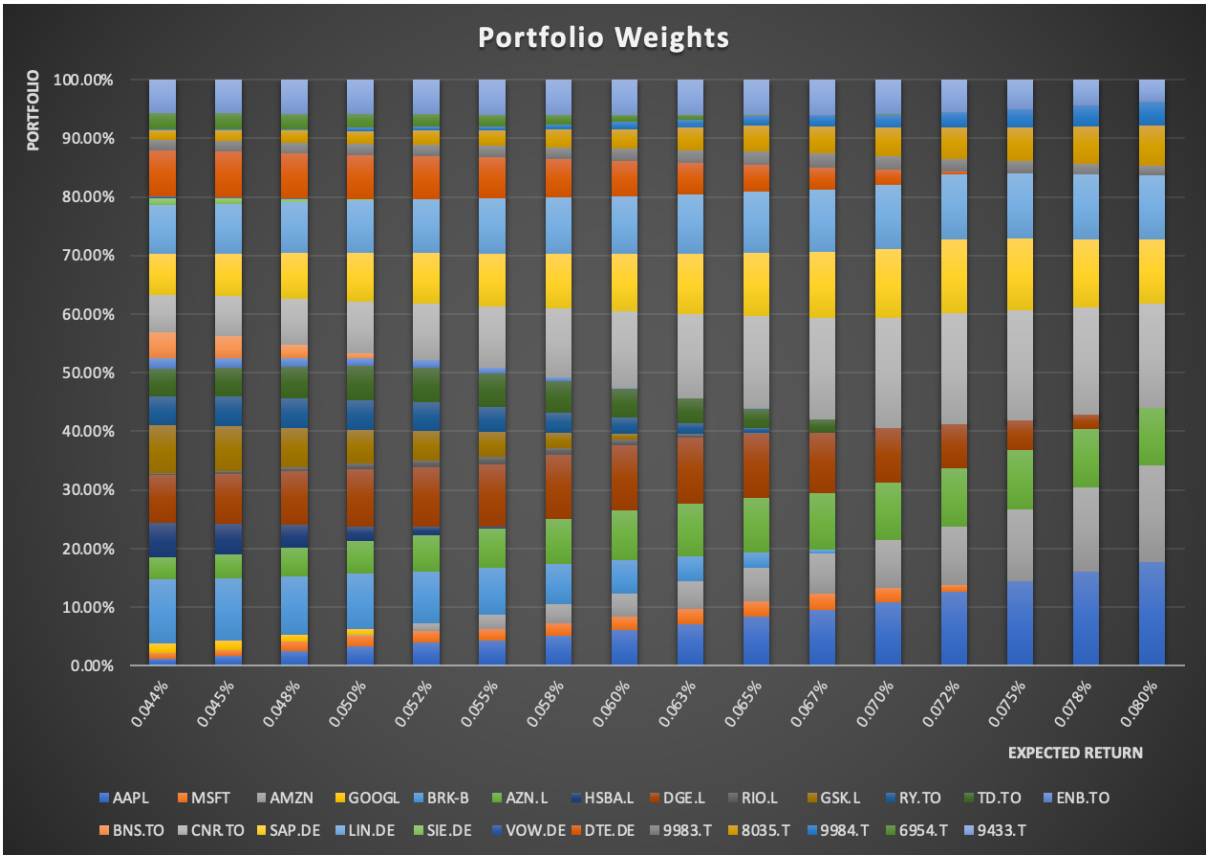

Figure 22: G5 Portfolio Weights

The chart above helps to see the weight of each stock more clearly in different expected returns. SAP.DE, LIN.DE and 9433.T have a constant weight regardless the expected return. On the other hand, it is evident to see the increasing weight of AAPL, AMZN and CNR.TO as long as the expected return increases.

5.2 Developing Countries

The developing countries selected to be part of this study are the BRICS, Brazil, Russia, India, China and South Africa. Inferences will be made for each one separately and a portfolio will be created at the end with the 5 stocks of each country evaluated together.

5.2.1 Brazil – Ibovespa

	VALE3.SA	PETR4.SA	BBDC4.SA	B3SA3.SA	ABEV3.SA
Daily Average Returns	0.0352%	-0.0010%	0.0303%	0.0312%	0.0482%
Standard Deviation	2.6746%	2.9814%	2.1873%	3.1175%	1.6627%
Number of obervations	2724				

Figure 23: Brazil Returns / Std. Deviation

The 5 stocks considered from Ibovespa index presented 2724 observations from the period. The results from the data collected showed a high standard deviation among the stocks. B3SA3.SA has the highest standard deviation, 3.12%, and it has an average return similar to stocks with far less volatility. Curiously, ABEV3.SA has the lowest standard deviation with the highest daily average return.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	VALE3.SA	PETR4.SA	BBDC4.SA	B3SA3.SA	ABEV3.SA
GMVP	1.5039%	0.0425%	13%	0%	15%	8%	64%
Efficient Frontier	1.5050%	0.0430%	13%	0%	13%	7%	67%
	1.5082%	0.0435%	13%	0%	11%	7%	70%
	1.5136%	0.0440%	12%	0%	8%	6%	73%
	1.5211%	0.0445%	12%	0%	6%	6%	76%
	1.5307%	0.0450%	12%	0%	4%	6%	79%
	1.5560%	0.0460%	11%	0%	0%	5%	84%
	1.5729%	0.0465%	9%	0%	0%	3%	88%
	1.5935%	0.0470%	8%	0%	0%	1%	91%
	1.6477%	0.0480%	2%	0%	0%	0%	98%

Figure 24: Brazil GMVP - Efficient Frontier

The GMVP presented an expected return of 0.043% and a standard deviation of 1.503%. However, in this scenario, ABEV3.SA occupied more than 60% of the weight in the portfolio, due to its high return with less volatility. The efficient frontier shows the increase of the expected return, which also increases the standard deviation and the participation of ABEV3.SA in the portfolio weight.

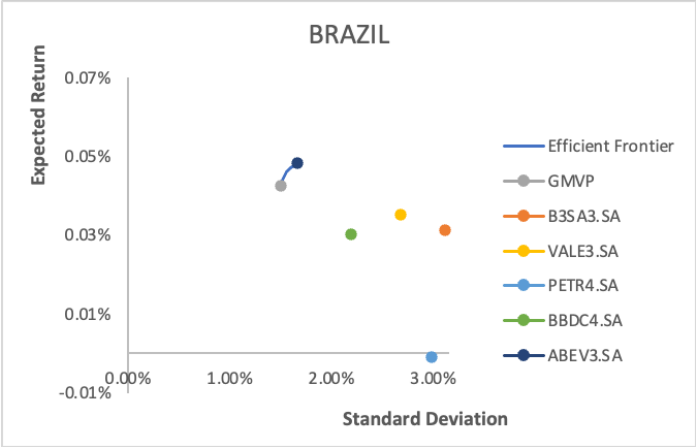

Figure 25: Brazil GMVP Chart

The chart illustrates perfectly how the minimum variance portfolio acts to bring the highest expected return with less risk. As previously mentioned, the stock ABEV3.SA presented a considerable result representing more than 60% of the GMVP. However, it demonstrated that mixed with a percentage of the other stocks, it brings a lower volatility.

5.2.2 Russia – MOEX

	GAZP.ME	LKOH.ME	GMKN.ME	ROSN.ME	PLZL.ME
Daily Average Returns	0.0316%	0.0672%	0.0937%	0.0370%	0.1004%
Standard Deviation	2.0163%	1.9413%	1.9692%	1.8508%	2.1766%
Number of obervations	2709				

Figure 26: Russia Returns / Std. Deviation

The stocks observed from MOEX index began in 04/03/2010 and finished in 31/12/2020. The reason of the first date is because Yahoo Finance does not provide the results prior that day. However, the 2709 observations is in line with the other stocks, which does not influence in the results. Polyus (PLZL.ME) shows the highest standard deviation and also the highest daily average return of the stocks. Is the opposite of presented by ABEV3.SA in the previous index analysis.

	Port Standard Deviation	Port Expected Return	Portfolio Weights				
			GAZP.ME	LKOH.ME	GMKN.ME	ROSN.ME	PLZL.ME
GMVP	1.3028%	0.0717%	12%	15%	23%	22%	28%
Efficient Frontier	1.3057%	0.0750%	8%	17%	26%	19%	30%
	1.3118%	0.0775%	6%	19%	28%	17%	31%
	1.3212%	0.0800%	3%	21%	30%	14%	32%
	1.3337%	0.0825%	1%	22%	32%	12%	33%
	1.3497%	0.0850%	0%	24%	34%	8%	34%
	1.3703%	0.0875%	0%	25%	36%	4%	36%
	1.3958%	0.0900%	0%	23%	39%	0%	38%
	1.4375%	0.0925%	0%	15%	43%	0%	42%
	1.4992%	0.0950%	0%	7%	48%	0%	46%

Figure 27: Russia GMVP - Efficient Frontier

The GMVP found in this analysis presented an expected return of 0.72% daily and a standard deviation of 1.30%. Furthermore, the distribution of weight is similar among the stocks, such as Gazprom (GAZP.ME) the lower representativeness of the portfolio. The construction of the efficient frontier shows the gradual increase of the expected return and standard deviation slightly until it reaches the 2.17% of volatility. The weights of Lukoil (LKOH.ME), Nornickel (GMKN.ME) and Polyus (PLZL.ME) are being increased when a higher return is expected.

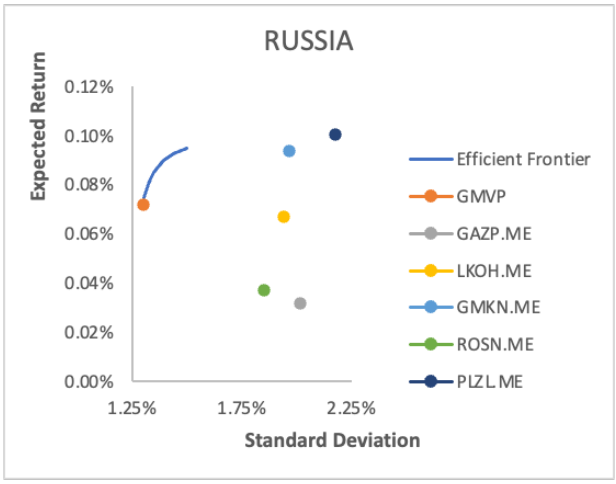

Figure 28: Russia GMVP Chart

The chart presents the location of each stock considering the standard deviation and return. The GMPV is located at the very left of the graph, demonstrating the lowest volatility. In addition, the return expected in the GMVP is below only two stocks, with the lowest standard deviation possible.

5.2.3 India – BSE Sensex

	RELIANCE.NS	HDFCBANK.NS	INFY.NS	HDFC.NS	ICICIBANK.NS
Daily Average Returns	0.0522%	0.0813%	0.0583%	0.0634%	0.0501%
Standard Deviation	1.8047%	1.4865%	1.7922%	1.8274%	2.1653%
Number of obervations	2709				

Figure 29: India Returns / Std. Deviation

The number of observations analysed in five stocks from BSE Sensex are 2709. HDFC Bank (HDFCBANK.NS) has the highest daily average return while Icici Bank (ICICIBANK.NS) has the highest standard deviation. It is plausible to say that the return and volatility of these 2 stocks are inversely proportional. The other three stocks present very similar results.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	RELIANCE.NS	HDFCBANK.NS	INFY.NS	HDFC.NS	ICICIBANK.NS
GMVP	1.2002%	0.0660%	20%	36%	30%	15%	0%
Efficient Frontier	1.2046%	0.0680%	16%	44%	28%	12%	0%
	1.2181%	0.0700%	12%	52%	27%	9%	0%
	1.2281%	0.0710%	10%	56%	26%	8%	0%
	1.2403%	0.0720%	8%	60%	25%	7%	0%
	1.2707%	0.0740%	4%	68%	24%	4%	0%
	1.2888%	0.0750%	2%	72%	23%	3%	0%
	1.3088%	0.0760%	0%	77%	22%	1%	0%
	1.3598%	0.0780%	0%	86%	14%	0%	0%
	1.4303%	0.0800%	0%	94%	6%	0%	0%

Figure 30: India GMVP - Efficient Frontier

The GMVP found an expected return of 0.066% with a standard deviation of 1.20%. Curiously, the Icici Bank (ICICIBANK.NS) does not have any participation in the GMVP or the efficient frontier, as it has a high volatility with low average return. HDFC Bank (HDFCBANK.NS) represents the highest weight, around 35% in the GMVP. As the expected return requires higher results, the volatility also increases to build up the efficient frontier.

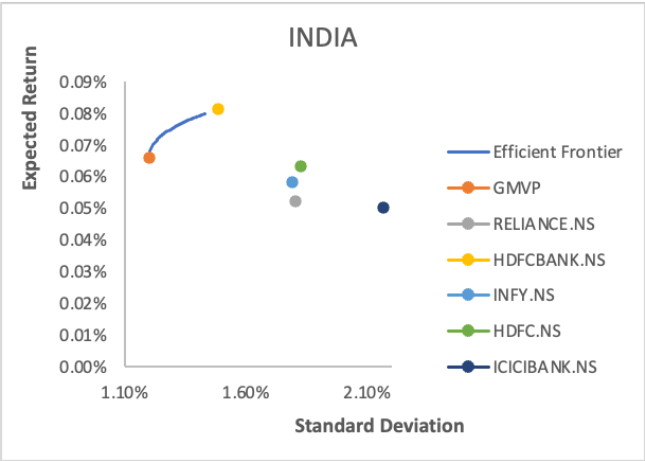

Figure 31: India GMVP Chart

The chart allows one to infer the discrepancy among the GMVP and the stocks individually. Also the position of Icici Bank (ICICIBANK.NS) in the chart clearly illustrates the reason that it is not a part of the weights in the minimum variance portfolio and the efficient frontier.

5.2.4 China – SSE

	600519.SS	601398.SS	600036.SS	601318.SS	601628.SS
Daily Average Returns	0.1089%	0.0167%	0.0505%	0.0499%	0.0136%
Standard Deviation	1.9438%	1.3509%	1.7861%	1.9753%	2.2291%
Number of obervations	2669				

Figure 32: China Returns / Std. Deviation

The number of observations used for the five stocks from China SSE index are 2669. China has the lowest number of observations among all the other countries analysed. However, it is still a considerable enough number to make inferences. China Life Insurance Company (601628.SS) has the highest standard deviation while Industrial and Commercial Bank of China (601398.SS) has the lowest. In contrast, the company with highest volatility has the worst performance in daily average return. China Life Insurance Company (601628.SS) delivers an average return of 0.01%. To the contrary, Kweichow Moutai Co. (600518.SS) has the highest average return with 0.11%.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	600519.SS	601398.SS	600036.SS	601318.SS	601628.SS
GMVP	1.2355%	0.0413%	27%	73%	0%	0%	0%
Efficient Frontier	1.2500%	0.0500%	35%	62%	2%	0%	0%
	1.2704%	0.0550%	40%	55%	5%	0%	0%
	1.2994%	0.0600%	45%	49%	6%	0%	0%
	1.3353%	0.0650%	48%	41%	11%	0%	0%
	1.3785%	0.0700%	53%	34%	13%	0%	0%
	1.4282%	0.0750%	57%	26%	16%	0%	0%
	1.4836%	0.0800%	62%	19%	19%	0%	0%
	1.6092%	0.0900%	70%	5%	25%	0%	0%
	1.7621%	0.1000%	85%	0%	15%	0%	0%

Figure 33: China GMVP - Efficient Frontier

The GMVP presented a result of 0.04% of expected return and 1.24% of standard deviation. However, it is the only analysis in which the GMVP consists of two of the five evaluated stocks. The efficient frontier over the increase of expected return adds the China Merchants Bank (600036.SS) to the portfolio.

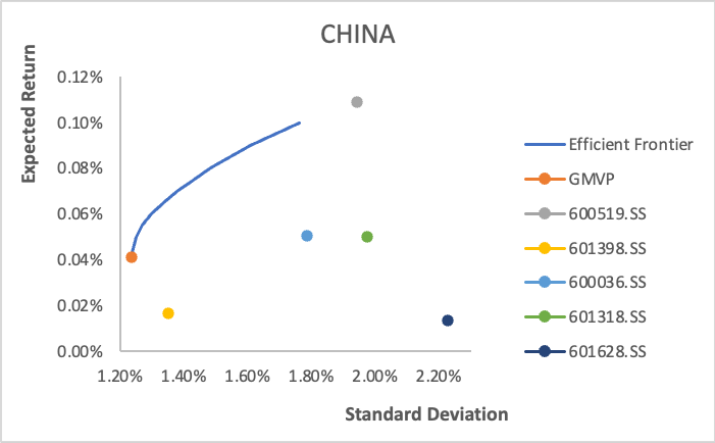

Figure 34: China GMVP Chart

The graph presents the lowest standard deviation by the global minimum variance portfolio and the efficient frontier towards the Kweichow Moutai Co. (600519.SS) which has the highest expected return. As long as the expected return increases, the standard deviation also increases and the weights of each company change in order to have access to the best result.

5.2.5 South Africa – JSE40

	BTI.JO	NPN.JO	BHP.JO	CFR.JO	AGL.JO
Daily Average Returns	0.0429%	0.1002%	0.0319%	0.0662%	0.0264%
Standard Deviation	1.4030%	2.0115%	1.9041%	1.6514%	2.3958%
Number of obervations	2831				

Figure 35: South Africa Returns / Std. Deviation

The South African stocks analysed in this study have 2831 observations. The results presented by the company Anglo American (AGL.JO) demonstrate the highest standard deviation, 2.39%, and the lowest daily average return, 0.02%. The Naspers (NPN.JO) has an average return of 0.10% which is 50% more than the results of any other stocks.

			Portfolio Weights				
	Port Standard Deviation	Port Expected Return	BTI.JO	NPN.JO	BHP.JO	CFR.JO	AGL.JO
GMVP	1.1233%	0.0559%	49%	17%	13%	20%	0%
Efficient Frontier	1.1299%	0.0600%	46%	22%	8%	23%	0%
	1.1554%	0.0650%	43%	28%	3%	26%	0%
	1.1797%	0.0680%	40%	32%	0%	28%	0%
	1.2012%	0.0700%	36%	36%	0%	28%	0%
	1.2751%	0.0750%	27%	44%	0%	29%	0%
	1.3739%	0.0800%	17%	53%	0%	30%	0%
	1.4927%	0.0850%	8%	61%	0%	31%	0%
	1.6274%	0.0900%	0%	70%	0%	30%	0%
	1.7953%	0.0950%	0%	85%	0%	15%	0%

Figure 36: South Africa GMVP - Efficient Frontier

The global minimum variance calculated showed an expected return of 0.056% and a standard deviation of 1.12%. Anglo American (AGL.JO) is not represented in the GMV portfolio and the remaining weights are distributed with 50% in British American Tobacco (BTI.JO) and the other 50% divided between the other three companies. The efficient frontier is basically compounded by three stocks, since the BHG Group (BHP.JO) decreases its participation sharply over the increase of expected returns.

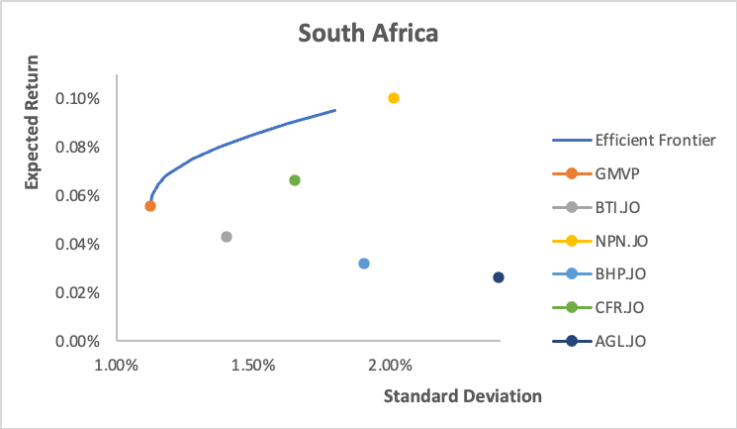

Figure 37: South Africa GMVP Chart

According to the chart, the GMVP presents an expected return in the middle of the highest and the lowest average returns of each stock. Furthermore, the diversification of the portfolio allows one to find a result that brings lower risk with a possible increase in the expected return.

5.2.6 BRICS

	VALE3.SA	PETR4.SA	BBDC4.SA	B3SA3.SA	ABEV3.SA	GAZP.ME	LKOH.ME	GMKN.ME	ROSN.ME	PLZL.ME	RELIANCE.NS	HDFCBANK.NS	INFY.NS	HDFC.NS	ICICIBANK.NS	600519.SS	601398.SS	600036.SS	601318.SS	601628.SS	BTI.JO	NPN.JO	BHP.JO	CFR.JO	AGL.JO
Daily Average Returns	0.0219%	-0.0144%	0.0182%	0.0265%	0.0428%	0.0216%	0.0603%	0.0870%	0.0320%	0.1051%	0.0540%	0.0754%	0.0531%	0.0543%	0.0397%	0.1089%	0.0167%	0.0505%	0.0499%	0.0136%	0.0532%	0.1070%	0.0242%	0.0607%	0.0118%
Standard Deviation	2.6876%	2.9855%	2.1739%	3.1323%	1.6411%	2.0181%	1.9348%	1.9771%	1.8477%	2.1686%	1.7987%	1.4809%	1.7939%	1.8230%	2.1619%	1.9438%	1.3509%	1.7861%	1.9753%	2.2291%	1.3789%	2.0030%	1.9126%	1.6241%	2.4122%
Number of Observations	2669																								

Figure 38: BRICS Returns / Std. Deviation

The analysis below consists the 25 stocks from the 5 developing countries. The data was collected from 01/01/2020 to 31/12/2020. However, in this scenario, the last days from Brazil, Russia, India and South Africa were omitted in order to keep the same number of observations for every stock. Therewith, the number of observations in the BRICS analysis was 2669. B3SA3.SA from Brazil presented the highest standard deviation, 3.13%, and Industrial and Commercial bank of China (601398.SS) the lowest, 1.35%. The highest daily average return is observed in Kweichow Moutai (600519.SS), 0.11%, and the lowest in PETR4.SA from Brazil, -0.014%.

	VALE3.SA	PETR4.SA	BBDC4.SA	B3SA3.SA	ABEV3.SA	GAZP.ME	LKOH.ME	GMKN.ME	ROSN.ME	PLZL.ME	RELIANCE.NS	HDFCBANK.NS	INFY.NS	HDFC.NS	ICICIBANK.NS	600519.SS	601398.SS	600036.SS	601318.SS	601628.SS	BTI.JO	NPN.JO	BHP.JO	CFR.JO	AGL.JO
VALE3.SA	1	0.4828	0.3992	0.2703	0.2763	-0.0052	-0.0265	-0.0090	0.0004	-0.0281	0.0922	0.0551	0.0321	0.0362	0.0572	-0.0414	-0.0107	-0.0148	-0.0032	-0.0008	0.0010	-0.0096	-0.0095	-0.0177	-0.0245
PETR4.SA	0.4828	1	0.5810	0.4013	0.3309	-0.0518	-0.0762	-0.0709	-0.0382	-0.0243	0.0694	0.0610	0.0292	0.0444	0.0762	-0.0238	-0.0301	-0.0375	-0.0531	-0.0754	-0.0008	0.0264	0.0272	0.0069	0.0358
BBDC4.SA	0.3992	0.5810	1	0.4450	0.4149	-0.0452	-0.0407	-0.0515	-0.0315	0.0013	0.0579	0.0427	0.0340	0.0205	0.0530	-0.0232	-0.0060	-0.0262	-0.0261	-0.0483	0.0041	0.0152	0.0026	-0.0068	0.0176
B3SA3.SA	0.2703	0.4013	0.4450	1	0.2378	-0.0258	-0.0211	-0.0117	-0.0118	0.0249	0.0581	0.0365	0.0366	0.0582	0.0508	0.0017	-0.0131	0.0103	0.0103	-0.0116	-0.0090	0.0232	0.0216	0.0118	0.0171
ABEV3.SA	0.2763	0.3309	0.4149	0.2378	1	-0.0112	-0.0060	-0.0246	-0.0112	-0.0011	0.0798	0.0280	-0.0129	-0.0053	0.0279	-0.0198	-0.0152	-0.0163	-0.0255	-0.0415	-0.0442	-0.0113	-0.0179	-0.0153	-0.0179
GAZP.ME	-0.0052	-0.0518	-0.0452	-0.0258	-0.0112	1	0.5630	0.3522	0.4811	0.1451	0.0251	0.0205	0.0367	0.0097	0.0514	0.0096	0.0091	0.0105	0.0041	0.0254	-0.0400	0.0155	0.0020	0.0140	-0.0102
LKOH.ME	-0.0265	-0.0762	-0.0407	-0.0211	-0.0060	0.5630	1	0.3150	0.5532	0.1362	0.0260	0.0388	0.0234	0.0096	0.0650	0.0040	0.0143	-0.0023	-0.0145	0.0013	-0.0351	0.0278	-0.0127	0.0307	-0.0132
GMKN.ME	-0.0090	-0.0709	-0.0515	-0.0117	-0.0246	0.3522	0.3150	1	0.3707	0.1408	0.0087	-0.0201	-0.0053	-0.0178	0.0145	-0.0179	0.0338	0.0248	0.0215	0.0551	-0.0273	0.0033	-0.0041	0.0116	-0.0059
ROSN.ME	0.0004	-0.0382	-0.0315	-0.0118	-0.0112	0.4811	0.5532	0.3707	1	0.1084	0.0350	0.0351	0.0162	0.0373	0.0628	0.0084	0.0143	0.0017	-0.0051	-0.0002	-0.0289	0.0152	-0.0202	0.0123	-0.0160
PLZL.ME	-0.0281	-0.0243	0.0013	0.0249	-0.0011	0.1451	0.1362	0.1408	0.1084	1	0.0095	-0.0389	0.0168	0.0043	0.0052	0.0077	0.0271	0.0461	0.0247	0.0377	-0.0177	-0.0023	-0.0475	0.0058	-0.0502
RELIANCE.NS	0.0922	0.0694	0.0579	0.0581	0.0798	0.0251	0.0260	0.0087	0.0350	0.0095	1	0.4423	0.2328	0.3279	0.4409	0.0068	-0.0103	-0.0174	-0.0108	-0.0160	-0.0263	-0.0188	-0.0108	-0.0018	-0.0263
HDFCBANK.NS	0.0551	0.0610	0.0427	0.0365	0.0280	0.0205	0.0388	-0.0201	0.0351	-0.0389	0.4423	1	0.2536	0.5580	0.5567	0.0228	-0.0200	-0.0183	-0.0212	-0.0177	-0.0102	-0.0148	0.0021	0.0072	-0.0150
INFY.NS	0.0321	0.0292	0.0340	0.0366	-0.0129	0.0367	0.0234	-0.0053	0.0162	0.0168	0.2328	0.2536	1	0.2159	0.2366	-0.0272	-0.0078	-0.0201	-0.0209	-0.0208	-0.0170	0.0031	-0.0070	0.0025	-0.0070
HDFC.NS	0.0362	0.0444	0.0205	0.0582	-0.0053	0.0097	0.0096	-0.0178	0.0373	0.0043	0.3279	0.5580	0.2159	1	0.4643	0.0199	-0.0003	-0.0004	-0.0026	0.0013	0.0079	-0.0149	-0.0005	0.0244	-0.0118
ICICIBANK.NS	0.0572	0.0762	0.0530	0.0508	0.0279	0.0514	0.0650	0.0145	0.0628	0.0052	0.4409	0.5567	0.2366	0.4643	1	0.0080	-0.0047	-0.0055	-0.0016	-0.0117	-0.0014	-0.0311	-0.0384	-0.0162	-0.0341
600519.SS	-0.0414	-0.0238	-0.0232	0.0017	-0.0198	0.0096	0.0040	-0.0179	0.0084	0.0077	0.0068	0.0228	-0.0272	0.0199	0.0080	1	0.2690	0.3530	0.4035	0.3453	-0.0285	0.0102	-0.0064	0.0110	0.0157
601398.SS	-0.0107	-0.0301	-0.0060	-0.0131	-0.0152	0.0091	0.0143	0.0338	0.0143	0.0271	-0.0103	-0.0200	-0.0078	-0.0003	-0.0047	0.2690	1	0.7022	0.5785	0.5471	0.0134	0.0225	0.0294	0.0142	0.0152
600036.SS	-0.0148	-0.0375	-0.0262	0.0103	-0.0163	0.0105	-0.0023	0.0248	0.0017	0.0461	-0.0174	-0.0183	-0.0201	-0.0004	-0.0055	0.3530	0.7022	1	0.7117	0.6291	0.0074	0.0187	0.0258	0.0144	0.0280
601318.SS	-0.0032	-0.0531	-0.0261	0.0103	-0.0255	0.0041	-0.0145	0.0215	-0.0051	0.0247	-0.0108	-0.0212	-0.0209	-0.0026	-0.0016	0.4035	0.5785	0.7117	1	0.7644	0.0006	0.0028	-0.0026	0.0036	0.0149
601628.SS	-0.0008	-0.0754	-0.0483	-0.0116	-0.0415	0.0254	0.0013	0.0551	-0.0002	0.0377	-0.0160	-0.0177	-0.0208	0.0013	-0.0117	0.3453	0.5471	0.6291	0.7644	1	0.0038	-0.0018	0.0025	0.0113	0.0208
BTI.JO	0.0010	-0.0008	0.0041	-0.0090	-0.0442	-0.0400	-0.0351	-0.0273	-0.0289	-0.0177	-0.0263	-0.0102	-0.0170	0.0079	-0.0014	-0.0285	0.0134	0.0074	0.0006	0.0038	1	0.1525	0.2075	0.3116	0.1260
NPN.JO	-0.0096	0.0264	0.0152	0.0232	-0.0113	0.0155	0.0278	0.0033	0.0152	-0.0023	-0.0188	-0.0148	0.0031	-0.0149	-0.0311	0.0102	0.0225	0.0187	0.0028	-0.0018	0.1525	1	0.3435	0.3187	0.3167
BHP.JO	-0.0095	0.0272	0.0026	0.0216	-0.0179	0.0020	-0.0127	-0.0041	-0.0202	-0.0475	-0.0108	0.0021	-0.0070	-0.0005	-0.0384	-0.0064	0.0294	0.0258	-0.0026	0.0025	0.2075	0.3435	1	0.4471	0.7955
CFR.JO	-0.0177	0.0069	-0.0068	0.0118	-0.0153	0.0140	0.0307	0.0116	0.0123	0.0058	-0.0018	0.0072	0.0025	0.0244	-0.0162	0.0110	0.0142	0.0144	0.0036	0.0113	0.3116	0.3187	0.4471	1	0.3677
AGL.JO	-0.0245	0.0358	0.0176	0.0171	-0.0179	-0.0102	-0.0132	-0.0059	-0.0160	-0.0502	-0.0263	-0.0150	-0.0070	-0.0118	-0.0341	0.0157	0.0152	0.0280	0.0149	0.0208	0.1260	0.3167	0.7955	0.3677	1

Figure 39: BRICS Correlation Matrix

The correlation matrix was generated to evaluate the correlation of the stocks in pairs. As previous cited in the G5 portfolio, the BRICS portfolio does not show a strong correlation among stocks from different countries. When compared by country, the stocks presents a weak, though positive, correlation. In general the stocks are positively correlated. However, the correlation is very weak, where the majority presents a number close to 0.

			Portfolio Weights																						
	Port Standard Deviation	Port Expected Return	VALE3.SA	PETR4.SA	BBDC4.SA	B3SA3.SA	ABEV3.SA	GAZP.ME	LKOH.ME	GMKN.ME	ROSN.ME	PLZL.ME	RELIANCE.NS	HDFCBANK.NS	INFY.NS	HDFC.NS	ICICIBANK.NS	600519.SS	601398.SS	600036.SS	601318.SS	601628.SS	BTI.JO	NPN.JO	BHP.JO	CFR.JO	AGL.JO
GMVP	0.5571%	0.0550%	1.62%	0.00%	2.28%	0.35%	10.31%	2.08%	2.55%	4.90%	3.66%	5.23%	3.10%	8.29%	6.69%	2.69%	0.00%	5.92%	13.64%	0.00%	0.54%	0.00%	14.64%	3.66%	2.86%	3.96%	1.01%
Efficient Frontier	0.5579%	0.0575%	1.56%	0.00%	2.03%	0.31%	10.33%	1.49%	2.93%	5.44%	3.21%	5.66%	2.96%	8.90%	6.66%	2.48%	0.00%	6.66%	12.32%	0.00%	0.72%	0.00%	14.63%	4.37%	2.59%	4.08%	0.68%
	0.5607%	0.0600%	1.45%	0.00%	1.77%	0.30%	10.35%	1.21%	3.03%	6.02%	2.62%	6.16%	2.80%	9.66%	6.60%	2.28%	0.00%	7.51%	10.94%	0.03%	0.73%	0.00%	14.56%	5.14%	2.11%	4.06%	0.67%
	0.5650%	0.0625%	1.42%	0.00%	1.55%	0.22%	10.32%	0.24%	3.67%	6.55%	2.25%	6.55%	2.67%	10.28%	6.59%	2.01%	0.00%	8.14%	9.74%	0.03%	1.00%	0.00%	14.50%	5.83%	2.11%	4.32%	0.00%
	0.5713%	0.0650%	1.36%	0.00%	1.25%	0.16%	10.33%	0.00%	3.89%	7.09%	1.63%	7.02%	2.53%	10.99%	6.53%	1.77%	0.00%	8.95%	8.36%	0.03%	1.12%	0.00%	14.47%	6.60%	1.47%	4.44%	0.00%
	0.5794%	0.0675%	1.26%	0.00%	0.95%	0.16%	10.32%	0.00%	3.99%	7.62%	0.93%	7.48%	2.39%	11.76%	6.45%	1.48%	0.00%	9.79%	6.94%	0.04%	1.22%	0.00%	14.43%	7.39%	0.79%	4.61%	0.00%
	0.5893%	0.0700%	1.20%	0.00%	0.62%	0.13%	10.32%	0.00%	4.16%	8.12%	0.22%	7.95%	2.21%	12.48%	6.37%	1.29%	0.00%	10.60%	5.44%	0.04%	1.42%	0.00%	14.42%	8.20%	0.11%	4.69%	0.00%
	0.6011%	0.0725%	1.06%	0.00%	0.36%	0.07%	10.25%	0.00%	4.01%	8.60%	0.00%	8.52%	2.00%	13.36%	6.26%	0.91%	0.00%	11.56%	3.58%	0.06%	1.61%	0.00%	14.27%	8.99%	0.00%	4.51%	0.00%
	0.6151%	0.0750%	0.89%	0.00%	0.20%	0.06%	10.05%	0.00%	3.77%	9.04%	0.00%	9.10%	1.90%	14.19%	6.10%	0.55%	0.00%	12.55%	1.39%	0.07%	1.97%	0.00%	14.06%	9.80%	0.00%	4.30%	0.00%
	0.6310%	0.0775%	0.64%	0.00%	0.00%	0.00%	9.82%	0.00%	3.42%	9.53%	0.00%	9.76%	1.63%	15.20%	5.90%	0.09%	0.00%	13.70%	0.00%	0.21%	1.67%	0.00%	13.73%	10.70%	0.00%	3.99%	0.00%
	0.6496%	0.0800%	0.10%	0.00%	0.00%	0.00%	9.17%	0.00%	2.81%	10.13%	0.00%	10.59%	1.08%	16.21%	5.41%	0.01%	0.00%	15.14%	0.00%	1.06%	0.00%	0.00%	12.95%	11.93%	0.00%	3.40%	0.00%
	0.6723%	0.0825%	0.07%	0.00%	0.00%	0.00%	8.10%	0.00%	2.17%	10.75%	0.00%	11.54%	0.24%	17.31%	4.84%	0.01%	0.00%	16.53%	0.00%	0.44%	0.00%	0.00%	11.93%	13.32%	0.00%	2.76%	0.00%
	0.6983%	0.0850%	0.00%	0.00%	0.00%	0.00%	6.90%	0.00%	1.44%	11.37%	0.00%	12.52%	0.03%	18.12%	4.09%	0.01%	0.00%	17.88%	0.00%	0.00%	0.00%	0.00%	10.82%	14.79%	0.00%	2.02%	0.00%
	0.7280%	0.0875%	0.00%	0.00%	0.00%	0.00%	5.47%	0.00%	0.70%	11.99%	0.00%	13.54%	0.03%	18.82%	3.21%	0.01%	0.00%	19.17%	0.00%	0.00%	0.00%	0.00%	9.52%	16.39%	0.00%	1.15%	0.00%
	0.7610%	0.0900%	0.00%	0.00%	0.00%	0.00%	4.07%	0.00%	0.00%	12.59%	0.00%	14.55%	0.03%	19.53%	2.36%	0.01%	0.00%	20.45%	0.00%	0.00%	0.00%	0.00%	8.24%	18.01%	0.00%	0.15%	0.00%
	0.7972%	0.0925%	0.00%	0.00%	0.00%	0.00%	2.31%	0.00%	0.00%	12.95%	0.00%	15.56%	0.03%	20.15%	1.23%	0.01%	0.00%	21.79%	0.00%	0.00%	0.00%	0.00%	6.35%	19.49%	0.00%	0.14%	0.00%
	0.8364%	0.0950%	0.00%	0.00%	0.00%	0.00%	0.54%	0.00%	0.00%	13.29%	0.00%	16.57%	0.03%	20.56%	0.54%	0.01%	0.00%	23.16%	0.00%	0.00%	0.00%	0.00%	4.16%	21.01%	0.00%	0.13%	0.00%
	0.8788%	0.0975%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	13.45%	0.00%	17.69%	0.00%	20.35%	0.00%	0.00%	0.00%	24.73%	0.00%	0.00%	0.00%	0.00%	0.93%	22.85%	0.00%	0.00%	0.00%
	0.9309%	0.1000%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.13%	0.00%	19.62%	0.00%	14.92%	0.00%	0.00%	0.00%	27.86%	0.00%	0.00%	0.00%	0.00%	0.00%	25.46%	0.00%	0.00%	0.00%
	1.0009%	0.1025%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	10.43%	0.00%	21.77%	0.00%	8.19%	0.00%	0.00%	0.00%	31.38%	0.00%	0.00%	0.00%	0.00%	0.00%	28.24%	0.00%	0.00%	0.00%
	1.0858%	0.1050%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	8.72%	0.00%	23.91%	0.00%	1.44%	0.00%	0.00%	0.00%	34.91%	0.00%	0.00%	0.00%	0.00%	0.00%	31.02%	0.00%	0.00%	0.00%
	1.2140%	0.1075%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.00%	0.00%	0.00%	0.00%	0.00%	0.00%	46.66%	0.00%	0.00%	0.00%	0.00%	0.00%	33.34%	0.00%	0.00%	0.00%

Figure 40: BRICS GMVP - Efficient Frontier

The global minimum variance portfolio considering the 25 stocks presented an expected return of 0.055% and a standard deviation of 0.5571%. Regarding the 25 stocks for 5 different countries, the result allows to infer that the standard deviation reached by the GMVP is less than a half that of the country which has the lowest if analysed individually. The addition of more options to the portfolio increases the possibility of find a lowest volatility. British American Tobacco (BTI.JO), from South Africa, presents the highest weight in the minimum variance portfolio, being responsible of 14.64%, followed by Industrial and Commercial Bank of China (601398.SS), with 13.64%. Petrobras (PETR4.SA), Icici Bank (ICICIBANK.NS) and China Merchants Bank (60036.SS) does not have any weight in the portfolio of the global minimum variance.

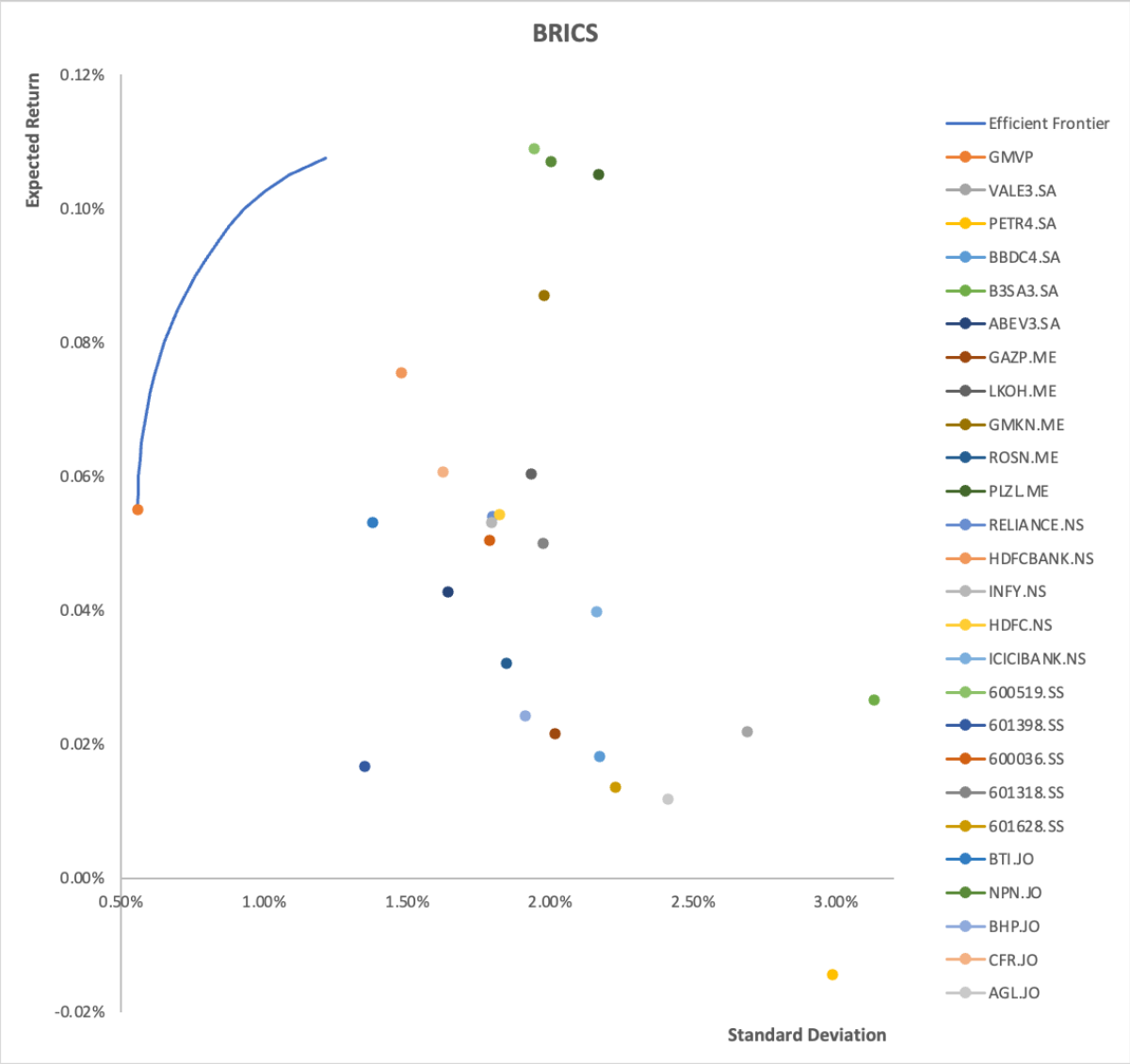

Figure 41: BRICS GMVP Chart

The chart allows to analyse that with the lowest standard deviation among the 25 stocks included, the global minimum variance portfolio has an expected return higher than the majority of the daily average returns. Also it is possible to infer that the efficient frontier increases the expected return, maintaining a standard deviation lowest than the stocks.

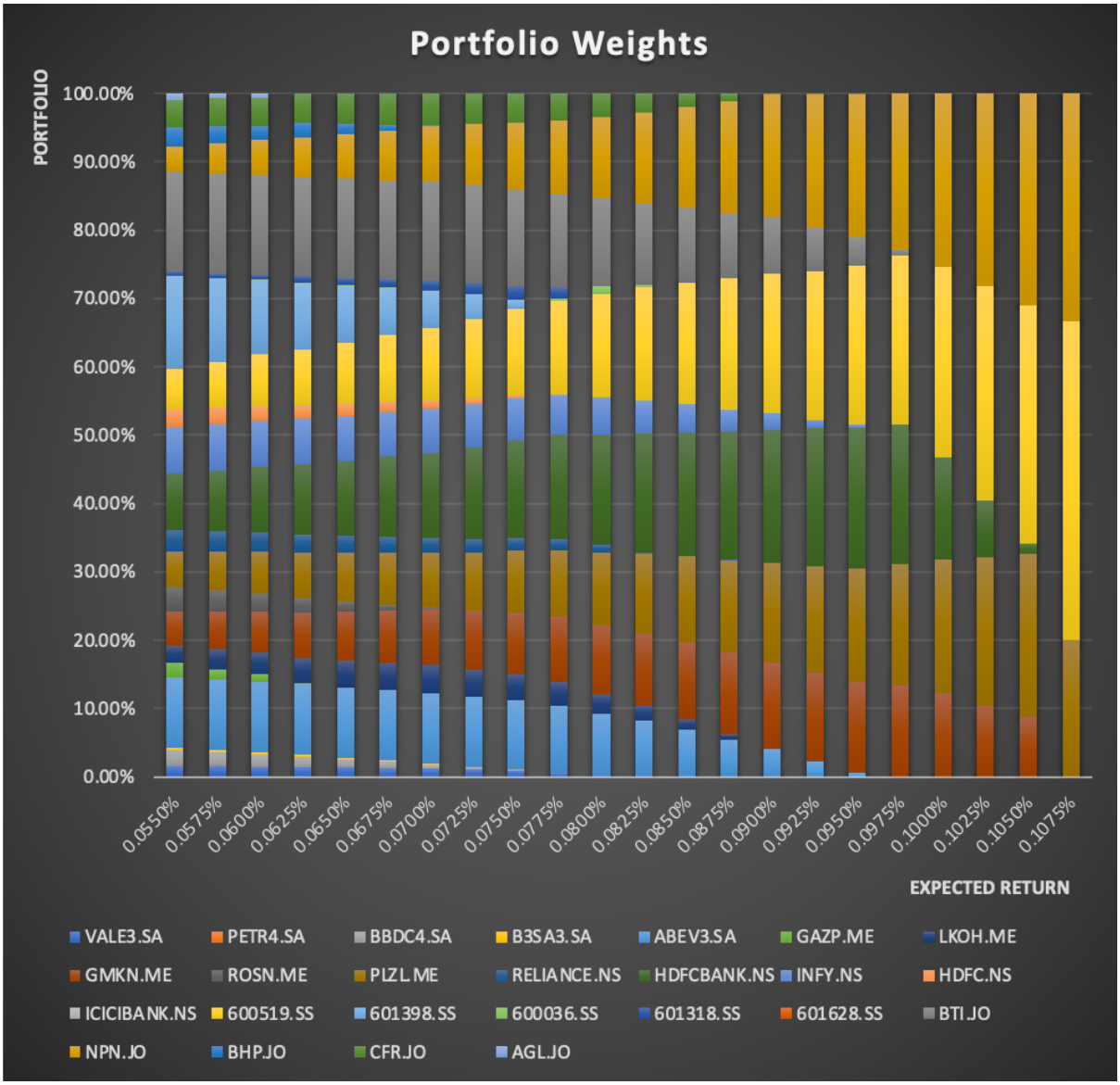

Figure 42: BRICS Portfolio Weights

The chart illustrates the weight of each stock over the increase of the expected return. The first expected return, 0.055% is from the GMVP and presents a well balance division among the stocks. As the expectation of the returns grow, determined stocks become predominant and its weight are more apparent.

5.3 G5 + BRICS

The G5 + BRICS analysis consist in 50 stocks from the ten countries previous stated individually and in their specific groups. The time frame used range from 01/01/2010 to 31/12/2020. However, to guarantee that the same number of observations have been done for the 50 stocks, all the stocks started its data in 01/03/2010, since the Russia has no previous data

in Yahoo Finance. Moreover, the last day is not the same for the stocks in order to achieve the same number of observations, in this case, 2669.

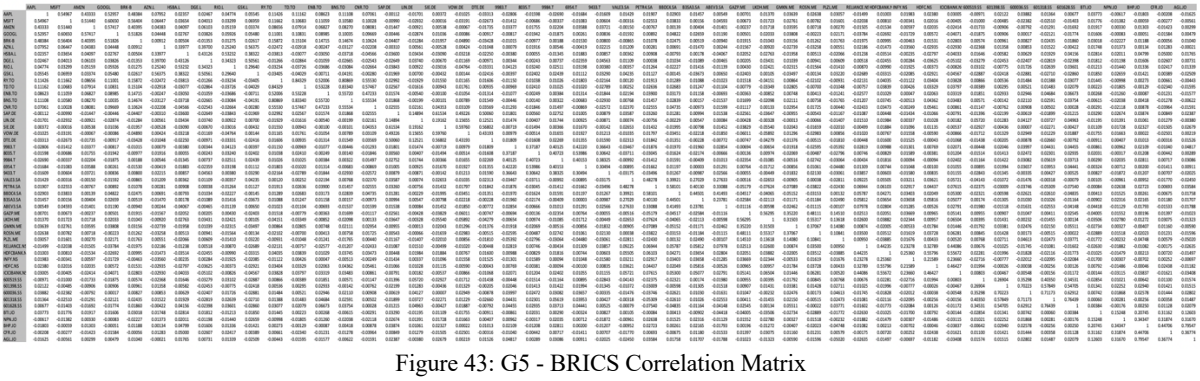

Figure 43: G5 - BRICS Correlation Matrix

The correlation among the 50 stocks are not strong and when compared to shares from different countries the majority presents a result below 0.01. However, even though the correlation is not strong, the curious fact is that stocks from developed countries presented a negative correlation to developing countries.

	Port Standard Deviation	Port Expected Return
G5-BRICS GMVP	0.3745%	0.0489%
	0.3747%	0.0500%
	0.3757%	0.0525%
	0.3783%	0.0550%
	0.3820%	0.0575%
	0.3868%	0.0600%
	0.3918%	0.0625%
	0.3991%	0.0650%
	0.4079%	0.0675%
	0.4183%	0.0700%
	0.4301%	0.0725%
	0.4439%	0.0750%
	0.4609%	0.0775%
	0.4802%	0.0800%
	0.5036%	0.0825%
	0.5287%	0.0850%
	0.5578%	0.0875%
	0.5876%	0.0900%
	0.6225%	0.0925%
	0.6580%	0.0950%
	0.6996%	0.0975%
	0.7408%	0.1000%
	0.7885%	0.1025%
	0.8404%	0.1050%
	0.9028%	0.1075%
	0.9818%	0.1100%
	1.4945%	0.1150%

Efficient Frontier

Figure 44: G5 - BRICS GMVP - Efficient Frontier

The Global Minimum Variance Portfolio reached considering the 50 stocks from the 10 countries analysed in this section presents an expected return daily of 0.049% with a standard deviation of 0.375%. The five weightiest companies in the GMVP are Industrial and Commercial Bank of China (601398.SS), from China, with 6.42%, British American Tobacco (BTI.JO), from South Africa, with 6.17%, Berkshire Hathaway (BRK.B), from the USA, with 5.69%, Glaxosmithkline Plc (GSK.L), from UK, with 5.22% and Ambev (ABEV3.SA), from

Brazil, with 4.84%. The five weightiest stocks presents a miscellaneous of developing and developed countries and are responsible for almost 30% of the portfolio. Furthermore, most of the stocks have its participation in the portfolio, except for 6 stocks, being 4 from China.

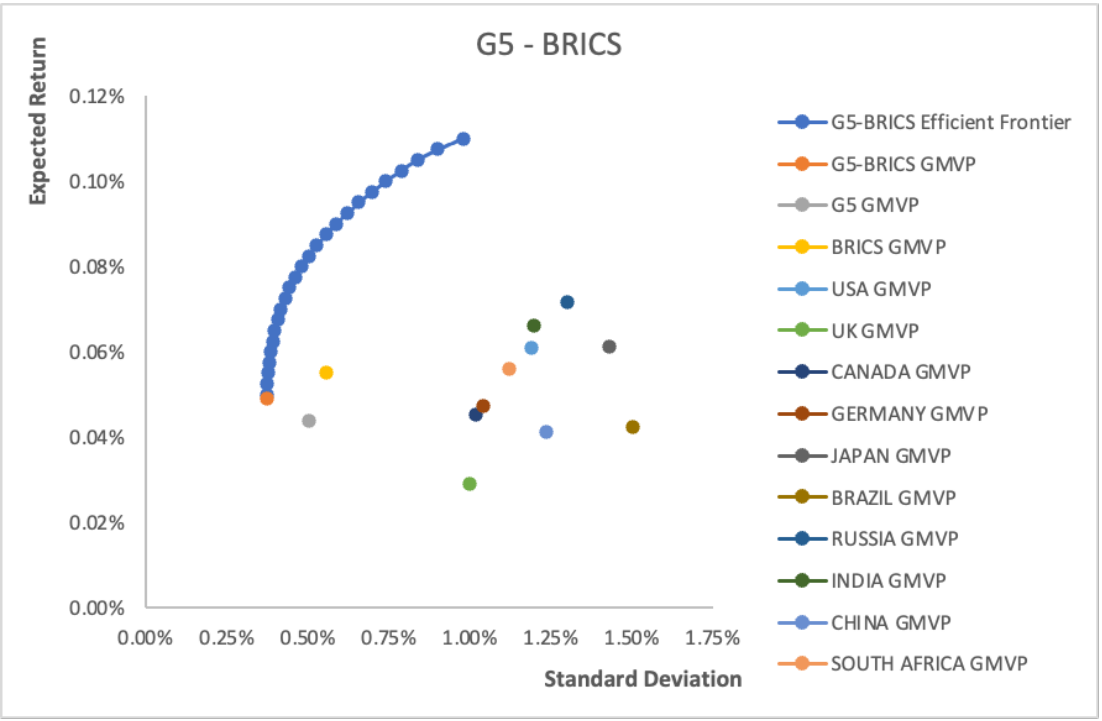

Figure 45: G5 - BRICS GMVP Chart

The chart above presents clearly the propose of the global minimum variance portfolio and the importance of diversification. The combination of G5 and BRICS stocks brings a considerable reduction to the standard deviation of the portfolio, achieving an expected return above G5 and close to BRICS. Considering the same standard deviation of G5 GMVP, 0.504%, the portfolio of 50 stocks would achieve an expected return close to 0.083%, against 0.044% found by G5. It is almost the double of the return, assuming the same risk. On the other hand, if assuming the standard deviation of BRICS GMVP, 0.557%, the expected return would be 0.088%, against 0.055% found. The efficient frontier from the minimum standard deviation until the highest risk assumed delivery an expected return much higher than analysed before.

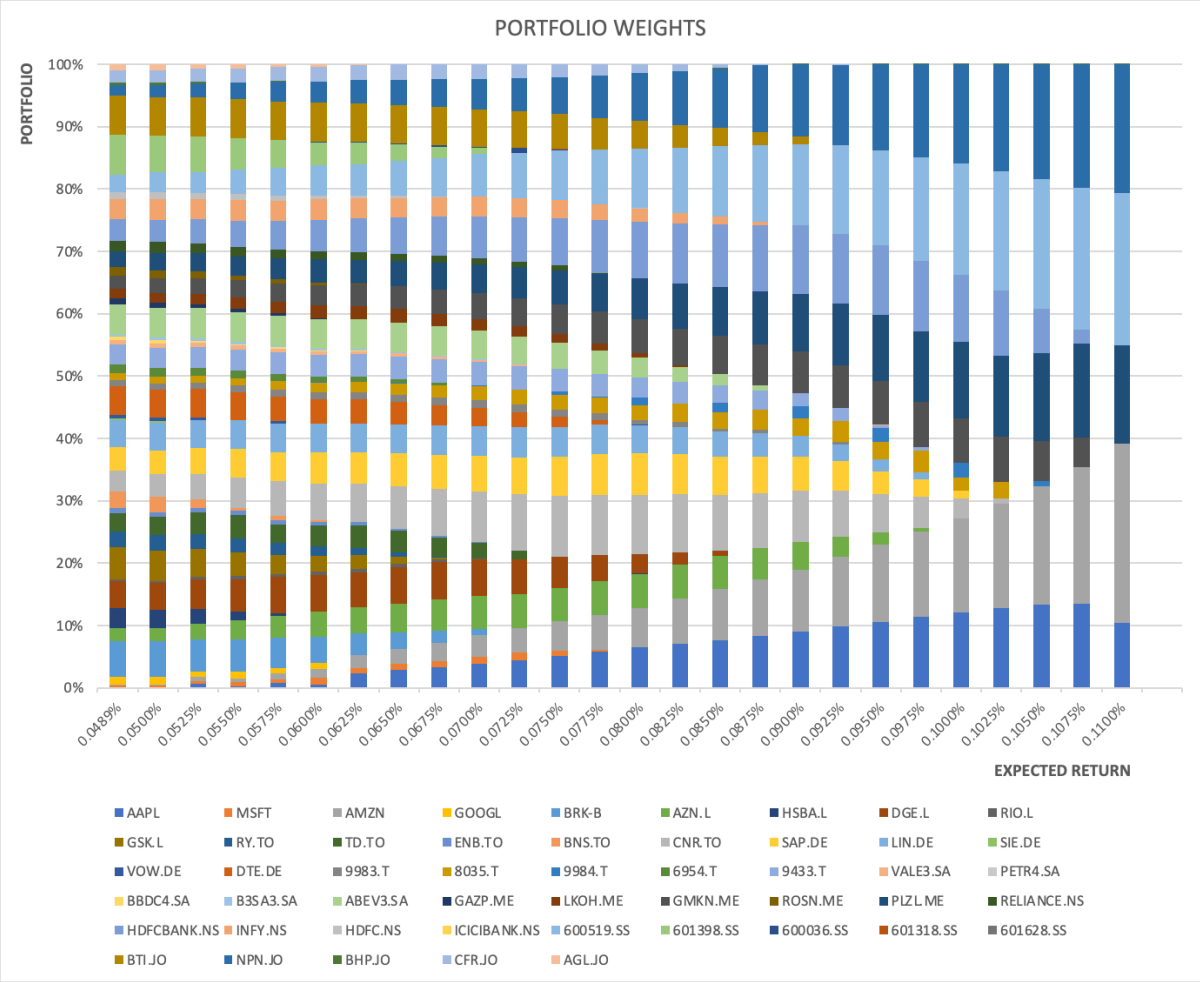

Figure 46: G5 - BRICS Portfolio Weights

The first expected return, which is the GMVP presents an homogeneous spread of weight among the stocks. As higher as the expected return goes few stocks assume a higher weight in order to achieve the return. It is visually possible to see the growth of weight among the expected returns of AAPL, PLZL.ME, 600519.SS and NPN.JO. These four stocks have the highest returns and for this reason increase their participation with the increase of the expected returns.

6. Conclusion

6.1 General Conclusion

The objective of this study was to achieve the global minimum variance portfolio (GMVP) from 5 developed countries and 5 developing countries, considering 5 stocks inserted in the main index of each country. Moreover, in order to more clearly illustrate how the GMVP behaves, they were also calculated individually as well as in groups. The data was collected for a 10 year period, from 01/01/2010 to 31/12/2020, to gather as many observations as possible and to reliably present the daily expected return and the daily standard deviation of each stock.

Individually by country, and in groups, the GMVPs were sorted considering the average daily returns of the stocks and the standard deviation over the period analysed. In addition, the correlation matrices were developed to evaluate whether there were strong or weak correlation between the stocks. Furthermore, the portfolio was created in order to achieve the weights and the volatility vector of each stock. In conclusion, a calculation has been made to find the portfolio standard deviation and the portfolio expected return. The GMVP has been highlighted by the lowest standard deviation of the efficient frontier.

Finally, the GMVP results of each of the scenarios were plotted in the same graph to allow to infer that as many as the number of assets included in the portfolio, the lowest the risk that may be found. In this study, considering stocks specifically, the daily average returns and standard deviation varied in many manners. However, the global minimum variance portfolio with the lowest standard deviation among the 13 portfolios created was that which included all of the countries. Even not all of the stocks were weighted in the portfolio with the lowest standard deviation, every country had its participation included.

The results in the research allow one to answer the research questions stated above. According to the findings, investing the wealth in the stock market, internationally, brought less risk and higher expected return to the portfolio. In addition, considering that the G5 – BRICS analysis gathered 50 stocks, the efficient frontier has been improved when compared to all other countries individually and in groups.

Markowitz stated in his model that there is not a specific number of assets in a portfolio which leads to decrease the risk and increase the expected return. However, he said that spreading the wealth across as many assets as possible will be beneficial to the risk averse profile that want

to diminish the volatility of the portfolio. The final result showed that considering 50 stocks, 44 were in the portfolio participation to achieve the minimum variance of the efficient frontier.

6.2 Recommendation for Further Studies

The number of stocks considered in this study was limited by 5 per country. In addition, 10 countries, among developed and developing economies, were cited in this research to achieve the findings. Other studies may choose more countries and more stocks to the analysis in order to find different perspectives of global minimum variance portfolio.

Although creating a portfolio with many assets is not feasible for individually investors due to costs, access to products, platforms, location, if considered by institutions, asset managers and investment managers, the higher the number of products, the lowest the volatility.

In addition, a different financial sector could be used for further research, ie: options, bonds, cryptocurrencies, currencies, real state. Or even a mix of the sectors by country considering an investment in different markets.

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