The Value at Risk Models in Times of Financial Crisis: Case Study of an Irish Equity Portfolio.

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Abstract

Value at Risk (VaR) is a risk measurement technique, that measures the risk associated with a portfolio at a given level of confidence for a certain time frame. It refers to maximum loss to a certain degree of confidence. It is widely accepted risk management tool and the use of the same has been made mandatory by the 'Basel Committee of Banking Supervision'. The aim of the Value at Risk is to measure the risk associated with a portfolio so as to enable investors about the risk associated to their investment.

The research therefore was conducted to investigate which VaR model Variance Covariance, Historical Simulation and Monte Carlo Simulation would best suit for an investor investing in an Irish equity portfolio. To ensure the validity of the models three back testing methods z, Kupiec and Christoffersen tests were implemented. The time frame constituted 11 years which include the Global Financial Crisis of 2008.

A model would be deemed best only if it could adopt to the changing market environment. The results however concluded that known of the models out of the three survived in extreme volatile period, although performed reasonable well during normal market scenario.

Keywords: Variance Covariance, Historical Simulation and Monte Carlo Simulation, Back Testing, Equity Portfolio.

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CHAPTER 1

Introduction

Risk Management that was once a secondary consideration is now a priority not only for the credit corporations and financial institutions but is also considered as a significant issue in the management of the business across the world (Kazlauskienė and Christauskas, 2007). An essential responsibility of the risk management is to minimise the chances of occurrence of losses (Zigid and Hadzic, 2012). In the era of globalization, there has been considerable growth in the financial services industry, markets are more connected than ever. While aggregate volatility appears to be lower, there has been sharp increase in volatility over last 20 years. There have been notable spikes such as Asian Crisis of 1997, Financial Crisis of 2008 and the very recent that occurred is COVID in early 2020.

The promotion of the capital markets has empowered the immense development in the growth and turnover of the stock market. Something that was once only confined to the banks and financial institution is now available to the general public domain, thus expanding the exposure of investments to their associated risks.

Capital markets across the world are now more connected. From 1980 onwards up till 2011 there has been a rise from 3% to almost around 30 - 32% in the international equity portfolio alone as a part of the global (GDP)Gross Domestic Product (Brusa et al., 2014). Thus, in present time investors are significantly concern with managing risk appropriately especially after the financial crisis of 2008. The benchmark tool to measure such risks is “Value at Risk” which is defined as *“it summarizes the worst loss over a target horizon that will not be exceeded with a given level of confidence”* (Jorion, 2007, p.17).

According to Hull (2015) Value at Risk (VaR) is an estimated single figure of total risk that is associated with the portfolio comprising financial assets. It may be used to quantify the market risk on a portfolio of a variety of asset types. Thus, VaR can be said as a measure of volatility. Christoffersen et al. (2001) through their paper mentioned that Value at Risk was introduced as one of the risk metric tool which could summarize entire loss in a single figure, this tool was largely found helpful by the investors and thus was considered as a benchmark quantitative technique of measuring risk.

According to Jorion (2007) the main aim behind the concept of VaR was to know the possible total risk of the portfolio as it represents leverage as well as diversification impacts. Initially the concept was limited in terms of measuring only the market risk but now it extends to measuring credit risk, operational risk as well as enterprise wide risk.

Effective management of risk has now been made obligatory by the "Basel Committee for banking supervision" which requires, the financial institutions to use Value at Risk so that these institutions can meet capital requirements as per the estimates made using the VaR model. Therefore, industry experts depend on VaR as a robust and coherent measure of risk. (Kellner and Rösch, 2016).

There are three types of models to calculate value at risk. Variance Covariance (parametric), Historical Simulation (non-parametric) and Monte Carlo Simulation.

Hull (2006) described Variance Covariance method which computes the standard deviation of price changes in any given investment by taking into account the normal distribution of stock returns at a specified confidence level to ascertain the maximum loss on a single day.

The Historical Simulation is based on the historical returns to ascertain the price changes in a portfolio. Historic simulation is limited insofar as the only possible risk factor changes that are considered are those that have already occurred.

Monte Carlo method involves generating random numbers to assess the returns of portfolio for a stated specified period. Monte Carlo techniques are a class of computational calculations that are based on rehashed calculation and random sampling.

The VaR models are proved to be beneficial only if they have the capacity to estimate correct risk figure. Thus, to authenticate the produced figure in terms of consistency, accuracy and reliability and further to decide as to which model out of the three displays most valid projections. These models are back tested to ascertain the best one for an Irish stock exchange. According to Halilbegovic and Vehabovic

(2016) back testing refers to method of comparing the calculated VaR estimates to the actual profits and losses.

Back testing generally should be carried out using various tests, mainly to affirm the accuracy of these models. Essential tests are utilized to check the total number of instances whereby the losses that have surpassed the estimated VaR. Proportion of failure models, compare number of times the realized losses exceed the VaR values for a given portfolio. If the number of losses which exceed the VaR values is consistent with the confidence level of the VaR model, the model is deemed to back-test appropriately. For the same, three different methods are used which include z test, Kupiec and Christoffersen.

This research intends to study the three models Variance Covariance, Historical Simulation and Monte Carlo Simulation and the various back-testing methods. For the purpose, three VaR models Variance Covariance, Historical Simulation and Monte Carlo Simulation are built, which are then back tested. The models would determine the amount of loss that can occur in one day as single value for an equity portfolio. These models would run over a time period of approximately 11 years.

Thus, the research aims to make the reader aware of the concept and practice of Value at Risk models and various back testing techniques in detail. Various authors are divided on opinion of selection of the model for various portfolio of assets. The author aims to investigate the suitability of the model for an Irish equity portfolio and select the best one out of the three models. The focus would particularly be on the performance of the models during the financial crisis. Ireland was one of the nations that was severely affected by the financial turbulence of 2008 and thus is significant to study and select the best model which could be put to use by the banks and the financial institutions in Ireland to deal with the modernized financial perspective. This certainly can be defined as an experimental study.

The objective of the research is to construct the three VaR models and test them to evaluate and assess at 95% confidence level, which model best suits for the Irish equity portfolio pre, during and post the time of financial crisis. The models are back tested using Z, Kupiec and Christoffersen tests. The time frame considered for the research is approximately 11 years beginning from 1st of January, 2002 to 31st of December, 2013.

The methodology adopted is quantitative and the models are built, tested and back tested using Microsoft Excel and the same is also performed in R-Studio.

Cheug & Powell (2012) said that using excel to build the three models would of great advantage to both the students as well as teachers.

The thesis is structured as follows: Chapter 2. Comprises literature review relating to Value at Risk and the three models Variance Covariance, Historical Simulation and Monte Carlo Simulation. It also describes three back testing methods z, Kupiec and Christoffersen along with its strengths and weaknesses. Chapter 3. Explains the research question. Chapter 4. Describes the methodology used. It tells in detail, step by step the manner in which three models are built. It also highlights the construction of back testing techniques. Chapter 5. Presents the findings of the research. Chapter 6. Discusses the various models Chapter 7. Reveals the insightful conclusion of the research undertaken.

CHAPTER 2

Literature Review

2.1 Value at Risk:

Value at Risk was founded and developed by JP Morgan in the late 1980’s and early 1990’s, as an essential method to measure the risk associated with an investment for a given time period. However, some literature contends and highlight that the underlying foundations can be followed back to the 1920’s. In between 1921 to 1928 New York Stock Exchange (NYSE) set out capital necessities convention, which were required to be followed by the US-listed companies (Gustafsson and Lundberg, 2009). The same was also discovered by Holton (2002). In addition, his paper stated that around the same time, ‘the Glass-Steagall agreement of 1933’ was passed, which required the banks to choose to be either a Commercial bank or an investment bank, this was done in the response of the US 1929 stock market crash. The Act was later weakening and as a result, was abrogated, which permitted both the categories of banks to trade in the securities market, this acted as the turning point in the broad reception of Value at Risk across the US financial institutions. Financial institutions, particularly banks, now trade in the capital markets, which was once restricted for them. Thus, exposing them to greater risks, which in turn demanded a uniformness amongst the association to measure the risk leading to more and more acceptance of the concept of Value at Risk.

The banking industry, insurance industry, individual investors as well as non-financial institutions now embrace VaR as a standard measure mainly due to its effortless requirements in both execution as well as interpretation. Giot and Laurent (2004) said that because of its simplicity feature, VaR is very popular. However, Taleb (1997) said although popular and widely used measure, it might end up encouraging unpractised people to pick up misled risks thereby losing the shareholders money, he referred to the financial institutions, who would use VaR as an assurance to document shareholders that losses incurred due to unforeseeable circumstances and would not disclose the real truth of large risk undertaken. He further added that the VaR approach is futile on the grounds that volatilities and correlations frequently change. Moreover, Duc et al. (2018) added VaR not only lacks properties of convex measurement but also lacks sub-additive.

Although VaR has been criticised mainly due to theoretical flaws but is still imposed as a mandatory obligation by the Basel Committee of Banking Supervision via Regulation (Basel Accord) I, II and III on all the financial institutions (Žiković and Filer, 2013).

Best (1998) says for a given period of time, Value at Risk is the worst or maximum portion of the money that can be lost in a portfolio.

Kimura et al. (2009) highlighted three essential elements of risk, which are the worst potential loss, the time period it considers both long as well as short horizon and the confidence level such as 90%, 95%, 99%. For instance, for one day holding period with the confidence level 99%, Value at Risk is 50 million dollars. This implies that the investor at the most can lose 50 million dollars in one day at a 99% confidence level and this loss cannot exceed. Thus, Value at Risk can be used to measure the risk associated with equity portfolio which permits any kind of investor to build up their own risk appetite.

2.2 Risk:

Risk in terms of investment could be defined as the uncertain and unsteady returns over a period of time, resulting in the loss of potential investment targets (Reilly and Norton, 2008). The following are the types of risk, which are commonly associated with financial firms:

2.2.1 Market Risk

According to Saunders and Cornett (2008) the market risk involves the risk that occurs due to changes in the prices of invested assets. Greater the volatility, more is the risk involved. For the banks and the other financial institutions, the market risk arises commonly due to fluctuations in interest rate, exchange rate, equity prices. However, another element is added to market risk, which is the outcome of the trading activity. It can be said as an incremental risk as it combines risks including interest rate, exchange rate as well as equity returns along with a trading approach particularly that is associated with short horizon, for instance, a day.

Barings PLC, a British Merchant bank which was nearly 233 year old bank went bankrupt in 1995 due to trading loses made by a trader Nicholas Leeson. He on behalf of the bank betted for around 8 billion dollars. He believed that the Japanese stock Index- Nikkei 225 would rise as a result, he bought futures on the index. However, the market fell by 15% and in a span of a month, the bank lost nearly 1.2 billion dollars, due to the number of reasons one major being earthquake in Kobe. Thus, it can be said that Barings was a victim of market risk. The bank was regarded as a conservative bank. Therefore the insolvency acted as a wakeup call, which in turn made financial institutions more concerned about the risk involved with the trading.

Jorion (2007) said that the market risk could very much be limited by VaR measures along with proper monitoring and supervision by the risk managers.

2.2.2 Liquidity Risk

Liquidity risk arises when financial institutions have to sell off their assets at a low price to resolve the creditors liquidity position in a short span of time. While insolvency risk refers to the state wherein, the institutions suffer the risk of losses arising mainly due to other risks such as interest rate, exchange, credit, market (Saunders and Cornett, 2008).

The Orange County,1994 is one of the case studies that set an example for liquidity risk. It apparently is regarded as the largest bankruptcy in the history of the US government. Bob Citron was authorized a portfolio of 1.7 billion dollars which belonged to the people of the county besides, he borrowed around 12.5 billion dollars and he added the amount as during the time the interest rates were falling. This was a leveraged strategy that worked amazingly well. However, the situation reversed and interest rates increased sharply. No sooner did the investors who included schools, districts, cities as well as the entire county itself, got to know about the losses, then they tried to pull back their money. This made Orange Country declare insolvency.

Liquidity risk may be incorporated into a Value at Risk model by ensuring that the projection horizon is at least as long as an efficient liquidation period (Jorion, 2007).

2.2.3 Credit Risk

Businesses are presented to credit risk through monetary transactions which requires the fulfilment of commitment by the counterparty. It refers to non-repayment of the cash amount that was credited to the client. It can be defined as the potential losses that occur due to denial or lack of customer's ability to payback either in full or partial (Nandi and Choudhary, 2011).

Hull (2018) stated that credit risk is by and large faced by the banking industry, indeed by the loans department, when the borrower defaults that is unable to pay back the principal and the interest amount. An ideal alternative to

.diminish the chance of this kind of risk is to have a collateral security. On the off chance that the counter party ends up being a default the banks can auction it to general public and thereby recover the losses.

2.2.4 Operational Risk:

Jorion (2007) this risk leads to loss which is outcome of deficient internal processes, individuals and the systems in the organisation or the outer events. It also takes into consideration the failure in information system and legal problems.

In 2002, John Rusnak a currency trader at Allied Irish Bank (AIB) brought the bank down by losing 691 million dollars. This accounted to 60% of bank's earnings. He used large options to engage in a type of arbitrage, endeavouring to take benefit out of the price discrepancies occurring between the currency options and forwards. He was bullish on the Yen currency and made one-way bets via forward contracts. Very soon, he started to lose the money and to compensate the same he created bogus options dealing in long positions. The trade was not approved by the bank. Although he had VaR limits, yet he went around with the weak risk management system. In addition, fake positions were fed in the system. All these in turn led to the forceful resignation of senior management. Therefore, AIB was the victim of operational risk.

This kind of risk can be controlled by segregating the responsibilities of the individuals in the organisation and enforcing sound internal control system.

2.2.5 Foreign Exchange Risk

This refers to the loss arising out of fluctuations in the exchange rate particularly transactions involving various currencies (Horcher, 2005).

2.2.6 Interest Rate Risk

Saunders and Cornett (2008) this risk is more relevant in case of banks and financial institutions whose profits are affected due to movements in the market interest rate. As compared to all other categories of risk, interest rate risk appear to be more sensitive to the shifts in the market and hence to minimise the same, market trends relating to interest rates should closely be observed (Hull, 2018).

2.2.7 Equity Price Risk:

This risk mainly impacts the corporate investors who generally invest either in equities or assets that directly or indirectly relate to the prices of equity. For instance, companies may be presented with equity risk in case of pension fund investments, mainly which depends upon the dividends and upward movement of share prices which leads to gains. Risk exposure could be related either to a stock or stocks or the entire sector or market. This risk also pertains to the firm's ability to supply capital to any kind of operations (Horcher, 2005).

Saunders and Cornett (2008) said financial institutions like to frequently trade in equities and there are two types of risks associated to it, systematic and unsystematic. Systematic refers to the movement of the stock beta as against the market movements. While, unsystematic pertains to the specific firm or an organisation.

Investors choose equity mainly due to the higher returns. There has been an established substantial relation between the stock returns and Value at Risk for various investment horizons (Bali and Cakici, 2004).

2.3 VaR measures the risk through the following approaches :

2.3.1 The Variance Covariance approach:

The first 'Risk Metrics' variance model was first published by JP Morgan in 1994. It included a covariance matrix procedure particularly for various risk factors and became suitable for various instruments including equities. Thus, gained popularity (Oanea and Anghelache, 2015)

According to Alexander (2008) this type of model has been given several names by various authors amongst which the popular is the Parametric VaR model. The name is so because it is based on estimation of various parameters such as standard deviation.

A Variance Covariance VaR model is based on the assumptions that the underlying asset returns are normally distributed. Further the underlying asset returns are required to be multivariate normal. This is essential so that the covariance matrix of asset returns can easily speak about the co-dependencies amongst them. Thus, stating that VaR for a portfolio can be computed as a linear function taking into consideration the standard deviation of the underlying returns they will be normally distributed around the curve of the probability that is they will adjust themselves and a fitted curve will then appear which would reflect VaR figure (Bozkus, 2005).

Changes in the share price of Paris stock Exchange appeared to be normal when central limit theorem was used to obtain a normal distribution for the movements in the share price (Bachelier, 1900). This assumption of normality has since then been into existence for the asset returns.

Variance Covariance method takes into consideration an approximate relationship between the portfolio value and the underlying market factors. Depending on the portfolio function it is divided into two categories, 'The Delta Model' and 'The Gamma Model'. The first model considers the above stated assumptions and only considers the linear sensitivity function. While the second was proposed to deal with the non-linear sensitivity function by various researchers (Xiao et al., 2014). They further added, it is due to the statistical properties that implementation of the model is relatively easy.

Jorion (2007) also highlighted the same and said that this method is convenient to execute as it involves very simple multiplication of matrix. He added saying that this method of computing is very fast even if the assets are in large quantity numerically. He further added saying *"VaR is easily amenable to analysis because measures of marginal and incremental risk are a by-product of the VaR computation."* Thus, portfolio risk can easily be managed.

However, according to Sollis (2009) Variance Covariance approach is based on the assumption of normal distribution, which in reality does not always hold true, having said that standardized returns of a portfolio though standardized cannot always be normal variables. Agreement about the incorrect assumption criteria was also found in the study carried out by Bohdalová (2005) who said that generally distribution of the daily asset returns have the peculiarity of fat tails which implies that extreme results might occur frequently than they would have under the conditions of normal distribution. This might produce an underestimated VaR figure. The author further reproved the model for their incorrect estimation in case of non-linear measures such as options or interest rates.

Fat tail implies kurtosis (a statistical property) greater than 3 however the normal distribution implies kurtosis equal to 3.

2.3.2 The Historical Simulation approach:

Richardson et al. (1998) and Barone-Adesi et.al (1999) published various papers through which historical simulation method was developed.

According to Linsmeier and Pearson (1996) and Hull (2006) this method takes into account the historical data which leads to what might happen in the future. The historical data is used to compute the 'Profit and Loss' distribution and exposing them to the real changes which are experienced in the market over the previous days, for a specific period. In other words, hypothetical mark-to market values for the portfolio are calculated which gives out hypothetical profit and loss, using the same VaR figure can be determined.

Jorion (2007) said this method removes the shortcoming of parametric approach that involves estimation through covariance matrix thus making the calculation process very easy for the portfolios involving large size of assets and shorter duration. Alexander (2008) further agreed in the same line and stated that unlike Variance Covariance model this one is not restricted to the linear function, thus can be applied to any type of a portfolio of assets. He further added, historical data can be directly used to capture the dependencies amongst the assets in a portfolio.

Although the model seems to be easy in terms of computation, it is accompanied with a baggage. This is can be concluded by the empirical evidences provided by various literature. Data considered for computation may be stale this is because the markets keep evolving in terms of technology, regulatory modifications and changes concerning economic and non-economic growth and expansion and various other such reasons. Thus, the market data that might be few weeks old might nor reflect the market today, past is not prolonged (Holton, 2014).

The model is also criticized by Pritsker (2006) who believed that historical simulation acknowledges less in case of conditional risk, in addition the response is also in a dissymmetry style. He further added risk expressed in figure increments in case of losses but does not in case of gains, experienced with reference to portfolio. The model is deemed to produce accurate estimate only for large scale data, implying not suitable when the sample size is small (Goorbergh and Vlaar, 1999). The same issue was faced by Hendricks (1996) who mentioned that with the percentile that are extreme such as 99% or 95%, accuracy is not obtained when dealing with small samples.

2.3.3 The Monte Carlo approach:

This model was invented in 1942 by Stanislaw Ulam at Los Alamos lab. Polish mathematician named the process Monte Carlo which was a the name of the popular Casino whereby her uncle gambled and thus in his honor, the method was named. Ulam believed numerical simulation had the capacity to evaluate functions that would assist in solving complex mathematical problems (Jorion, 2007).

As stated by Alexander (2008) the Monte Carlo VaR has two significant elements, the first being sampling algorithm and the second, the model to which this algorithm is applied too. Monte Carlo techniques are a class of computational calculations that are based on random sampling to acquire numerical returns. Monte Carlo involves generating random numbers using Pseudorandom number generation. Its main objective is to create numbers between 0 and 1. These numbers are uniformly distributed and in addition are independent as well as non-periodic. Random Number are generated taking into consideration the volatility and correlation estimates, that is risk manager must model both the aspects each asset's returns and dependency amongst the each assets returns. Random numbers created are used to build the hypothetical profits and losses for the portfolio. Following which these hypothetical profits and losses are distributed. VaR is than determined from it taking into consideration the set parameter such as confidence level (Saita, 2007). Each Scenario is recorded which reflects the portfolio value risk over a specified period (Aniūnas et al., 2009).

According to Choudhry (2006) this model is more realistic and practical than any other model. Therefore, the computed Value at Risk estimate is likely to be more accurate.

Xinrong and Jianhui (2011) said the method is also called Stochastic Simulation as it takes into account the past figures to analyse the fluctuations in the market. He further added that this method does not count upon the normal distribution and thus is an effective and efficient problem solver of the challenges of non-linear distributions. Thus, the research conducted by them proved this method to be not only flexible and reliable but also powerful in terms of practical application. Jorion (2007) also reflected upon the dynamism and powerfulness of the model in the computation of VaR. Bohdalová (2005) agreed in the line and said that this model is persuasive out of all the models. He further added it is also flexible as it does not limit itself to any assumptions. This model is suitable for non-linear portfolios and at the same time it incorporates all alluring distributional features such as fat tails and also the changing volatilities experienced over the period of time. It is also suitable for long horizon period.

However there is a downside to this methods as well. It has be said that the model is very time consuming that generally takes long hours as it require calculation of the portfolio value over and over at end of each simulated price path (Jiménez and Arunachalam, 2011). The model is also very sensitive to the certain-parameters and incurs huge computational charges (Pasieczna, 2019). 'Model Risk' is another weakness and this arises as the model is based on certain stochastic processes for the assets returns, this requires the assumptions to be specified properly or else it might turn out to be wrong and distorted VaR figure, thus will be estimated. (Jorion, 2007). Besides, Dowd (1998) identified the need for an expertise to deal with the complex challenges associated with this model. In addition, senior administration may along these lines have tough time staying up to date with how VaR figures are determined with Monte Carlo model.

On comparing three models various researchers had different opinion. According to McNeil et al. (2005) Variance Covariance model has the capacity to produced cogent results without simulation, on the other hand Linsmeier and Pearson (1996) stated that the great advantage of historical simulation is no involvement of assumptions and the only requirement of historical series which makes it more intuitive to function. He further added Historical simulation and Monte Carlo can successfully capture portfolio risk while variance-covariance lacked the ability. Alexander (2008) mentioned that out of the three models, Monte Carlo method acted as *"last Resort"* in absence of analytical clarification or certainly when other approaches break down. Danielsson &Vries (2000) said that at 95% confidence level, Monte Carlo performs the best.

Researches conducted by Pritsker (1997); Jadhav and Ramanathan (2009) and Lechner and Ovaert (2010) all conclude no particular method can be termed as the best. This was proved by Cheung & Powell (2012) who examined both the sides of the models stating that though variance-covariance is easy to implement in case of normal distribution however suffers from "asymmetry". He further added statistician generally find the Variance Covariance method restrictive and distorted. In case of historical simulation particularly during the times of crisis, it tries to evade problems in the specification of "probability density function" of various risk elements. In addition although the model has the capacity to estimate past returns but may fail in case of future predictions when changes in the markets are experienced. Moving on to the next one he said monte carlo though enjoys the merit of increasing the data in terms of observations however it is computer oriented and involves a lot time consumption. Further, Mandaci (2003) said that using more than one method of VaR leads to better and objective results.

According to Beder (1996) VaR models seem to be easy however each model has its own uses and limitation. Stambaugh (1996) said since each method has its own merits and demerits thus instead of comparing them, they should be looked at as an alternative that might prove to be the best in particular circumstances or situations. Therefore, the best model answer is difficult, as these strategies differ in their capacity to quantify risk which depends on the type of instrument being used, ease of execution and explanation, flexibility to tackle with the changes that might be made to the assumptions and further the reliance upon the outcome generated (Radivojević et. al., 2017). Therefore, comparison should only be made considering one of the criteria stated above. According to Blanco and Oks (2004) to check how accurate the models are, all the three should be frequently back tested and substitute models should be taken into account. Akkaya et al. (2008) in their study inferred that VaR approaches can be used jointly with back testing, to get better results.

2.4 Back Testing:

"Value at Risk is only as good as its back test. When someone shows me a VaR number, I don't ask how it is computed, I ask to see the back test" (Brown, 2008, p.20).

According to Angelovska (2013) Back testing is the act of comparing the real profits and losses generated from trading activity to the model produced risk estimates. He further added, the following two questions are very essential in determining the correct model:

1. How accurately does the model measure risk for a defined percentile of or the Profit and Loss distribution as a whole?
2. How accurately does the model anticipate the size and recurrence of losses?

Three VaR approaches are very independent, they have their own uses and shortcomings. Therefore, back testing is essential to check the accuracy of the risk figure generated by the models. In addition, Basel Committee for Banking Supervision has made it mandatory for the banks and other financial institutions to imply back testing mechanism so that the risk estimates calculated via internal VaR models do not underestimate the capital requirement calculations (Jorion, 2007).

The simplest and popular technique to measure VaR involves counting the number of VaR failures, exceptions or exceedance which could be either days, weeks or months. If this count turns out to be less than the chosen confidence level it would announce overestimation of risk or else the event would display an opposite scenario, was found in the literature of (Baciu, 2014).

As correctly said by Haas (2006) if one test is able to produce reasonable outcome, the outcome must always be verified and reviewed with other tests. Therefore, the research takes into consideration three tests z, Kupiec and Christoffersen to evaluate the accuracy of the models.

2.4.1 Unconditional Coverage back testing

1. Z-Test: It takes into consideration the binominal distribution, which tests the accuracy of VaR. This is a kind of hypothesis test, thus it accepts or rejects the model to testify whether it is good. It faces two types of errors either 'Rejection' of the correct model or 'non-rejection' of the inadequate model. This test requires fulfilment to three criteria which includes stating null and alternative hypothesis, z-score that is the critical value and alpha for instance when confidence level is 95% the alpha is 5%.

Easy to compute and capacity to overcome errors mentioned above. However, z test is only suitable for a larger sample size (Jorion, 2007).

2. Kupiec Test: This test is also known as 'POF-test' (proportion of failures). This test takes into consideration the likelihood ratio (LR).

Halilbegovic and Vehabovic (2016) said that the test basically counts or checks whether or not the number of exceedances is in accordance with the specified confidence level. The number of exceedances must be consistent along the specified confidence level. This test requires fulfilment of three criteria stating the confidence level, total number of observation and total number of exceedances. Correct selection of confidence level is very essential so that the errors mentioned previously that is under z test are well balance. Under null hypothesis the model is deemed to be correct. This happens when either the likelihood ratio is equal or less than the critical value of the chosen confidence level.

The model is also criticized mainly because it only considers the recurrence of losses and leaves out the time period of occurrence. This method alone cannot be relied for back testing.

Unconditional Coverage sets a benchmark for checking the verity of the VaR models, yet it has certain imperfections, major being the inability to detect the VaR models which systematically underestimate VaR risk and report the same. Therefore, this may led to underestimation of capital requirements for the market risk (Campbell, 2005).

2.4.2 *Independence back testing:*

1. Christoffersen's Interval Forecast Test: Christoffersen (1998) gets the credit to develop this test. The test seeks to measure the possibility of observing

whether today's exceedance occurrence, has occurred on any of the previous days or not. This technique thus enables to understand the spread of the distribution is even over a certain time period or does it occur in clusters. Campbell (2005) said if clustering of VaR exceedances happens than it certainly indicates that the model used cannot generate correct VaR figure as it would not reflect the changing market risk.

The model is deemed to be accurate if VaR estimates are not greater than the critical value of the chosen confidence level.

However, the test only considers the successive days to measures the dependencies amongst the exceedances is the only drawback of this test (Jorion, 2007). The back testing process can be easily explained in the research (Nieppola, 2009).

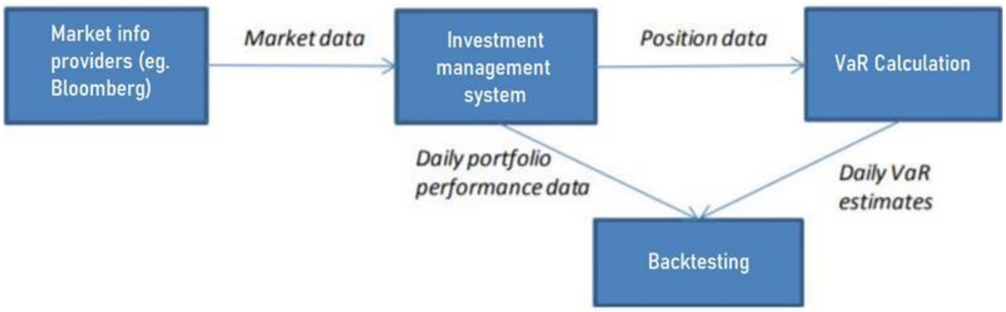


Figure 1. Back Testing Process.

2.5 Equity Portfolio:

Investing in equities have been increasing seen over the years, mainly due to the growth potentials as against the bonds. In addition, high returns appear to be the main attraction for the investors although the risk involved is also potential high which could be minimised using diversification. According to Markowitz (1952) through diversification the risk is significantly reduced in the portfolio. Portfolio Manager must consider investing in stocks of different sector. Value at Risk Models have been used on various stock markets.

Sarma et al. (2003) used different VaR models on different Stock Market which included America's S&P 500 and India's NSE-50. In case of S&P 500 parametric model weighed over historical simulation. For NSE none of them turned out as per the expectation.

Tas and Iltuzer (2008) said that Calculated VaR on Istanbul(ISE-30)index through Monte Carlo Simulation method and found that value generated could be used to measure the portfolio risk. Huang and Tseng (2009) compared the VaR models and found that Historical simulation performed better both in case of developed as well as emerging economies. Shah and Raza (2014) used variance and covariance and historical simulation models on Karachi stock Exchange and found that the former gave better results.

The literature highlights the various studies done on different stock exchanges and the different choice of the model however, no study in respect to Irish stock Exchange and the three classical models have been carried out in particular. Therefore, this research would consider Irish equity portfolio for calculating and comparing three models taking into consideration the financial crisis.

Ireland was one of the countries that was severally impacted by the financial crisis of 2008. In fact, it was the worst crisis in the history of Ireland. The crisis was the consequences of the property bubble which aroused due to growing population, low interest rate and the expanding fiscal regime. Besides, the banking regulatory system at the time was very liberalized and therefore, a rise in market funding took place mainly due to the cheap credit which added fuel to the already existing blazing property market. This led to a drastic increase in the property rates leading to credit default which ultimately led to a huge crash (O'Sullivan and Kennedy, 2010).

In turn, the stock markets crashed leading to sharp decline in the share prices. Therefore, the risk needs to be managed efficiently as well as effectively to reduce the risk of loss on the potential investment by controlling and avoiding such disastrous events in the future through process risk management technique.

CHAPTER 3

Research Question

The literature displayed a lot of various researches, involving the three models which highlights the peculiarities, suitability and limitations of each of them. Literature also stated, the comparison would be best made amongst the three when only one criteria out of all is selected. Therefore, equity is the only instrument considered for the research purpose.

Equity prices are sensitive to shifts in the market. They are directly impacted by the extreme events such as financial turbulence. Hence, considering such an instrument would help in comparing and identifying the best model decision. The research aims to answer the following question.

Which Value at Risk model would produce better results, for an investor investing in an Irish equity portfolios at a given level of confidence being 95% for the period of 11 years which includes the global financial crisis of 2008?

The research takes into consideration Irish equity portfolio which comprises 10 stocks listed on Irish Stock Exchange (ISEQ). The three models Variance-Covariance, Historical simulation and Monte Carlo are constructed using Microsoft Excel and also R-Studio. The two platforms are used to ensure the construction of the models is carried out effectively and compare the results. The models are then back-tested using three test z, Kupiec and Christoffersen.

The timeframe considered is 11 years which is divided as 2002 to 2007 and 2008 to 2013 that is pre, during and post the time of financial crisis. The financial crisis of 2008 is the focus. The period of 11 years is undertaken to get a clear idea of which model in normal circumstances would perform better and which one during the extreme events.

It also aims to understand that can historical data give an idea about the future prices so as the investors can judge based on the history.

CHAPTER 4

Methodology

4.1 Research Type:

The research can be categorised as an experimental and longitudinal, thus falls under quantitative type, which can be defined as gathering essential numerical information or data and analysing the same with mathematical techniques especially statistics (Creswell, 1996).

Quantitative approach includes collecting both statistical data as well as numerical data and using tools such as linear and visual presentation in the form of graphs and various charts presented in tabular form along with comparative analysis to scrutinize the data (watkings and Gioia, 2015).

This research aims to build three models Variance-covariance, Historical simulation and monte-carlo simulation. These models shall calculate and display one day specific amount at risk of equity portfolio. To check the authenticity and accuracy of models back testing via three tests z, Kupiec and Christoffersen is carried out. As the work involved requires using statistical method to build the models and then apply tests to analyze the same. Therefore, the author considers quantitative approach to be robust.

4.2 Data Collection:

The research uses secondary data, which is collected from yahoo finance and Irish stock exchange websites. The time frame involved is around 11 years from 1st January, 2002 to 31st December, 2013. The main focus would be on the period during which financial crises occurred.

The portfolio comprising 10 stocks mentioned in the table below are included. These are listed on ISEQ (Irish Stock Exchange). Equal delta is assigned to each of them. The total portfolio amount is 10 million euros. The stocks are diversified that is they are carefully selected from different sector which includes banking and finance, retail, technology, airline and energy.

STOCKS	AMOUNT INVESTED
CPL RESOURCE PLC	€1,000,000
ALLIED IRISH BANK	€1,000,000
KINGSPAN GROUP PLC	€1,000,000
TESCO PLC	€1,000,000
UTV MEDIA PLC	€1,000,000
CHL	€1,000,000
RYANAIR HOLDING PLC	€1,000,000
BANK OF IRELAND	€1,000,000
PERMANENT TSB	€1,000,000
DONEGAL INVESTMENT GROUP PLC	€1,000,000
Total	€10,000,000

Table No. 1

Alexander (2008) said that 95% confidence level along with one day range can be set for any kind of "trading limit". Therefore, the research is performed with 95% level of confidence and 5% level of significance. The holding period is 1 trading day. The calibration period for which VaR construction has taken place is 3 years it would be as one year, two year and three year. The research computes a time series over a period of 11 years of 1 year VaR, 2 year VaR and 3 year VaR. It records what the VaR would have been over the period of 11 years if the VaR is computed using 1 years' worth of data at any given point in time. Similar fashion for 2 year and 3 year are also considered.

The procedure in case of 1 year starts exactly after completion of a year from the time the data sample initiates. In case of 2 year and 3 year, it begins after two and three years respectively from the time data is considered. Elaborate explanation about the same is highlighted with the model building.

Back testing also at 95% is considered appropriate so that reasonable number of deviations can be observed to validate the models (Jorion 2007).

In spite of a lot of software available to construct models. For this research, author chose Microsoft Excel for building the three models and back testing. Jackson and Staunton (2001) and Choudhry (2006) said excel is not only simple but also is ideal way of constructing models. In addition, the construction of models is also performed on R-Studio only to verify and check that the constriction has been carried out without errors. R-Studio has been chosen for verification because of its versatility and quickness to compute the models. Back-testing shall be carried out only using spreadsheet.

4.3 Construction of Models:

A. Variance Covariance Model :

The model measures the standard deviation and correlation of the various components which multiplies by appropriate sensitivity weights, assuming the normality. The selection of the data to form equity portfolio was the initial process. Followed by which natural log function assisted in computation of returns. Delta in equal segment is assigned to each asset in the portfolio. SUMPRODUCT function realise simulated Profit and Loss (P&L). STDDEVP function and CORREL function computes a series of correlated pairs. Finally the risk analysis is computed with the following formula. Alexander (2008) used the formula to compute the risk measures.

$$\sigma^2_{\text{portfolio}} = X_1^2\sigma_1^2 + X_2^2\sigma_2^2 + X_3^2\sigma_3^2 + 2X_1X_2\rho_{12}\sigma_1\sigma_2 + 2X_1X_3\rho_{13}\sigma_1\sigma_3 + 2X_2X_3\rho_{23}\sigma_2\sigma_3$$

X refers to the Delta

σ refers to the standard deviation

p refers to the correlation pairs

B. Historical Simulation Model :

The model is relatively easy to build. The portfolio comprising Deltas is multiplied to the sum of the returns computed by natural log (ln FUNCTION), which produce Hypothetical Profit and Loss (P&L). In case of 1 year calibration period the modelling begins exactly after one year of data considered. It produces the outcome at a chosen

level of percentile. In this case it displays the maximum loss for the top fifth percentile. Considering the same pattern for 2 year and 3 Year are also computed, however the 2 year begins after the two year and 3 year after three years from the time the data is considered. The process of these calibration period is standard in all the three models.

C. Monte Carlo Simulation Model :

This method uses Cholesky decomposition which is based on matrix algebra. Matrix can be defined as an array of the numbers. An **g x h** matrix is an array of g rows and h columns. Cholesky Factorization maps matrix A which is the result of $L * L'$.

The former L represents the lower triangular matrix while the latter represents its transposed for that is upper triangular form. Parker (2017) made the use of matrix substitution to explain Cholesky decomposition.

$$\begin{pmatrix} A_{11} & A^*_{21} & A^*_{31} & A^*_{41} \\ A_{21} & A_{22} & A^*_{32} & A^*_{42} \\ A_{31} & A_{32} & A_{33} & A^*_{43} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{pmatrix} = \begin{pmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{pmatrix} * \begin{pmatrix} L_{11} & L^*_{21} & L^*_{31} & L^*_{41} \\ 0 & L_{22} & L^*_{32} & L^*_{42} \\ 0 & 0 & L_{33} & L^*_{43} \\ 0 & 0 & 0 & L_{44} \end{pmatrix}$$

Figure No. 2

The code for the Cholesky decomposition has been provided in (Appendix 1).

The process begins with calculation of average return (μ), standard deviation (Sigma) of each asset return price along with the covariance matrix. The covariance matrix are used to generate Cholesky Decomposition which leads to Pseudo-Random Number Generation that is generation of sequence of numbers between 0 and 1 that are l be uniformly distributed, independent and non-periodic. Independent Random numbers or Sequence of standard normal simulation(z) are generated using the inverse transform of a given normal distribution NORMSINV(RAND ()) function. Thereafter correlated Random Correlated Numbers are generated via multiplication of Independent Random Numbers (z) to the Cholesky decomposition of the covariance matrix. Using the Visual Basic Application (VBA) Code given in (Appendix 2) macros are run to record the number of maximum loss. However the process begins with computing a single simulation VaR with 1 day risk horizon and 95% confidence level in excel which in turn would enable the creation of a Macro in VBA.

R-studio computation involves inserting the data, writing an appropriate code which is mentioned in (Appendix 3) and running the code.

4.4 Back Testing Methods:

Blanco and oks (2004) emphasized the use of back testing, to understand whether the model succeeded or failed . It displays the performance and reliability of the model. For an equity portfolio these are the chosen back test performed by (Baciu, 2014).

A. z test:

   After the VaR calculation is completed using the above stated models. Back testing begins with the calculation of exceedances. It compares the simulated Profit and Loss and the computed VaR estimate, if P&L is exceeds VaR then it is assignment 1 or else 0. The number of such exceedances are counted which are regarded as observations and using the SUM function a total amount is obtained.

   It is performed using the following formula.

$$z = \frac{Total\ Number\ of\ Exceedances - Expected\ Number\ of\ Exceedances}{Standard\ Deviation\ for\ the\ Binominal\ Distribution\ of\ 1\ and\ 0}$$

   The z critical value for 95% confidence level is ±1.65.

B. Kupiec test:

   Another type of back testing method is Kupiec test and is executed through Visual Basic Application and the code is mentioned in (Appendix 4). Although the initial function remain same as that of z. It takes into consideration the total number of observations, exceedances and the significance level. Kupiec Statistic is distributed according to a Chi Squared distribution and that the result is compared to the Chi squared distribution with one degree of freedom. (Alexander, 2008). The critical value is 3.8415

C. Christoffersen Test:

   The initial process remains the same. This test is also executed through Visual Basic Application and the code is mentioned in (Appendix 4). It takes into consideration Simulated P&L, VaR estimates, Confidence level which is 95% and significance level which is 5%. If the computed value is less than the critical value of 3.8415, the model is said to pass the test.

Hypothesis tests the performance and efficacy of the VaR Models. The VaR Model sets bound on potential losses such that the exception to this VaR Model should be independent event that occur with the probability alpha.

In case of z and Kupiec, the alpha will be considered and the null and alternative hypothesis would be as such:

$H_{0:}\, VaR\ Model\ produces\ exceptions\ with\ the\ probability\ 0.05$

$H_{a:}\, Probability\ to\ exception\ is\ different\ to\ 0.05$

While in case of Christoffersen, the element if independence is considered and the null and alternative hypothesis would be as such:

$H_{0:}\ Exceptions\ are\ the\ independent\ events$

$H_{a:}\ Exceptions\ are\ not\ the\ independent\ events$

CHAPTER 5

Findings and Analysis

The chapter comprises two sections. The first one highlights the three models and their computational results in the form of graphs performed on excel as well as R-Studio. The second part revolves around the results produced via three back testing methods and a detailed analysis on the period pre, during and post the crisis is carried out. The models performance and reliability is examined with 95% confidence level. This would enable a transparent and firm grasp of the outcome obtained, so as to make a distinctive decision regarding the best model for an Irish equity portfolio.

5.1 Computation Results:

Variance Covariance Results:

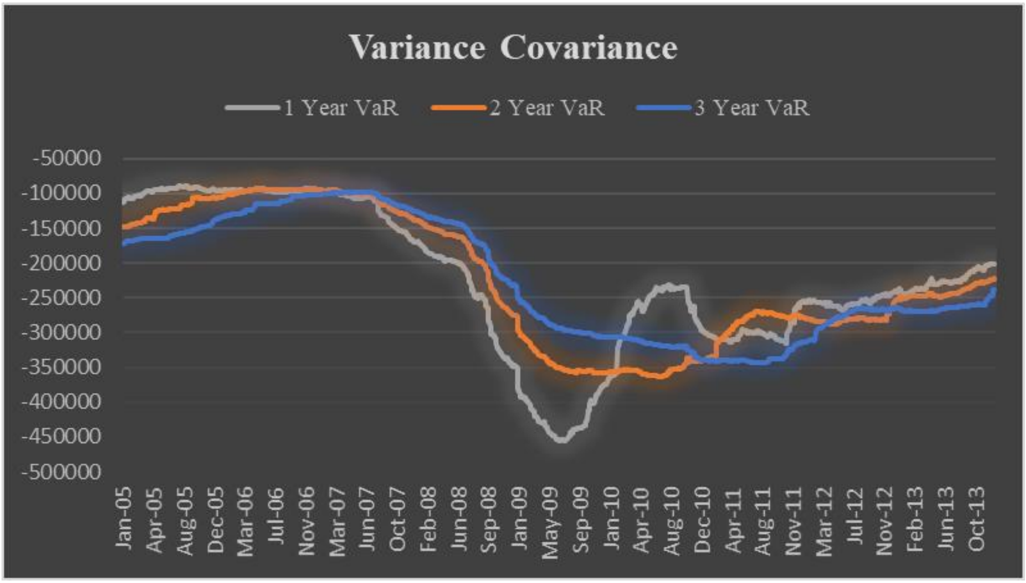


Figure No. 3

The graph displays the 1 year, 2 year and 3 year VaR computations performed by Variance Covariance model. The 1 year is represented by the grey colour while the 2 and 3 year are represented by the orange and blue colours respectively. The grey line displays Sharpe moves as compared to the other two.

The grey line is moving sharply downwards from June 2007 to June 2009, this is mainly because when volatility appears, 1 year VaR model tends to react quickly and displays higher VaR levels, as seen in the graph the amount estimated went almost to

450,000. As a result there are fewer exceedances. The scenario appears to be the same in case when volatility reduces, the 1 year VaR model reacts very quickly and is less likely to overestimate VaR. In comparison 2 year and 3 year all though reacts to the market fluctuation but relatively in a sedate manner to the market volatility. Amongst the two, the 3 year is slower than the 2 year.

Therefore, since 1 year reacts quickly in both the cases of volatility period as well as non-volatility period it could be deemed as better performer as compared to the other two.

Historical Simulation Results:

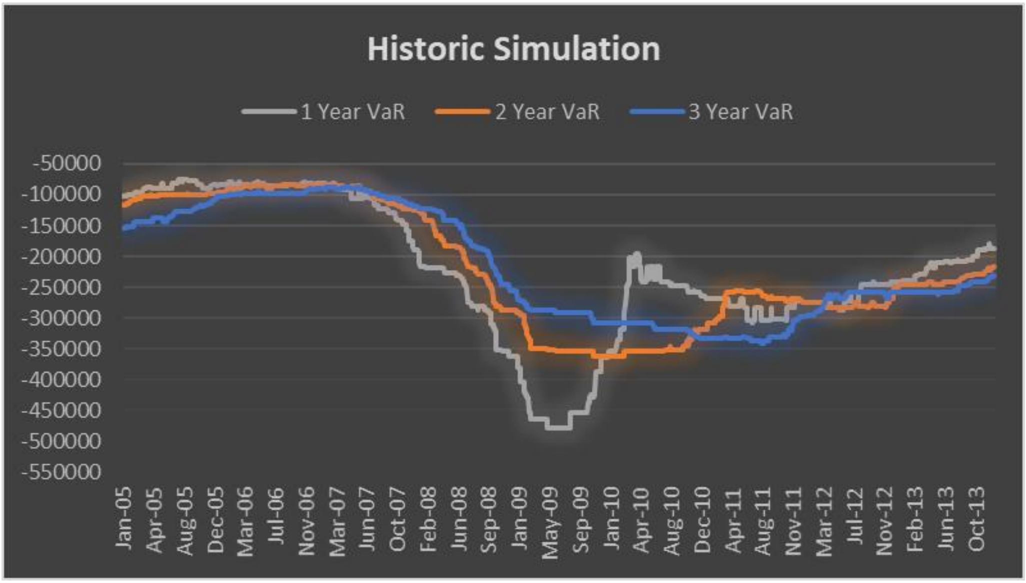


Figure No. 4

The graph presents the results of the VaR figures produced using Historical Simulation model. It follows the same colour sequential as that of Variance Covariance. However in this case the grey line appears to decline but not as sharply as that of Variance Covariance. The distance between grey and orange is also slightly more than what appeared in case of Variance Covariance.

1 year graph as expected reacts quickly to the changing market climate as compared to the 2 year and the 3 year. At 95% confidence level the 1 year graph shows higher VaR level as seen by the grey line ranging between 450,000 and 500,000 euros in mid-2009 and the scenario completely turned opposite in early 2010 when the grey line went as low as roughly around 195,000.

Therefore quickly adjusting itself to the any market condition. While the 2 year and the 3 year exhibit gradual adoption to the market conditions, they have more exceedance which may likely overestimate risk especially in case of the 3 year. Around march 2012 all the three overlap one another.

Monte Carlo Simulation Results:

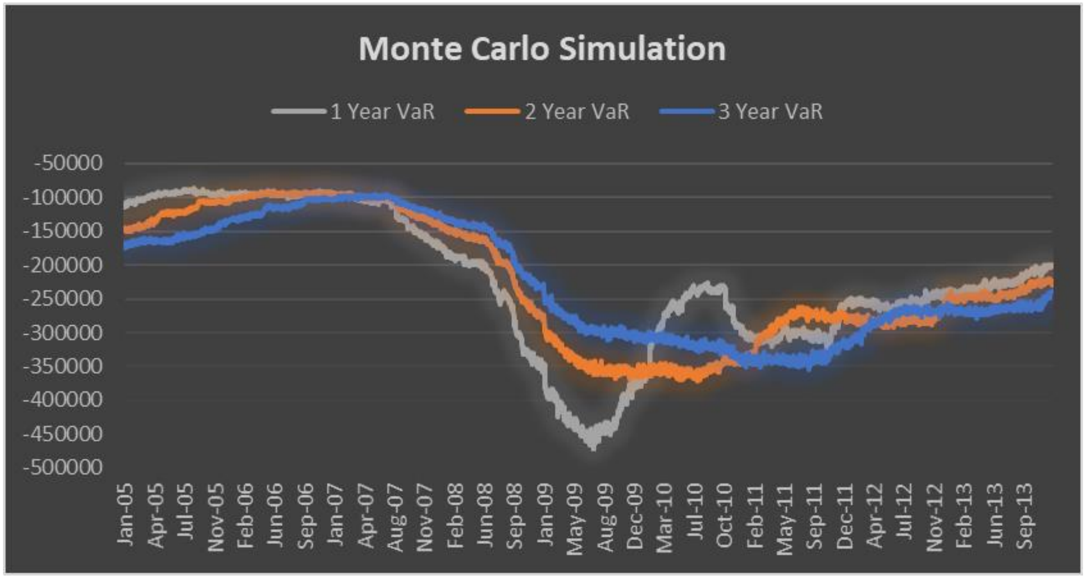


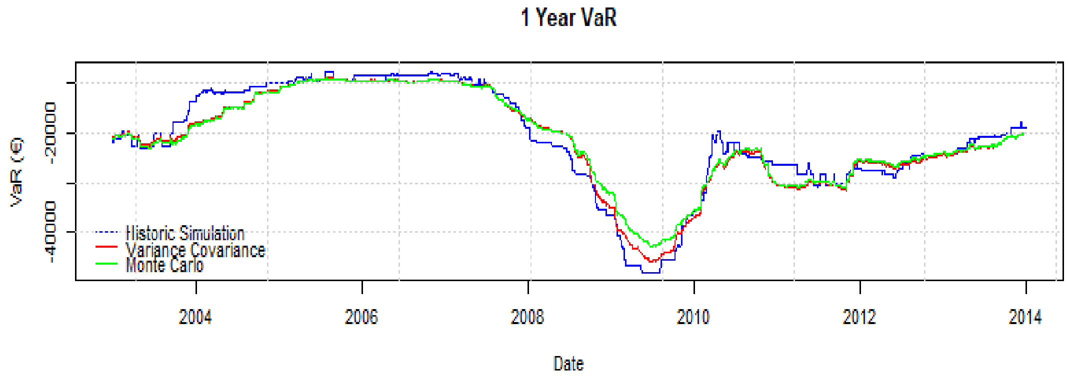
Figure No. 5

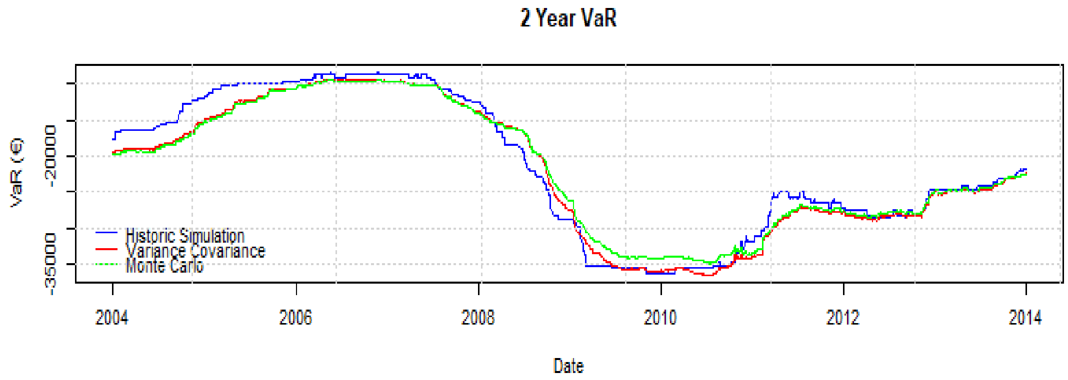
The graph of Monte Carlo Simulation VaR results appears to be similar to that of Variance Covariance. Particularly adhering to this research Monte Carlo Simulation falls under the category of Parametric VaR.

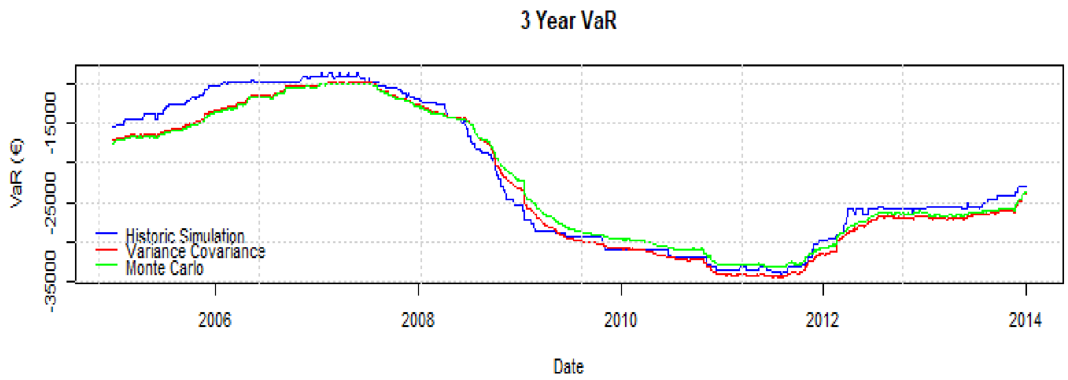
In fact, it represents the peculiarities of both Variance Covariance and Historical Simulation. In case of the 1 year the VaR level goes as high as 470,000 somewhat similar to that of Historical Simulation and the very next year goes down and ranges between 2000,000 and 250,000 which falls in line with Variance Covariance.

The graph too displays the same movements, the 1 year quickly adopts and reacts to the market fluctuations than the 2 year, which than when compared to 3 year tends to highlight the better performance. Therefore, it would be right to state that 1 year Monte Carlo model underperformed 2 year which than underperformed 3 year.

Following Charts are the snippet of outcome produced in R-Studio.







The results appear to be same for aggregate graphs as well as for the individual graphs. Therefore, it can be concluded that models are built well and are error free. The graphs produced in Excel are attached in (Appendix 5) which appear to be similar and follow the same trend for 1 year, 2 year and 3 year.

The three graphs represent the computed maximum loss computed via three models. All the three graphs display that the Variance Covariance and Monte Carlo simulation supersede each other. While, Historical Simulation tends to move separately. The former takes into consideration standard deviation and correlation amongst the assets in the portfolio whereas the latter solely relies on the historical data.

The calibration period considered is 1 year exhibited in the first graph, 2 year and 3 year in the second and the third graphs respectively. The 1 year reacts quickly to the changing market factors, during the time of volatility it quickly falls down, followed by which during normal condition it quickly moves up, the same has been portrayed in first graph which forms 'V' like a shape. In comparison for all the three models the 2 year takes more time to endorse itself as per the market scenario, as seen in the second graph which forms somewhat 'u' like a shape for the period lying in between 2009 and 2011. While, the third one exhibits Historical Simulation, Variance Covariance and Monte Carlo Simulation are all in line which took relatively longer time to react, when compared to 2 year graph. It starts to move down post 2008 and continues until early 2012. It forms somewhat 'u' like a shape however it is stretched over the period due to inability to quickly respond to the changing circumstances.

5.2 Back Testing Results:

As seen above the graphs move in the same direction, indicating that the simulations produced by all the three models for 1 year, 2 year and 3 year are in the similar range. Thus, all the three could be deemed as reliable. However, by only spectating the models presented cumulatively for all the years, it is potentially possible that Value at Risk figures have been standardised and normalised over a long period of time. Therefore, the back testing has been carried out by three tests. These three tests have been performed year by year which would ascertain which model failed and which passed in a particular year. In the case of z test if the value is between + and – 1.65 than the model is deemed as pass. In case of Kupiec and Christoffersen the model is considered as an apt and passed the test only if the value is less than 3.8415. The critical value have been chosen from the z table and Chi-square table.

The Results are presented in the following manner: It begins with 1 year three models and respectively for 2 year and 3 year. The green highlight indicates the failure of the model that is the value generated did not range within the stated critical values. While, the other non-highlighted values indicate the passing.

In Case of z and Kupiec for 5 % significance level, the null and alternative hypothesis are stated as:

$H_{0:}$ $Observed\ Number\ of\ Exceedance = Expected\ Number\ of\ Exceedances.$

$H_{a:}$ $Observed\ Number\ of\ Exceedances \neq Expected\ Number\ of\ Exceedances.$

Whereas for Christoffersen with the same alpha, the null and alternative hypothesis are stated as:

$H_{0:}$ $Exceedances\ are\ the\ independent\ events.$

$H_{a:}$ $Excedances\ are\ not\ the\ independent\ events.$

Therefore, z and Kupiec values highlighted in green states the decision of 'Rejection of null hypothesis' and the non-highlighted values leads to the decision of 'Fail to Reject the null hypothesis' .

Whereas in case of Christoffersen the green highlighted values show cluster and therefore are not the independent events. The decision formulated therefore is 'Rejection of Null Hypothesis', while for the non-highlighted figures the decision is 'Fail to Reject the null hypothesis'.

1 year VaR Test Results:

Variance Covariance			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	0.627	0.387	15.685
2003	-1.718	3.526	0.442
2004	-2.580	9.038	0.155
2005	-1.423	2.332	0.573
2006	-0.828	0.741	3.990
2007	4.981	18.247	2.136
2008	7.255	35.225	0.347
2009	-0.739	0.584	0.915
2010	1.238	1.392	13.772
2011	-0.476	0.237	0.989
2012	-1.065	1.258	0.736
2013	-2.505	8.468	0.160
Critical Value	±1.65	3.8415	3.8415

Table No.2

Historical Simulation			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	2.270	4.875	16.791
2003	-1.434	2.373	0.442
2004	-0.595	0.374	0.875
2005	-0.285	0.083	0.719
2006	0.314	0.096	2.179
2007	5.844	24.182	1.735
2008	5.244	19.995	2.618
2009	-1.028	1.168	0.344
2010	2.390	4.811	13.849
2011	-0.188	0.036	1.094
2012	-1.065	1.258	0.736
2013	-1.641	3.194	0.339
Critical Value	±1.65	3.8415	3.8415

Table No. 3

Monte Carlo Simulation			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	0.627	0.387	14.566
2003	-1.718	3.526	0.442
2004	-2.580	9.038	0.155
2005	-1.423	2.332	0.719
2006	-0.828	0.741	3.990
2007	5.269	20.152	2.136
2008	7.255	35.225	0.407
2009	-0.739	0.584	0.746
2010	1.238	1.392	10.542
2011	-0.764	0.628	0.908
2012	-1.065	1.258	0.736
2013	-2.495	8.397	0.160
Critical Value	±1.65	3.8415	3.8415

Table No.4

2 year VaR Test Results:

Variance Covariance			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	1.571	2.368	16.863
2004	-3.146	15.170	0.046
2005	-2.846	11.601	0.094
2006	-0.828	0.741	3.990
2007	6.420	28.490	1.483
2008	10.416	65.154	0.461
2009	0.710	0.475	1.299
2010	-1.929	4.586	2.500
2011	0.101	0.010	1.298
2012	-1.065	1.258	0.736
2013	-2.793	11.127	0.096
Critical Value	±1.65	3.8415	3.8415

Table No.5

Historical Simulation			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	2.479	5.768	12.625
2004	-2.863	11.760	0.093
2005	-1.707	3.478	0.444
2006	0.029	0.001	2.179
2007	7.572	37.890	1.196
2008	8.979	50.719	0.455
2009	0.999	0.922	3.193
2010	-1.929	4.586	2.500
2011	0.389	0.147	1.762
2012	-1.065	1.258	0.736
2013	-2.505	8.468	0.160
Critical Value	±1.65	3.8415	3.8415

Table No.6

Monte Carlo Simulation			
Dates	**z(95%)**	**Kupiec(95%)**	**Christoffersen(95%)**
	1.848	3.256	16.852
2004	-3.146	15.170	0.046
2005	-2.846	11.601	0.094
2006	-0.828	0.741	3.990
2007	6.708	30.744	1.843
2008	10.703	68.195	0.551
2009	0.710	0.475	1.299
2010	-1.929	4.586	2.500
2011	0.101	0.010	1.298
2012	-1.065	1.258	0.736
2013	-2.505	8.468	0.096
Critical Value	±1.65	3.8415	3.8415

Table No.7

3 Year VaR Test Results:

Variance Covariance			
Dates	z(95%)	Kupiec(95%)	Christoffersen(95%)
	3.680	12.312	47.055
2005	-2.846	11.601	0.094
2006	-2.257	6.574	10.059
2007	7.284	35.445	2.692
2008	14.438	112.028	1.194
2009	1.868	3.035	4.011
2010	-1.065	1.258	4.750
2011	-2.207	6.259	0.161
2012	-1.353	2.096	0.587
2013	-2.793	11.127	0.096
Critical Value	± 1.65	3.8415	3.8415

Table No.8

Historical Simulation			
Dates	z(95%)	Kupiec(95%)	Christoffersen(95%)
	4.159	15.553	33.494
2005	-2.846	11.601	0.094
2006	-0.828	0.741	3.990
2007	8.147	42.959	1.466
2008	13.576	101.239	0.620
2009	1.578	2.209	4.718
2010	-1.065	1.258	4.750
2011	-1.918	4.532	0.341
2012	-1.353	2.096	0.587
2013	-2.793	11.127	0.096
Critical Value	± 1.65	3.8415	3.8415

Table No.9

Monte Carlo Simulation			
Dates	z(95%)	Kupiec(95%)	Christoffersen(95%)
	3.393	10.535	45.404
2005	-2.846	11.601	0.094
2006	-2.257	6.574	10.059
2007	7.284	35.445	2.692
2008	14.438	112.028	1.194
2009	1.578	2.209	4.718
2010	-1.353	2.096	1.470
2011	-2.207	6.259	0.242
2012	-1.353	2.096	0.587
2013	-3.080	14.474	0.048
Critical Value	± 1.65	3.8415	3.8415

Table No.10

5.3 Analysis

Pre global crisis: The period from 2003 to 2007

Overall Irish stock market performed very strongly, there has been stable increase in the stock prices from 2002 to 2007. Thereafter it started to fall shortly. In case of 2005 – 2006 the three models experience extremely fewer exceedances as a result of which the models are failing on the downside of back tests.

In case of 1 year VaR, the Variance Covariance model could not pass z test for the year 2003, 2004 and 2007 while it passed for the year 2005 and 2006. With Kupiec test, however it passed in all the stated years except in 2004 and 2007. While for Christoffersen test the model only failed in 2006. Whereas the Historical Simulation the model passed all the test from 2003 to 2006 however in 2007 it failed z and Kupiec. While, the Monte Carlo Simulation model could only pass all the three tests in 2005 however it failed to pass the z and Kupiec test in 2003, 2004 and 2007. In 2006 it failed to pass Christoffersen test.

In case of 2 year VaR, the Variance Covariance model could only pass the test in 2006 for z and Kupiec whereas in the same year it failed Christoffersen test. While for rest of the year 2004, 2005, 2007 it cleared Christoffersen test. The Historical Simulation model failed z test for all the years except 2006. In case of Kupiec it only cleared in 2005 and 2006. While it cleared Christoffersen in all the years. The Monte Carlo could only clears z and Kupiec in 2006 and in the same here it failed Christoffersen. While for rest of the years it passed Christoffersen.

In case of the 3 year, the Variance Covariance failed all the years for z and Kupiec while it only failed in 2006 for Christoffersen and for the rest of the years it cleared. Historical Simulation model only passed in 2006 by z and Kupiec, in the same year it failed Christoffersen. The Monte Carlo model failed z and Kupiec in all the years while passed Christoffersen in all the years except 2006.

During and post crisis: The period from 2008 to 2013

Nearly during the end of 2007 and beginning of 2008 exceedances start to appear as a result of which re-emergence of volatility occurred but VaR models couldn't adopt quickly enough and as consequence the models failed on the upside due to a lot of exceedances than expected. Therefore, from 1 year to 3 year z and Kupiec value increase in the period of 2008, all ranging outside the critical value which resulted in failing of all the three models. However, in case of all 1, 2 and 3 year VaR for 2008 all the three models cleared Christoffersen.

The models started to adopt volatility in 2009 and by 2010 abets somewhat and the models again start to overestimate risk.

For 1 year VaR period ranging from 2009 to 2013, the Variance Covariance failed z Kupiec and Christoffersen in 2010. In addition, in 2013 it failed z and Kupiec, for all the other years it cleared it all. Whereas, the Historical Simulation VaR model only failed all the three tests in 2010 while for rest of the years it cleared it all. Monte Carlo VaR model shows failure for the z and Kupiec only in 2013. While it failed Christoffersen in 2010. For all other years it cleared all the three tests.

For 2 year, the Variance Covariance again failed z test and Kupiec both in 2010 and in 2013 while cleared for all the other years. While the model cleared Christoffersen for all the years. In case of Historical Simulation the results appear to be in line with Variance Covariance failure in case of first two test for the same time frame. Whereas Christoffersen cleared it all the years. Monte Carlo Simulation too exhibits the same results as of Variance Covariance and Historical Simulation.

For 3 year VaR, the Variance Covariance exhibits failure in 2009, 2011 and 2013 by z test whereas for Kupiec the failure is displayed only in 2011 and 2013. The model failure in 2010 by Christoffersen only. For result of the years it cleared the tests. Historical Simulation VaR model failed z and Kupiec in 2011 and 2013 while Christoffersen in 2009 and 2010. Monte Carlo VaR model failed only in 2011 and 2013 by z and Kupiec and in 2010 by Christoffersen.

It is observed generally for the same years, models failed to clear the tests. In addition z and Kupiec seemed to produce similar results in terms of rejection and acceptance. z could be termed as a crude approximation to Kupiec, by and large both of them correspond. However, the three models seem to pass Christoffersen test for most of the years, particularly for 2008 with respect to the 1 year, 2 year and 3 year data. Although scrutinised in detail for the individual years. It is essential to have an aggregate results examined too.

Variance Covariance model in case of 1 year passed both z and Kupiec with results 0.627 and 0.387 both the values lying within the range of respective critical values however failed Christoffersen test as the result produced was 15.685 outside the range of the critical value of 3.8415. In case of 2 year the first two tests again produce results within the range of critical value with z 1.571 and Kupiec 2.368 while for Christoffersen the model failed with results of 16.863. In case of 3 year the results for z, Kupiec and Christoffersen are 3.680, 12.312 and 47.055 respectively. Hence, it is inferred:

Null (H_0) hypothesis fails to reject for the z and Kupiec tests for 1 year and 2 year.

Null (H_0) hypothesis is rejected for the z and Kupiec tests for the 3 year.

Null (H_0) hypothesis is rejected for the Christoffersen for 1, 2 and 3 year.

Historical Simulation failed all the three tests in case of 1 year, 2 year and 3 year. The results obtained for 1 year VaR were z 2.270, Kupiec 4.875 and Christoffersen 16.791. 2 year and 3 year produced z 2.479, 4.159 and Kupiec 5.768, 12.625 and Christoffersen 12.625, 33.494 respectively. Therefore, the following conclusion is drawn:

Null (H_0) hypothesis is rejected for all the three tests.

In case of z, Kupiec and Christoffersen it is over and above the critical value.

All though Historical Simulation performed unsatisfactorily, the 1 year VaR estimates of z and Kupiec were better than 2 year and 3 year, indicating when small size sample of 1 year calibration used for Historical Simulation proved to be better than the 2 year, similarly 2 year was better than the 3 year. The results in the context contradict with the claim made by Hendricks (1996) and Pritsker (2006) who stated that the model performs poorly for a small size sample.

In case of first two tests exceedances are higher thus, the model may overestimate the risk computations. In case of Christoffersen exceedances produced could not hold the property of independence.

Monte Carlo Simulation in case of 1 year clears both z and Kupiec with 0.627 and 0.387 however fails the Christoffersen with 14.566. In case of 2 year failure is experienced with z 1.848 and Christoffersen 16.852. However, Kupiec test was passed by 3.256. In case 3 year all model fails in all the three tests with z 3.393, Kupiec 10.535 and Christoffersen 45.404. Thus, it is concluded:

Null (H_0) hypothesis fails to reject for the z and Kupiec tests for 1 year.

Null (H_0) hypothesis is rejected for the z in case of 2 year.

Null (H_0) hypothesis fails to reject for the Kupiec for 2 year.

Null (H_0) hypothesis is rejected for the Christoffersen for 1, 2 and 3 year.

When examined year on year basis, all the three models mostly cleared Christoffersen test. However, on examining the aggregate results all the models failed the test. The results produced ranged out of the critical value 3.8415 this indicates clustering.

Considering the z, Kupiec and Christoffersen values mentioned in table above. It is concluded that when the values are above the significance level it acts as weak evidence and as a result null hypothesis fails to reject and when the test values are below the alpha it leads to rejection of null hypothesis. Therefore, it can be stated that Variance Covariance models for 1 year and 2 year pass the unconditional coverage test where as in case of Monte Carlo 1 year passes the test and in case of 2 year only Kupiec clears whereas z fails the test.

Christoffersen interval forecast test fails in all the three cases. Thus, Partly optimistic results of z and Kupiec suffers from the impact of clustering.
As seen generally 1 year results are the best as compared to the 2 year and 3 year this is mainly because 1 year VaR model quickly reacts to bullish and bearish market. However the back testing results for Historical Simulation appear to be worst which is unexpected. The Variance Covariance model and the Monte Carlo Simulation (particularly in this case) assume normality of risk and returns. However, Historical Simulation does not make such assumptions and it is expected in times of enhanced volatility, models that rely on normal assumptions would underperform those that do not. Whereas, the results found Variance Covariance and Monte Carlo outperformed the Historical Simulation VaR model.

The main aim of the research was to build and test all the three models to evaluate the performance, reliability and efficacy so as to select the best for an Irish equity portfolio. However all the three models could not survive the financial turbulence and instable market conditions at 95% confidence level. The risk computation by all the three models appeared to be in line considering longer span of time however when tested individually by the three tests for year on year basis, only for a handful number of years all the three tests were cleared. Moreover, when examined on an aggregate basis such as 1 year, 2 year and 3 year, the z and Kupiec results were reasonable however, the Christopherson test failed for all of models. Therefore it can be concluded that the models deem to function well in normal market conditions. However fail in the climate of extreme market volatility due to non-adaptation of significant market shifts as the models are set and function according to their own parameters

CHAPTER 6

Discussion

The research displayed the constructed models and back tested results. The models performance and efficacy have been exhibited so as to select the best one for an Irish equity portfolio. The Variance Covariance, Historical Simulation and Monte Carlo Simulation give out different results when back tested. In general, the z and Kupiec tests correspond in many instances however Christoffersen highlighted totally different results, where the former two agreed the latter disagreed. Therefore, it could be said that the results are mixed, which does not interpret and answer the decision regarding the best model.

The results therefore appear to be in line with the literature that mentioned no model can be deemed as best, all of them have their own peculiarities, uses and limitation. The results of the research are pursuant to the claims made by Pritsker (1997); Jadhav and Ramanathan (2009); Lechner and Ovaert (2010) and Cheung & Powell (2012) that no particular method can be termed as the best.

The section also aims to make the reader aware of the limitations of the three models and the various back testing methods executed. It should be noted that although limitations but they may also be extending into future research.

VaR Models Limitations:

Although the research has been carried out using different classic methods to estimate future Value at Risk, it hasn’t considered advanced and extended version termed as Conditional Value at Risk. Conditional Value at Risk is the shift which Basel Committee is working on. It imbibes the qualities of sub-additivity and coherence (Lim et al., 2011). However, it is an extended version therefore, computation of classic VaR models is an obligation.

Another aspect involves describing the models itself as one and extending advanced upgradation of the same for instance Variance Covariance involves computation through ARCH, GARCH. Whereas Historical Simulation uses Filtered Historical Simulation and Monte Carlo through Principal Component Analysis.

The advances models could be applied to mix portfolio's such as equities, commodities and currencies altogether. In addition it could also be applied to non-linear portfolios such as derivatives that depend on number of factors such as spot price, maturity time and volatility, which the research does not consider. This could be a potential for further research with requires large data size and relatively longer time.

In addition, equity portfolio has been considered using equal delta. Change in the proportion of deltas or totally changing the same to weights could test efficacy and efficiency in a better manner. Moreover the risk horizon considered only limits to 1 trading day at 95% confidence level. The scope could have been broadened using long term horizon such as a year or a mix of two involving short as well as long. The limitation is also suspected in case of confidence level, 99% and 97.5% have not been considered.

As seen in the research the answer of best model was difficult one of the potential reasons for the same could have been the data size comprising only 10 stocks and also the time flame which was limited to only 11 years.

In terms of simulation the number is only restricted to around 3,000. Increase in simulation up to 10,000 would have been difficult on Excel as the computation would have very time consuming. On the other side use of R-studio though was only undertaken for the verification purpose it proved to be very quick and flexible. Therefore, future research could be conducted using such advance platform and to verify the results another advanced platform such as SPSS or MATLAB could be used.

Back Testing Limitation:

Back testing is also restricted to 95% confidence level. 99% or 97.5% or 90% would have given clearer picture as the with the change in level of significance the p-values change. Research could also be done on the VaR and back testing methods with different level of confidence. For instance, VaR could be measured at 95% whereas back testing at 99% or both 95% and 99%. Back Testing has been carried out with three tests z, Kupiec and Christoffersen. The first two are unconditional coverage type of back testing methods while the third one is independence test. The research has not used joint of Christoffersen and Kupiec tests.

According to Christoffersen (1998) a back testing method should satisfy two qualities which include unconditional coverage and independence property. With regards to first property the realised loss should always be more than the VaR estimate which is reported. The loss in addition should be exactly what alpha multiplied by 100% would be. In respect to the independence property unconditional coverage should be able to put a limit in relation to the number of violations generated.

The research has been limited to only the use of unconditional coverage and independence test, conditional coverage has not been considered. However the results in any case would have been different as all the tests consider different assumptions. Although the future research can be done on the conditional coverage back testing, it will require the use of unconditional coverage and independence back testing. Hence, immense scope lies in further study with regards to not only conditional coverage test but also other advanced back testing methods. In addition, back testing is the area where a lot of experiments can be carried out by using a mix of various tests to measure the performance and validity of the VaR models.

CHAPTER 7

Conclusion

The main aim of the research was to select the best model out of the three Variance Covariance, Historical Simulation and Monte Carlo Simulation suitable for an investor investing in Irish equity portfolio. The time frame considered was 11 years from 2002 to 2013. The period was so selected as the main focus was on the financial crisis of 2008. To attain the objective, three models were built on Excel and R-Studio. Two platforms were used only for the verification purpose. In addition, three back testing methods z, Kupiec and Christoffersen were also implemented to validate the models.

The models were built considering the risk horizon of 1 trading day at 95% confidence level. The same confidence level was used for back testing. The equity portfolio comprised 10 stocks listed on Irish Stock Exchange (ISEQ), which were carefully selected from different sectors so as to display the true representation of Irish Stock Exchange. Equal investment amounting to 1 million euros were invested in each asset in the portfolio. The models were well constructed and were error free as both the platforms produced the same results. The 1 year VaR model reacted quickly at the time when volatility increased as well as when it decreased when compared to the 2 year and 3 year VaR models.

To validate the same back testing was done by three methods and results were analysed year on year basis. The conclusion drawn was all the tests gave differing results in most cases z and Kupiec tests corresponded whereas Christoffersen did not. All the three models failed to survive in times of extreme market volatility as inferred by z and Kupiec although they survived the Christoffersen test which only indicated that the exceedances generated were independent. Therefore, it could be strongly concluded that three models are unfit during the extreme market events such as financial turbulence.

When the aggregate performance was examined based on 1 year, 2 year and 3 year for all the models. 1 year results validated Variance Covariance and Monte Carlo Method by z and Kupiec however was not by Christoffersen. Similarly, 2 year results validated Variance Covariance by z and Kupiec. In case of Monte Carlo, z test was cleared whereas Kupiec wasn’t. Models failed the Christoffersen test.

In case of 3 Year all the models failed all the three tests. Historical Simulation failed all the three tests for 1 year, 2 year and 3 year. This was precipitous as Historical Simulation approach has one of the distinct advantages, it is thought to better account for non-normality of data whereas Variance Covariance and Monte Carlo Simulation (particularly adhering to this research) by their construction assume normality of returns however, the back testing results did not show that because when a model not bounded by normality assumption back tests better in chaotic times whereas that did not occur. In addition, none of the models 1 year, 2 year and 3 year Historical Simulation were produced satisfactory results, although 1 year performed better than 2 year. Whereas when compared 2 year and 3 year. The 2 year performed better. The results in turn were contrary to the results founded by Hendrick (1996) and Pritsker (2006) who stated that small size data leads to poor performance of the model.

Although, for all the three models 1 year tends to outperform the 2 year, which when compared to the 3 year outperforms but on comparison made amongst the three the Variance Covariance and Monte Carlo Simulation outperforms Historical Simulation. Over all, none of the models can be termed as best because all the three performed well during the normal market scenario but none of them could survive the period of financial crisis. The three models fundamentally failed to produce Value at Risk figure beyond their own criteria.

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Appendices

1. Cholesky decomposition

```
Function Cholesky(Mat As Range) As Variant
Dim A() As Double, L() As Double, sum As Double, sum2 As Double
Dim m As Byte, i As Byte, j As Byte, k As Byte
'Ensure matrix is square
    If Mat.Rows.Count <> Mat.Columns.Count Then
        MsgBox ("Correlation matrix is not square")
        Exit Function
    End If
    m = Mat.Rows.Count
'Initialize and populate matrix A of values and matrix L which will be the lower Cholesky
    ReDim A(0 To m - 1, 0 To m - 1)
    ReDim L(0 To m - 1, 0 To m - 1)
    For i = 0 To m - 1
        For j = 0 To m - 1
            A(i, j) = Mat(i + 1, j + 1).Value2
            L(i, j) = 0
        Next j
    Next i
'Handle the simple cases explicitly to save time
    Select Case m
        Case Is = 1
            L(0, 0) = Sqr(A(0, 0))
        Case Is = 2
            L(0, 0) = Sqr(A(0, 0))
            L(1, 0) = A(1, 0) / L(0, 0)
            L(1, 1) = Sqr(A(1, 1) - L(1, 0) * L(1, 0))
        Case Else
            L(0, 0) = Sqr(A(0, 0))
            L(1, 0) = A(1, 0) / L(0, 0)
            L(1, 1) = Sqr(A(1, 1) - L(1, 0) * L(1, 0))
            For i = 2 To m - 1
                sum2 = 0
                For k = 0 To i - 1
                    sum = 0
                    For j = 0 To k
                        sum = sum + L(i, j) * L(k, j)
                    Next j
                    L(i, k) = (A(i, k) - sum) / L(k, k)
                    sum2 = sum2 + L(i, k) * L(i, k)
                Next k
                L(i, i) = Sqr(A(i, i) - sum2)
            Next i
    End Select
    Cholesky = L
```

2. Macro VBA Code for Monte Carlo Simulation.

```
Sub Run_1y_MC_VaR()

For i = 1 To 2816
    Sheets("Monte_Carlo").Range("A1").Value = i
    my_date = ActiveSheet.Range("a2").Value
    my_vcv_var = ActiveSheet.Range("R4").Value
    my_mc_var = ActiveSheet.Range("R5").Value
    ActiveSheet.Range("T1").Offset(i, 0).Value = my_date
    ActiveSheet.Range("T1").Offset(i, 1).Value = my_vcv_var
    ActiveSheet.Range("T1").Offset(i, 2).Value = my_mc_var
Next i
End Sub


Sub Run_2y_MC_VaR()

For i = 1 To 2555
    Sheets("Monte_Carlo2").Range("A1").Value = i
    my_date = ActiveSheet.Range("a2").Value
    my_vcv_var = ActiveSheet.Range("R4").Value
    my_mc_var = ActiveSheet.Range("R5").Value
    ActiveSheet.Range("T1").Offset(i, 0).Value = my_date
    ActiveSheet.Range("T1").Offset(i, 1).Value = my_vcv_var
    ActiveSheet.Range("T1").Offset(i, 2).Value = my_mc_var
Next i
End Sub


Sub Run_3y_MC_VaR()

For i = 1 To 2293
    Sheets("Monte_Carlo3").Range("A1").Value = i
    my_date = ActiveSheet.Range("a2").Value
    my_vcv_var = ActiveSheet.Range("R4").Value
    my_mc_var = ActiveSheet.Range("R5").Value
    ActiveSheet.Range("T1").Offset(i, 0).Value = my_date
    ActiveSheet.Range("T1").Offset(i, 1).Value = my_vcv_var
    ActiveSheet.Range("T1").Offset(i, 2).Value = my_mc_var
Next i
End Sub
```

3. R code used for building 3 Models in R-Studio.

```
S <- as.data.frame(StockPrices)
my_1y_HS_var <- vector()
my_2y_HS_var <- vector()
my_3y_HS_var <- vector()
my_1y_VCV_var <- vector()
my_2y_VCV_var <- vector()
my_3y_VCV_var <- vector()
my_1y_MC_var <- vector()
my_2y_MC_var <- vector()
my_3y_MC_var <- vector()
delta_vect <- vector()
N <- 10000
for (i in 1:10){
  delta_vect[i]=1e5 # Assume a holding of 100k in each asset
}
for(i in 1:(length(S$Date)-260)){
  my_1y_HS_var[i]=HSVaR(S,i,i+260,delta_vect,0.05)
  my_1y_VCV_var[i]=VCVVaR(S,i,i+260,delta_vect,0.05)
  my_1y_MC_var[i]=MCVaR(S,i,i+260,delta_vect,0.05,N)
}
for(i in 1:(length(S$Date)-260*2)){
  my_2y_HS_var[i]=HSVaR(S,i,i+260*2,delta_vect,0.05)
  my_2y_VCV_var[i]=VCVVaR(S,i,i+260*2,delta_vect,0.05)
  my_2y_MC_var[i]=MCVaR(S,i,i+260*2,delta_vect,0.05,N)
}
for(i in 1:(length(S$Date)-260*3)){
  my_3y_HS_var[i]=HSVaR(S,i,i+260*3,delta_vect,0.05)
  my_3y_VCV_var[i]=VCVVaR(S,i,i+260*3,delta_vect,0.05)
  my_3y_MC_var[i]=MCVaR(S,i,i+260*3,delta_vect,0.05,N)
}
par(mfrow=c(3,1))
plot(S$Date[261:length(S$Date)],my_1y_HS_var,type='l',main='1 Year VaR',xlab='Date',ylab='VaR (Eur)',col='blue')
lines(S$Date[261:length(S$Date)],my_1y_VCV_var,col='red')
lines(S$Date[261:length(S$Date)],my_1y_MC_var,col='green')
legend("bottomleft",legend = c("Historic Simulation","Variance Covariance","Monte Carlo"),col=c("blue","red","green"),
grid()
plot(S$Date[521:length(S$Date)],my_2y_HS_var,type='l',main='2 Year VaR',xlab='Date',ylab='VaR (Eur)',col = 'blue')
lines(S$Date[521:length(S$Date)],my_2y_VCV_var,col='red')
lines(S$Date[521:length(S$Date)],my_2y_MC_var,col='green')
legend("bottomleft",legend = c("Historic Simulation","Variance Covariance","Monte Carlo"),col=c("blue","red","green"),
grid()
plot(S$Date[781:length(S$Date)],my_3y_HS_var,type='l',main='3 Year VaR',xlab='Date',ylab='VaR (Eur)',col = 'blue')
lines(S$Date[781:length(S$Date)],my_3y_VCV_var,col='red')
lines(S$Date[781:length(S$Date)],my_3y_MC_var,col='green')
legend("bottomleft",legend = c("Historic Simulation","Variance Covariance","Monte Carlo"),col=c("blue","red","green"),
grid()
```

4. VBA code for Kupiec and Christoffersen

```
'KupiecTest
PI_obs = sumexcept / sampsize
LL_Null = Log((1 - bin_p) ^ (sampsize - sumexcept) * bin_p ^ sumexcept)
LL_Alt = Log((1 - PI_obs) ^ (sampsize - sumexcept) * PI_obs ^ sumexcept)
kup = -2 * (LL_Null - LL_Alt)
Kup_crit = WorksheetFunction.ChiInv(alpha, 1)



'ChristTest
Pi01 = n01 / (n01 + n00)
Pi11 = n11 / (n10 + n11)
LL_Null = LL_Alt
LL_Alt = Log(Pi01 ^ n01 * (1 - Pi01) ^ n00 * Pi11 ^ n11 * (1 - Pi11) ^ n10)
Chr = -2 * (LL_Null - LL_Alt)
```

5. <u>The Excel Generated Graphs</u>