Historical Simulation Value at Risk and Expected Tail Loss: A test of reliability in a modern financial climate

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Abstract

In 2016, the Basel Committee for Banking Supervision intends to implement mandatory changes to the way in which financial institutions and banks measure the risk associated with capital requirements. This shift will move the focus from Value at Risk models to Expected Tail Loss models. This report set out to determine if this shift is warranted. To do this, Value at Risk and Expected Tail Loss models were created using the Historical Simulation methodology at both 95% and 99% confidence levels. The Expected Tail Loss model was constructed as an extension of the VaR model. These models were then divided into smaller models based on individual years.

All models were then back-tested using simple hypothesis tests in order to establish their reliability. It was found that VaR models are not reliable in periods of market volatility. Expected Tail Loss however was found to be reliable at both the 95% and 99% confidence levels.

These results would seem to support the shift from VaR to Expected Tail Loss to some extent although considering the ETL model is built from the Value at Risk model, it may be more beneficial to use both tools instead of one individual.

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Introduction

Risk management was once a secondary thought in most financial institutions. There were certainly no dedicated departments that focused on risk as a fundamental function of the organisation. According to Covello and Mumpower (1984, p.1) however, the first instances of a risk analyst can be traced back to 3200 B.C. in the Tigris – Euphrates valley. Regardless of whether there has been designated functional departments for risk, people have been dealing with risk in measured and quantitative ways for a very long time. As civilization has developed and trade has moved beyond tangible goods to complicated financial products, the need for more complex risk models has also developed.

As companies are now no longer bound to their own domicile countries due to the rise of globalisation, financial transactions have increased in risk and value. Multi-National banking and financial regulators such as the Basel Committee for Banking Supervision and the European Central Bank have made risk management mandatory. In previous years, financial institutions were free to choose their own risk measure although this has changed with the more frequent fluctuations in markets and thus these regulators moved to give some level of uniformity to how risk is managed and handled throughout the companies and countries that fall in to their jurisdiction.

Initially Value at Risk was the model that the Basel Committee for Banking Supervision insisted on banks and financial companies using when managing risk across their organisations. Value at Risk is a statistical measurement used to assign a value to quantifiable levels of risk that a portfolio may be vulnerable to. It is usually expressed across a certain timeframe with a specific percentage of confidence, usually either 95% or 99%. As Linsmeier and Pearson (2000, p.48) describe it, “VaR is a single, summary statistical measurement of possible portfolio losses.” However, when the latest financial crash took place in 2007/2008, these VaR models were heavily criticised. As recent as January 2016, the Basel Committee for Banking Supervision declared that there would be a shift from Value at Risk to Expected Tail Loss. Expected Tail Loss is the average expected losses beyond VaR. With the ever increasing criticisms of VaR, this research will seek to determine whether the criticism of a model that has been an industry standard for almost 25 years is really as unfit for purpose as current opinion would suggest. This research will also use that same process to determine whether Expected Tail Loss is indeed as robust and coherent as current opinion does suggest.

This research will look to construct multiple VaR and expected Tail Loss models from a Historical Simulation methodology and test whether these models can be deemed reliable in the context of the modern financial landscape. It is expected that by doing this, there will a strong position to take in either manner with regards to the reliability of both models and how they have performed over the last ten years. Particular attention will be placed on the performance of both models during the years of the financial crisis and the years immediately preceding.

Academics and industry experts are now seeking to move away from VaR as a risk management model. This research will look to deduce whether that shift is warranted or is it an almost sub-conscious reaction to the financial crash in which the financial industry is seeking to rationalise the irrational.

Literature Review

A History of Value at Risk and Expected Tail Loss

Value at Risk (VaR) was introduced in the early 1990's as a method of easily quantifying the possible losses a trading portfolio may encounter over a specific time to a specific certainty. Some however argue that Value at Risk as it is now can trace its lineage even further back. Glyn Holton (2002, p.1) believes that Value at Risk as it is now can be traced back to the 1920's when the New York Stock Exchange set capital requirement protocols on all listed companies. In fact, Holton (2002, p.14) further believes that one of the major turning points in the widespread adoption of Value at Risk throughout financial institutions across the United States was the weakening and eventual repeal of the Glass-Steagall act which removed the divide between commercial and investment banking and allowed banks to adopt far greater levels of risk. Banks who normally would not have dealt in the securities markets began to invest in securities from which they would have before been prohibited from entering. This greater adoption of risk within these banks prompted greater need for a uniformity across organisations in the way in which risk was quantified and thus Value at Risk became more and more embraced.

While the repeal of Glass-Steagall was undoubtedly a turning point for the embrace of Value at Risk throughout the financial industry, Darryll Hendricks (1996) believes that JP Morgan introduction of its RiskMetrics database that allowed outside users conduct their own Value at Risk calculations was a watershed moment for Value at Risk. This is a sentiment that is echoed by Linsmeier and Pearson (2000, p.47) who believed that Value at Risk truly gained industry wide usage when JP Morgan introduced its RiskMetrics system in 1994 which it hoped would become an industry standard. As Linsmeier and Pearson (2000, p.48) also highlight, the use of Value at Risk became so widespread across the financial and banking industries that regulators such as the Basle Committee on Banking Supervision and the Securities and Exchange Commission began to insist on banks using Value at Risk as a method of calculating the risk associated with their capital requirements. It is undoubtedly true that while Value at Risk can trace its origins back to the 1920's in some form or other, it was the late 1980's and early 1990's that saw Value at Risk become the preeminent risk measure in the financial and banking industries.

Expected Tail Loss in comparison to Value at Risk, a relatively new tool that is used by risk management teams. Expected Tail Loss (ETL) can also be known as Expected Shortfall, Conditional Value at Risk (CVaR) or Average Value at Risk (AVaR). For the purpose of this report however, it shall only be referred to as Expected Tail Loss. As highlighted above, from the 1980's right through to the mid to late 1990', Value at Risk was the risk measure of choice and to some extent has remained so through to modern day. This does not mean that Value at Risk is without its critics and to some extent was the reason for the introduction of ETL. Research papers published by Artzner et al. in 1997 and 1999 called in to question that validity of Value at Risk as a reliable measure of risk in real world practise. As Acerbi and Tasche (2001, p.2) believed, the gap between academic theory in relation to Value at Risk and its application and validity in real world scenarios was greatly widening and thus another risk measure with more stringent properties needed to be found. This need would be filled in some part by Expected Tail Loss. In fact, while the use of Value at Risk as a risk measure was formally recommended and to some degree required by the Basel Committee of Banking Supervision as a risk measure on Capital Requirements, a report published in January 2016, the fundamental review if the trading book, seeks to shift away from Value at Risk and move to an Expected Tail Loss Model. As the report by the Basel Committee (2016, p.1) stated, "Use of ES will help to ensure a more prudent capture of "tail risk" and capital adequacy during periods of significant financial market stress."

With this shift, there is a belief that the adoption of ETL by the Basel Committee of Banking Supervision will make Value at Risk redundant in much the same way the adoption of Value at Risk by regulators made Value at Risk the risk measure of choice. This report seeks to test the validity of Value at Risk and ETL and show that despite the academic and regulatory belief that Value at Risk is no longer reliable, it should be used in conjunction with ETL to produce a fuller and more dependable risk profile.

Historical Simulation Value at Risk and its strengths

The purpose of the following sections in which the strengths and weaknesses of both models are highlighted is to show that prior to the analysis conducted of both models, there will be an understanding that both models have underlying flaws and that where one model may be weak, the other will perhaps be able to account for this weakness. It is hoped that by sufficiently highlighting these strengths and weaknesses, the reader will be able to draw their own conclusions as to the reliability of both models regardless of this reports findings.

When Value at Risk was first adopted, it was a major improvement on how risk exposure was communicated throughout financial institutions. Investment managers for example soon realised that they could apply Value at Risk models to a range of financial instruments and the model did not break down. This was one of the main reasons why Value at Risk became so popular and as Žiković (2008, p.2) explained, that when looking at Value at Risk and its performance and uses against previous techniques, Value at Risk could be used to compare risk in equity portfolios and fixed income portfolios. This allowed investors to compare the risk exposure associated with a multitude of different portfolios and develop their own personal risk appetites. The ease at which Value at Risk can be communicated is also down to the way in which VaR is reported. Value at Risk can be reported with a simple quantifiable monetary or percentage value that even those who do not understand the methods of calculating Value at Risk can easily understand. It assigns a simple value with which non-quantitatively minded people can base investment or portfolio decisions.

One of the most important things to consider when discussing Historical Simulation Value at Risk is that it does not make assumptions on the distribution of returns. The reason that this research was conducted solely on Historic Value at Risk is that Value at Risk based on a Variance-Covariance model are generated from simulations that involve the standard deviation of the returns over time that have been assumed to follow a normal distribution. One thing that can be said without any doubt is that returns do not follow a normal distribution. In fact, when graphing the realised Profit and Loss of the US Treasury rates portfolio, the presence of fat tails can be seen which is in direct contradiction to normal distribution. This can be seen in Figure 1 below.

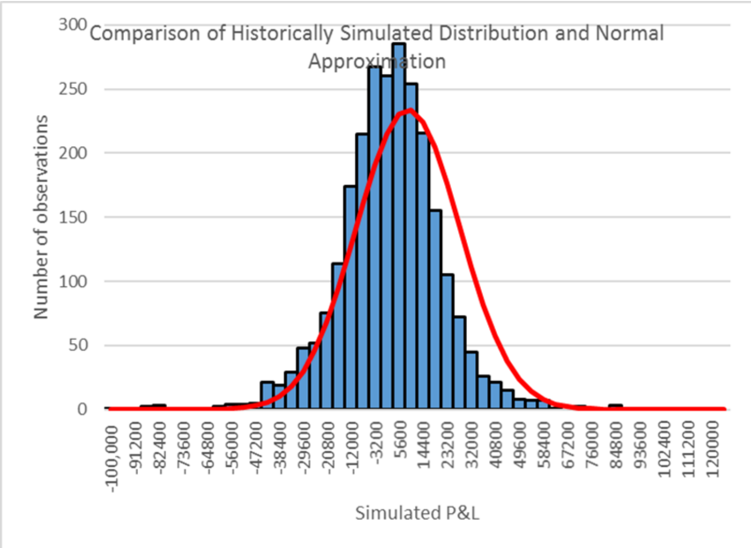

Figure 1: Comparison of Historical Simulation Distribution -v- Normal Distribution

This sentiment is one echoed by many academics when discussing Historic Simulation Value at Risk. In fact, Carol Alexander (2008, p.44) states that "one great advantage of historical Value at Risk is that it makes few distributional assumptions. No assumption is made about the parametric form of the risk factor return distribution, least of all multivariate normality". Linsmeier and Pearson (1996, p.7) also make reference that because of the lack of assumptions made about the distribution in Historical Simulation, it is a simpler and more intuitive model to run.

Another strength of Historic Simulation Value at Risk is that it is easy back-tested and therefore its validity is easier to prove or disprove as the case may be. This may also be referred to as elicitability by some academics. Johanna Ziegel (2014, p.4) believes that one of the most import characteristics of a risk measure should be its elicitability and it is in this characteristic that Value at Risk performs excellently. Ziegel (2014, p.1) gives greater definition to the idea of elicitability when she states that "in statistical decision theory, risk measures for which such verification and comparison is possible, are called elicitable." In fact, Bellini and Bignozzi (2014, p.2) also echo this sentiment where they state that elicitability is almost the key characteristic of risk measures as it creates a natural path to a back-test. Historic Simulation Value at Risk allows the user to measure times the realised profit and loss exceeded the expected profit and loss and conduct a simple hypothesis test in order to measure the significance of the results. All Value at Risk models are elicitable, however Historic Simulation is conducted with far greater ease and computing time.

Finally, one further strength of Historical Simulation Value at Risk is the ease at which it can be computed with limited sample data sizes. Unlike other risk measures, Value at Risk does not need large sample sizes to calculate relatively accurate results. This means that the information costs associated with Historic Simulation Value at Risk are significantly lower and yet it does not lose much of its accuracy. Emmer et al. (2015, p.23) have noted this distinction that even with smaller data samples Value at Risk can still function as intended to a significantly accurate degree. Yamai and Yoshiba (2005, p.1012) whose report is somewhat critical of Value at Risk also make this distinction in favour of Value at Risk. They go on to explain that when compared to other risk measures, Value at Risk has a significantly lower estimation error when dealing with distributions of returns with fat tails. The idea of these fat tails shall be further explored below.

Historical Simulation Value at Risk and its weaknesses

The first criticism that can be levelled at Historical Simulation Value at Risk and in fact all Value at Risk models is that it is limited by its own defined parameters. Value at Risk by its very definition is a measure of risk that puts a value on possible losses that a portfolio may suffer over a specified time period with a specified confidence level. An example of these parameters would be a one-day VaR with a confidence level of 95%. This means that beyond its own calculations, Value at Risk cannot distinguish whether a loss may be slight or catastrophic to the value of an investment. This limitation is another example of a flaw that may lead to over-confidence amongst investors that do not fully comprehend what Value at Risk can and cannot deduce. Yamai and Yoshiba (2005, p.998) further expand on this sentiment where they claim that rational investors whose strategies are based around a small risk appetite may make decisions based on Value at Risk that are not based on accurate risk representation. This parameter limitation that affects Value at Risk is also something that may be exacerbated during volatile markets with a greater level of fluctuation. This is obviously due to the fact that Historic Simulation Value at Risk that has been calculated using data sampled from previous periods where there was possibly a more stable market will have no reference point in regards to the instability being encountered at that time.

This parameter limitation has also been heavily discussed by Nassim Taleb in his book Black Swan. Taleb defines a black swan event as having three distinct characteristics:

1. They are unpredictable events.
2. It completely upsets the market in a way that takes time to recover.
3. Experts try to create a rationale that allows it to be explained and claim that it was someone not regulating the market properly and it could have been avoided. (2007, prologue)

Value at Risk would predict that these great market fluctuations should happen at most once every twenty years which falls into the 95% confidence level and the most frequently used version of Value at Risk. However as can be seen, these market fluctuations tend to happen twice every ten years and that is when observed Value at Risk exceedances tend to greatly outnumber expected Value at Risk exceedances. This flaw of Value at Risk also tends to weaken its position as a coherent and robust risk measure.

One of the major reasons financial institutions become active in portfolios is for the benefits of sub-additivity. Sub-additivity means that as a portfolio diversifies, it reduces the risk associated with the portfolio across all asset classes. Danielsson and Jorgenson define sub-additivity as such:

> Subadditivity ensures that the diversification principle of modern portfolio theory holds since a sub-additive measure would always generate a lower risk measure for a diversified portfolio than a non–diversified portfolio. (2005, p.4)

The idea of simple portfolio theory not applying to Value at Risk is a serious criticism that can definitely undermine the reliability and trust worthiness of the Value at Risk model. Artzner et al. (1998, p.6) concurs that a natural requirement of a portfolio or any diversification strategy should be that it should not create extra risk for the investor. Johanna Ziegel (2014, p.2) believes that this lack of sub-additivity by Value at Risk ensures that it cannot be considered a coherent measure of risk. This lack of sub-additivity is something that Danielsson and Jorgenson identified in their paper as having knock on effects to investors that may not be fully aware of the limitations of certain models. This point may be one of the more crucial aspects raised in regards to this report on the reliability of Value at Risk and its place in a modern financial world. As Danielsson and Jorgenson clarify:

> it can lead a financial institution to make a suboptimal investment choice, if Value at Risk, or a change in Value at Risk, is used for identifying the risk in alternative investment choices. (2005, p.2)

It is with this statement by Danielsson and Jorgenson that Value at Risk as a risk measure may be deemed to be not as reliable as other models. Danielsson and Jorgenson are not the only researchers who believe that when it comes to portfolio diversification benefits, Value at Risk comes up very short. As Frey and McNeil exclaim:

> while it is admittedly not very likely that we will observe the worst features of Value at Risk for some randomly chosen portfolio, the picture changes, if investors optimize the (expected) return on their portfolios under some constraint on Value at Risk, as the portfolios resulting from such an optimization procedure do exploit the conceptual weaknesses of Value at Risk. (2002, p.5)

One final limitation that is somewhat limited to Historic Simulation Value at Risk is that past performances of a portfolio are in no way a representation of what may happen in the future. As time passes, flaws and weaknesses that may have been in a market may be regulated away or made redundant through advances in financial technologies. This may not be a weakness that is solely limited to Value at Risk and may be somewhat be applicable to all models of risk that depend on Historic Simulation in order to produce results. As investors believe that previous market flaws have been imbedded in to their model by the data they are sampling, new market frailties may appear. This may lead rationale investors to make irrational decisions based on a Value at Risk calculation that is not up to date. While this should be a concern for any investor using a historic simulation model, it is probably also one of the more intuitive flaws with Value at Risk and as such may not be as big a problem as some of the other flaws mentioned above. It is also worth noting that Historical Simulation Value at Risk cannot be used on portfolios where an asset is a new product that has never been sold before. This would have been of particular annoyance when Electronically Traded Funds that allowed a spread exchange were released. Felix Salmon gives a brief description of these financial products:

> Factor Advisors, a New York-based asset management firm, announced today the launch of FactorShares, the first family of spread exchange traded funds (ETFs) that allow sophisticated investors to simultaneously hold both a bull and a bear position in one leveraged ETF. (2011)

These products were new and therefore there would not have been past data to sample if they were added to a portfolio and thus Historical Simulation would not have been appropriate. There is also that matter of the fact that this product allows the investor to hold simultaneous positions and thus makes the risk associated with such an asset more difficult to compute. This also disagrees with the sentiment that Historical Simulation Value at Risk can work on any asset in a simply manner.

Historical Simulation Expected Tail Loss and its strengths

Expected Tail Loss is a measure that has been proposed as a replacement for Value at Risk by not only academics and industry experts but by the Basel Committee for Banking Supervision. The reason for this is that they believe Expected Tail Loss is a more robust and cohesive risk measure and in most ways they are correct. One thing worth mentioning before further examining the strengths of Expected Tail Loss as a risk measure is that many of the advantages of Expected Tail Loss are in direct competition with the weaknesses in Value at Risk models.

Expected Tail Loss has its strongest characteristic in the fact that it is indeed sub-additive in nature. As mentioned previously, sub-additivity is one of the most important aspects of any risk measure model as without it, the model will directly contradict basic concepts of portfolio theory. The very fact that Expected Tail Loss satisfies these criteria is a substantial improvement on the lack of this characteristic from Value at Risk. As Žiković (2008, p. 7) states, the idea that Expected Tail Loss is sub-additive is the most important aspect of any logical risk measure specifically in relation to portfolios. Žiković then adds that in a practical environment beyond academia, the most important property of any risk measure is that it is sub-additive. As Acerbi (2003, p.5) further reflects on the importance of sub-additivity to investors where he adds, "sub-additivity is even more important when we turn to decision-making through risk measures." This also does not bode well for Value at Risk as this model can only be seen to be sub-additive in a normal distribution and as mentioned previously, returns do not follow a normal distribution. It is worth noting that as stated above, if there was a decision to be made in regards to one model over another, the fact that Expected Tail Loss has this sub-additive nature would put it above Value at Risk, regardless of other flaws or strengths either may possess.

Another example of an advantage of Expected Tail Loss is that unlike Value at Risk, it is better at assigning a value to risk in low probability events which Value at Risk cannot predict as they fall beyond its own pre-determined parameters. As mentioned previously in relation to Nassim Taleb and black swan fait tails, not only does Expected Tail Loss not assume normal distribution much the same as Historic Simulation Value at Risk but it also is able to take into account the nature of fat tails in the returns of a portfolio. This ability to estimate possible losses in these black swan events makes Estimated Tail Loss more applicable to real world application according to many academics. Emmer et al. (2015, p.10 – 12) make this distinction that since Value at Risk does not offer any prediction to the losses attributed in the fat tails, Expected Tail Loss is a far more practical model for use. As mentioned in a previous section, the Basel Committee for Banking Supervision have requested that in future risk associated with capital requirements be conducted using ETL models because it gives greater protection against fat tail risk.

Historic Expected Tail Loss and its weaknesses

As mentioned above, many of Expected Tail Loss advantages are where Value at Risk models have weaknesses and to some extent, it is the same for Expected Tail Loss weaknesses. Many of the positive characteristics of Value at Risk models are areas where the Expected Tail Loss models do have limitations and weaknesses. While these limitations will be discussed further below, it is important to make this assessment as the report proceeds so that as judgements are made about one model over the other or the model with greater reliability, a balanced approach can be taken.

Expected Tail Loss is difficult to back-test and prove its reliability as a measure of risk. This is due to the fact that it is not elicitable and in order to back-test Expected Tail Loss, it must be done through a method not dissimilar to the method of back-testing Value at Risk although this is only really possible if the Expected Tail Loss has been calculated as an extension of the Value at Risk model. There are some other methods for back-testing Expected Tail Loss although they too are difficult and delicate. Emmer et al. (2015, p.6) believes his method of splitting the model in to two separate sub-models would offer Expected Tail Loss what he calls conditional elicitability. This method as expressed by Emmer et al. themselves is a long delicate process that can be difficult to compute by anyone not completely comfortable with the method.

Expected Tail Loss also needs far larger data samples than Value at Risk in order to maintain the same level of accuracy. In order to predict portfolio risk vulnerabilities to 95% and 99% confidence levels in the way that Value at Risk does, nearly double the amount of data must be sampled with can greatly increase both computing time and informational costs to the user of the model. This need for a larger data sample also makes Expected Tail Loss far more sensitive to any data that made be added over time to the model that the user may feel will add to the reliability of the model. This is not always the case as extra data points can cause a break down in the model which may result in the model producing a risk level that is not a fair reflection of the investments true position. This is one aspect that Emmer et al. feels can greatly harm the reliability of Expected Tail Loss models. This sensitivity to extra data points however was initially seen as an improvement on Value at Risk models that were deemed to static. Emmer et al. (2015, p.12) explains that "the notion of ETL was introduced precisely as a remedy to the lack of risk sensitivity of Value at Risk."

Overall Academic opinions on both Models

As can be seen by some of the literature mentioned above, some academics and indeed industry experts feel that Value at Risk has become unfit for use. They shift towards Expected Tail Loss has even begun to move beyond academia and will be required on all risk management associated with capital requirements by the Basel Committee on Banking Supervision. In their report published in 2016, they highlight the deficiencies with regards to the Value at Risk models when it comes to what they perceive to be greater tail risk in the modern financial climate. One of the main reasons that both academics are shifting away from Value at Risk and moving to a focus on Expected Tail Loss models is the fact that the ETL model can look to evaluate low probability events that occur in the fat tails of the returns distribution of a portfolio. As Acerbi and Tasche (2002, p.16) explain, "simply taking a conditional expectation of losses beyond Value at Risk can fail to yield a coherent risk measure."

Sub-additivity is another characteristic that seems to appear in a lot of research about both models of risk measure. In fact, as stated above, most researchers believe that when it comes to portfolios in particular, there is no characteristic of a model greater than sub-additivity as without it, a model completely contradicts portfolio theory and what is now known to be fact. Diversification should nearly always mean reduced risk or else there is an underlying correlation between the assets that may not be intuitive. The idea that a lack of sub-additivity greatly reduced Value at Risk models appeal is something that has been echoed throughout numerous research papers. Yamai and Yoshiba (2005, p.998), Žiković (2008, p.2), Acerbi and Tasche (2002, p.1), Acerbi (2003, p.5) and Artzner (1998, p.6) are all in agreement that sub-additivity is crucial to any risk measure. In fact, this lack of sub-additivity has some researchers willing to completely disregard Value at Risk completely. Caillault and Guegan (2004, p.3) are determined in their appraisal of Value at Risk when they state that Value at Risk is totally unfit as a risk measure for use due to its underlying numerous flaws.

This evidence is pretty damning in regards to Value at Risk and its place in a modern financial landscape. This research however hopes to not only show Value at Risk to be reliable as a risk metric on its own, but show that by using a Value at Risk model in co-ordination with an ETL model, by expanding the Expected Tail Loss model as a branch of the Value at Risk model, can produce far greater accuracy and therefore produce a more transparent view of the risk associated with a randomly generated portfolio.

Methodology

In this section, a detailed description shall be given of the processes used in this research to conduct Historic Simulation Value at Risk and Historic Simulation Expected Tail Loss. The data used were US Treasury Yields ranging from three years to thirty years. The rates used were from the dates of 15th May 2006 to 13th June 2016. The following methodology will detail not only the processes used in calculating each model but also the formulas for each calculation. The results produced in this report will be calculated using Microsoft Excel with certain calculations relying on Microsoft Excel Visual Basic for Applications. All Macros used in this report shall be detailed in the appendices should further research be required and to ensure uniformity in approach.

As already stated, for the purpose of this report, Historic Simulation was used to calculate both the Value at Risk model and the Expected Tail Loss model. The reason Historic Simulation was chosen over other models was to allow an examination of Value at Risk and Expected Tail Loss models using actually data across a number of years and to not only test their reliability in general, but to test them against years where there is a known volatile market. It was felt that Historic Simulation models would avoid the possibility of volatile moments becoming normalised by periods of stability and thus giving an impression that neither model would actually fail.

The first process was the selection of the data which would be used to form the basis of a fixed income portfolio based on US Treasury Yields. The bonds range from three month bonds to thirty year bonds. This research did not see any value in assigning a nominal value to the portfolio as this would make the model more static when it came to running multiple simulations. Instead, a sensitivity index was created and used to return a realised Profit and Loss for the portfolio. The sensitivity index was randomly generated in excel by using the random number function multiplied by an assigned value of 2000. The value assigned was not important in that it could have been any number and the model would not have been affected. The only reason behind choosing a larger number such as 2000 was to allow the values in the realised profit and loss to be more substantial in value and make the results more intuitive.

Calculating Value at Risk and Expected Tail Loss

Initially before multiple simulations were run, there was just two models created using the sampled data. The Portfolio was created using 8 different US Treasury Yields. The portfolio assumed equal weighting initially and then multiplied the sum of these yields by the sensitivity index in order to produce the realised profit and loss. The use of a Historical Simulation model meant that the Value at Risk on the model was easily calculated. The formula can be seen below in equation 1.

$$VaR_{1-\alpha} = \mu(R) - R_{\alpha}$$

Equation 1: Historical Simulation Value at Risk

The model starts one year after the data sample begins and simply looks for the assigned losses associated with the percentile chosen. The model was tested with both a 99% and a 95% confidence. This meant that the Value at Risk models were calculated looking for the top 5th percentile and the top 1 percentile. These two Value at Risk models only ran one simulation each initially. The Value at Risks for both the 95% and 99% were then weighed against the realised Profit and Loss. This allowed the model to be back-tested at a later stage.

For the expected Tail Loss, this was calculated as an extension of the Value at Risk model and this was the same for both the 95% and 99% confidence levels. The Historic Simulation Expected Tail Loss was calculated as the average of all the losses that exceeded VaR. The formula can be seen below which has been sampled from Carol Alexander (2008, p.38).

$$ETL_{h,a} = -E\left(X_h \mathrm{I}\, X_h < -VAR_{h,a}\right) * P$$

Back-testing Value at Risk

When looking to gather whether the model built has failed or not, it is important to back-test which will determine whether or not the model is reliable. The back test involves creating a simple logical test in which the realised profit and loss is compared to the Historical VaR. If it exceeds the VaR, it is assigned a value of one. If it does not exceed VaR, it is assigned a value of zero. This creates an observational exceedance that can be weighed against the expected exceedance. The formula for the logical text is pictured below.

$$I_{t+1}(\alpha) = \begin{cases} 1 \text{ if } x_{t,t+1} \leq -VaR_t(\alpha) \\ 0 \text{ if } x_{t,t+1} > -VaR_t(\alpha) \end{cases}$$

The formula for the simple hypothesis test in order to test the failure rate of the VaR can be seen below.

$$\frac{Eo - Ee}{\sigma}$$

Where: Eo is observed exceedances
Ee is Expected exceedances
σ is the standard deviation.

These calculations are then used to generate a thousand simulations using a Macro created in Visual Basics for Application. This produced a thousand portfolios with in each of the 95% and 99% confidence level models of VaR and ETL. The code used for the Macro can be seen in the appendices below.

Finally, the models shall be tested by year of data. The models will only be tested for years where there was a full year's data. The process for this is still the same as generating the other models with the only exception being that the data samples are smaller which may possibly increase the standard error.

Results and Analysis

In this results and analysis section, the results shall be broken down in to multiple segments in order to not only five an overall result of both models but the result of both models in individual years. As well as this, the models and their reliability will be examined through both the 95% and 99% confidence levels. This will present a more transparent picture of the results which should allow a definitive decision on the validity and reliability of both models based on Historical Simulation.

Value at Risk and Expected Tail Loss - 95%

Initially, a single simulation of the Value at Risk one-day 95% was created in Excel. This was to enable a Macro to be created in Excel Visual Basics for Applications to be created that would take this model and simulate it one thousand times.

The 95% Value at Risk model that ran from 2006 – 2016 performed beyond expectation. The singular model that was constructed initially, along with its subsequent Expected Tail Loss was constructed with minimal difficult in relation to other models and back-tested using a simple hypothesis test. The Z statistic in this hypothesis test was calculated by subtracting the expected exceedances from the observed exceedances and dividing the answer by the standard deviation. This produced a Z statistic that can be seen below in the Appendices in Table 1. of 0.46. This did not reach the rejection point of 1.96 and therefore in the single model test, the Value at Risk model was proven to be reliable.

It is important to state the process by which these singular models were generated, as the Macro would run a thousand of these same simulations and back-tests without producing the same format of results. Only stating whether the model failed or not failed. This was however only one simulation and therefore in order to conduct a more comprehensive test, a thousand historical simulations of both the Value at Risk and Expected Tail

Loss Models were run. These results as mentioned previously were unexpected in that Value at Risk performed quite well overall. The results of the 1000 simulations can be seen below in the appendices in table 2.

While this seems to show Value at Risk as a reliable model, it does not dissect the model year by year and therefore may only Value at Risk working as a result of large corrections taking place following economic crisis taking place. By only looking at Value at Risk and Expected Tail Loss as a one ten-year model, it creates the possibility that Value at Risk failings have been normalised by large periods of Value at Risk perhaps being overly cautious. In order to correct any possible normalisation of volatile markets and possible failings of Value at Risk, the models were broken down year by year. This would allow to see whether the models failed in any particular years and over-performed in others. From this data below it can be seen that the Expected Tail Loss Models for all the years involved perform as expected.

Historical Simulation Value at Risk and ETL (95%)			
2007 - 2008		**2011 – 2012**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	80%	VaR Failings (%)	9%
ETL Failings (%)	3%	ETL Failings (%)	0%
2008 - 2009		**2012 – 2013**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	63%	VaR Failings (%)	15%
ETL Failings (%)	2%	ETL Failings (%)	0%
2009 - 2010		**2013 - 2014**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	53%	VaR Failings (%)	8%
ETL Failings (%)	1%	ETL Failings (%)	0%
2010 - 2011		**2014 - 2015**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	3%	VaR Failings (%)	5%
ETL Failings (%)	1%	ETL Failings (%)	0%

Table 1: Historical Simulation VaR & ETL (95%) 1000 Simulations

The largest percentage failings experienced by the Expected Tail Loss Model was in the periods between 2007 – 2008 and 2008 – 2009. These exceedances however are not substantial enough to discard the model at the 95% confidence level and therefore the Historical Simulation ETL model appears to remain reliable. It remained reliable throughout the most volatile years of the financial crash. Based on the scope of this research, these results are enough to warrant a decision that the one-day Expected Tail Loss Model at 95% is indeed a valid a reliable model. This seems to echo the belief placed on this model by the Basel Committee of Banking Supervision that the model can indeed cope with the anomaly that is fat tails in the distribution of returns on a portfolio.

The Historical Simulation one-day Value at Risk model when ran as a model across ten years performed beyond expectation. As can be seen from the table above, when 1000 simulations were running, the Value at Risk model only failed with 3.9% of the 1000 portfolios. This result was unexpected given that the first two years' data sampled were arguably the most volatile years on record with the global financial crash. On first glance and without seeking to further investigate, it would seem that at a 95% confidence level, the VaR model was a reliable and adequate model. However, in order to fully validate the model, it was needed to run the model across a number of years in order to remove the possibility of failures being lost amongst many years of stability.

It can be seen in table.6 located below, that in the first three years tested, VaR has a high failure rate and therefore must be completely discarded. In the period 2010 – 2011 however, the model performs in such a way that it only exceeds three percent of the time in one thousand simulations. This however is the nature of Historical Simulation in that the VaR model will unintentionally correct itself as larger losses in the realised profit and loss inflate the top 5th percentile boundary and therefore VaR becomes a substantially larger figure less likely to fail. The 95% VaR model then increased in failure rate again in 2011 – 2012, although this was due to VaR being too cautious and significantly overstating the risk associated with the portfolio. While VaR was not exceeded more times than what was expected at this time, the over-cautious position that the model had taken was enough to fail the model on that fact. As stated already, a problem with Historic Simulation Value at Risk is that it can be slow in correcting the VaR threshold as it uses historical data from the previous year and even when markets do stabilise, VaR can still overstate the risk profile of a portfolio. This is indeed a weakness in this Value at Risk model although one that is almost intuitive in nature.

Value at Risk and Expected Tail Loss - 99%

The 99% single simulation Value at Risk model that ran from 2006 – 2016 performed poorly in comparison to the 95% model. Once again the singular model was constructed, this time using a 99% confidence level. The Expected Tail Loss model was also generated using a 99% confidence interval. The same method of back testing was used with a simple hypothesis test generating a Z statistic of 2.82. This placed the model in the rejection zone of the hypothesis test and as such the VaR model was rejected. However, as with before, this single simulation model was not comprehensive enough to either reject or fail the model as a whole. The results of these single simulation VaR and Expected Tail Loss models can be seen in as Table 4. in the appendices.

Once again the Macro generated a thousand of random portfolios based on the single data used in the single simulation models. As stated, the single simulation of the VaR model failed and so too did the model as a whole when simulated a thousand times generating random portfolios. From 1000 simulations, the VaR model failed an overwhelming 40% of the time. This was somewhat expected due to the nature of Value at Risk although a failure rate that high was not expected. As the Confidence level increases, the model becomes far more sensitive to volatility in the markets and thus is more likely to fail. The results of the 1000 simulations can be seen below in the appendices in table 5.

Historical Simulation VaR and ETL (99%)			
2007 - 2008		**2011 - 2012**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	76%	VaR Failings (%)	9%
ETL Failings (%)	4%	ETL Failings (%)	0%
2008 - 2009		**2012 - 2013**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	25%	VaR Failings (%)	8%
ETL Failings (%)	3%	ETL Failings (%)	0%
2009 - 2010		**2013 - 2014**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	6%	VaR Failings (%)	12%
ETL Failings (%)	1%	ETL Failings (%)	0%
2010 - 2011		**2014 - 2015**	
No. of Simulations	1000	No. of Simulations	1000
VaR Failings (%)	6%	VaR Failings (%)	14%
ETL Failings (%)	0%	ETL Failings (%)	0%

Table 2: Historical Simulation VaR & ETL (99%) 1000 Simulations

The Historical Simulation 99% one-day VaR was rejected as a single simulation model and it was rejected and found to be not valid when it was generated across a ten-year period with 1000 simulations of randomly generated portfolios. The nature of that failure though was surprising as a 40% failure rate did seem unexpectedly high even with the presence of the financial crash in the sample data. The size of the failure rate regarding VaR in the 1000 randomly generated portfolios allowed for a rejection of the model as it cannot be deemed reliable with a failure rate of 40%. The expected Tail Loss did perform as before and managed to have a failure rate of just 0.5%. This works out at only 5 failures of Expected Tail Loss in 1000 randomly generated portfolios.

Much like the previous models that focused solely on individual years, the 99% VaR model significantly failed in both its first and second years' data although this was expected due to the high volatility experienced in the market between 2007 – 2009. However, this VaR model recovers in its third year to a failure rate of 6% although it still leaves the model in a position to be rejected. This recovery is not easy to understand although on further examination of the realised profit and loss, it can be seen that the large losses in the most volatile years have once again inflated the top 1 percentile of the Value at Risk model. This also has the repeated effect of driving the VaR too high and making the model overly cautious as the data is slow in being processed.

Overall Analysis

The main objective of this research was to evaluate the reliability and validity of Historical Simulation Value at Risk and Expected Tail Loss models. It can be seen that in times of market instability, the Value at Risk models at both 95% and 99% confidence fail. It is worth noting that when used over a longer period of time, the 95% VaR model was effective. This however is not enough to warrant a decision that the VaR model is reliable and it is for this reason that it must be rejected based on the research conducted. While the model may be rejected as an individual risk measure, there is some issues in fully discarding VaR. The reason that the Value at Risk model cannot be discarded fully is due to the fact that the Expected Tail Loss model in this research is built as an extension of the VaR. The Expected Tail Loss model was a far better performing model and when basing a decision on the validity of both the 95% and 99% Historical Simulation ETL model based on the research conducted, the decision must be that the model is valid and reliable.

Finally, the VaR and the Expected Tail Loss models for both 95% and 99% were graphed together. These can be seen in the appendices below in figure 2 and figure 3. What is worth noting is that in times of relative stability, VaR seems to mirror Expected Tail Loss to the extent that it almost appears that there is no difference between the movement of the models or their performances, only that Expected Tail Loss accounts for losses at a slightly higher level. What does separate these models through the graphs though is when looking at the times when it is already known that the VaR model fails to perform. It can be seen that in both the 95% confidence level graph and the 99% confidence level graph, VaR fails to estimate the losses beyond its own parameters and therefore cannot function as intended. The gap between the losses that Expected Tail Loss and VaR predicts is instantly noticeable. These graphs excellently highlight one of the key flaws of VaR mentioned in a previous section. VaR is limited by its own parameters and as such has failed in almost every back-test it has faced during this research. Expected Tail Loss on the other hand can be shown to quickly adapt to fundamental shifts in the market which allows it to function as intended. These graphs allow the results to become more tangible in that it is possible to see these limitations of VaR when graphed with Expected Tail Loss.

Discussion and Further Reflection

While it has been seen that there was mixed results for the Value at Risk models, the Expected Tail Loss models performed as expected and did not break. Even during the most volatile periods of market fluctuation the Expected Tail Loss model worked. While this research has achieved what it set out to do with regards to the measuring of reliability of Historical Simulation Value at Risk and Historical Simulation Tail Loss, there are several issues that do need to be raised in order to fully understand the limitations of the tests conducted. Not only are there limitations, there are areas which may be expanded into for further research. In this section it is hoped that these limitations can be sufficiently highlighted but that the reader is made aware of how to negate these limitations should they wish to further research this topic.

Limitations of One-model Value at Risk and ETL on Excel

There are many ways to build Value at Risk models and although this research focused solely on Historical Simulation, there are other methods such as Variance-Covariance and Monte Carlo simulation. Variance-Covariance involves the examination of the fluctuations of the returns of a portfolio and the corresponding correlations between the assets that comprise the portfolio. It does however make assumptions about the distributions of these returns in that the model works by assuming normality. As stated before and highlighted with the use of a graph, it can be seen that this is not always so.

Monte Carlo Simulation is a more flexible model than both the Variance-Covariance and Historical Simulation in that it is able to build both Value at Risk and Expected Tail Loss models with a degree of randomness to the returns. This does allow the model to behave less stationary in that while it does depend on standard deviations of previous years, it creates returns using these standard deviations but in a more random pattern and does not assume that the past performance of a portfolio will reflect future performance. One of the more desirable aspects that some have commented on is that Monte Carlo Simulation can be extended out of a large period of time. Mária Bohdalová (2007, p.4) explains "Monte Carlo simulations can be extended to apply over longer holding periods, making it possible to use these techniques for measuring credit risk."

These two methods show that there are limitations to the research undertaken in this report in that while the findings in this report are indeed valid in determining the validity of the Value at Risk and Expected Tail Loss models, it is only in the context of Historical Simulation and does not fully answer any questions on Value at Risk or ETL models as a whole. In order to fully validate either model, the models would have to be constructed by every possibly method. Not only that, with models such as Monte Carlo Simulation, a far greater number of simulations may need to be run in order to reduce the standard error of the tests. This increase in the number of simulations would not have been possible on Excel as its computing power would not be capable of running the desired number of simulations without being far too time consuming.

Further research undertaken may not only seek to further develop these models but also perhaps create these models on a different computing program such as MATLAB or R-Studio. These would allow for a far greater number of simulations to be ran in a much reduced timeframe.

Back-Testing Limitations

In this report, the method of Back-testing Value at Risk used was a simple exceedance test which was stated in the methodology above. This test measures the times the realised Profit and Loss has exceeded the Value at Risk calculated and then creates a simple hypothesis test. If the observed exceedances are greater than the expected exceedances, the hypothesis test allows the user to determine whether the disparity between observed and expected is significantly different enough to warrant a failure of the model. This method is simply yet relatively effectively, especially when constructing

Value at Risk models based on Historical Simulation. Christoffersen argues that the validity of this back-test can be broken down to whether test satisfies two properties. These are:

> 1. Unconditional Coverage Property - The probability of realizing a loss in excess of the reported Value at Risk must be precisely α*100%.
> 2. Independence Property - The unconditional coverage property places a restriction on how often Value at Risk violations may occur. (1998)

Without satisfying both of these properties, Christoffersen states that a Value at Risk model is invalid. The thing he argues however is that a simple back-test such as the one used in this report may not necessarily satisfy both properties. Christoffersen believes that more sophisticated back-tests must be used in order to fulfil all the necessary criteria that he believes adds validity to a Value at Risk model and its back-test.

It is important to note the limitations in the back-testing used in this report in order to ensure that the reader is made aware that while this research has evaluated and analysed the reliability of both models, there are criticisms to be made of the back-testing. There may be underlying flaws that are contained in the back-testing models that are difficult to determine without conducting further research in to multiple back-testing models. This however was beyond the scope of this research and may warrant its own individual research to determine the best model for back-testing these models.

One test that can be recommended for further research can test for both unconditional coverage and Independence and combines both Christoffersens Markov Test and Pelletiers duration test. Sean Campbell describes the process involved with this test:

> The joint Markov test examines whether there is any diff erence in the likelihood of a Value at Risk violation following a previous Value at Risk violation or non-violation and simultaneously determines whether each of these proportions is significantly diff erent from α. (2005, p.10)

Any additional research done in to the reliability of back-testing models would benefit from first examining this method as it seems to satisfy Christoffersens criteria of Unconditional Coverage and Independence.

Possible Combination of both Models

This research set out to measure the reliability of both Historical Simulation Value at Risk and Historical Simulation Expected Tail Loss. This research measured the reliability of both models by running simulations and back-testing. While the Expected Tail Loss model in this research was calculated as an extension of the Value at Risk, it remained a separate model for the purposes of back testing and the accuracy at which it functioned. This is where there seems to be a gap for further research and development of both models. Saša Žiković in his address to the Young Economists Seminar organised by the Croatian National Bank stated that neither model should be overly relied on. Many of the faults and limitations laid at Value at Risk were always limitations and the Value at Risk number produced was never designed to be an absolute. As he further went on to explain, he believed that there were many possibilities for the future of risk management and although he was aware of the many limitations of Value at Risk, he believed it still had a place in modern financial risk management. Žiković explained it as such:

> Research in Value at Risk estimation should by no means be discouraged, but instead intensified, because it could now serve a dual purpose – improving Value at Risk estimates but also improving ETL estimates. The focus of future research should be on improving both Value at Risk and ETL estimation techniques as well as finding optimal combinations of Value at Risk-ETL models, because only such complete information can serve as a solid basis for decision making in financial institutions and reveal actual risk exposure both to investors and regulators. (2008, p.18)

It was this idea of combining both models that should raise further interest for future research. As stated in previous sections, the Basel Committee for Banking Supervision have recommended the move to an Expected Tail Loss model for measuring Capital Requirement risk. This shift will soon be mandatory for all financial institutions, but as these institutions and banks become more knowledgeable of these Expected Tail Loss models and the different ways in which they can be constructed, it may mean Expected Tail Loss becomes the industry standard in much the same way Value at Risk did. However, while Value at Risk was a model that survived almost 25 years before it was replaced with Expected Tail Loss, the nature of how finance is constantly evolving suggests that it may not be 25 years before the next model will need to replace Expected Tail Loss. Further research could be undertaken to look to create a model that combines the strengths of both models and eradicates their weaknesses. This may be possible due to the fact that when looking at these models separately, the characteristics that usually define them as good models are the characteristics the other model lacks.

Overall, the research in this report has achieved what it initially set as its objectives. A test of the reliability of Value at Risk and Expected Tail Loss through Historical Simulation was conducted. It was important however the note the limitations of these models and in turn have the ability to recommend further research that would negate these limitations. It has been noted that to fully avoid the limitations of the flaws associated with these models would require further research beyond the scope of this report.

Conclusion

This research was conducted in order to measure whether the Historical Simulation Value at Risk and Expected Tail Loss models were valid and reliable in measuring the risk vulnerability of a portfolio. The reason this research was undertaken was in no small part due to the report published by the Basel Committee of Banking Supervision and its proposed shift from Value at Risk models to Expected Tail Loss models when measuring the risk associated with Capital Requirements. At present there are no plans by that regulatory body to completely shift all risk measurement away from Value at Risk although as financial institutions implement Expected Tail Loss models for capital requirement, it may create a need for them to implement Expected Tail Loss Models across all aspects of their risk management strategies to help uniformity. This is in essence what made Value at Risk models such an industry standard.

A hypothetical portfolio of US Treasury Yields was created and a sensitivity row vector used in order to generate a realised Profit and Loss. Value at Risk models and Expected Tail Loss models at both 95% and 99% confidence intervals were created and then back-tested. Initially it seemed that at 95% the Value at Risk model performed well although on closer inspection when broken up in to separate models based on each year it failed in known high volatility years. This led to a decision that in times of large market volatility and fluctuation, the VaR model may not be fit for purpose. It is when financial institutions most need their risk models and methodologies to perform that it seems VaR fails. The Expected Tail Loss model at 95% based however performed reliably and maintained its credibility as a coherent and robust risk measure.

The Value at Risk one-day 99% confidence level ten-year model performed quite poorly compared to its 95% confidence model in that, even the ten-year model failed 41% of the 1000 simulations when back-tested. As mentioned above, although 99% confidence levels would seem to guarantee a better performing model, in reality it usually performs worse. When the

99% confidence level Value at Risk model was divided into models for each year, the results were somewhat surprising, in that while the models still failed they performed better in some years than models at the 95% confidence level. The 99% confidence level Expected Tail Loss model did stand up to the scrutiny of the back testing. It has in most ways proven itself to be a reliable risk measure that far outperforms any Value at Risk model.

Risk management is a constantly evolving process which has seen the widespread adoption of VaR as a risk management tool, to the present day where VaR is now seen as an unfit and non-robust model which is not fit for purpose. Expected Tail Loss is now the risk model that will soon be an industry standard, although this report believes that VaR still has a role to play in any risk management department. For one, the Historic Simulation Expected Tail Loss model built in this report was created by extending an already built VaR model. While the VaR model was overall insufficient in years where there was high market instability, the Expected Tail Loss model was more than capable of functioning as desired. The Expected Tail Loss model was not rejected although it is derived from the VaR model which may show that while VaR is flawed, it can be used as part of a multi-model risk management system. VaR is no longer the model that should be used as a sole tool for risk management but its use alongside Expected Tail Loss will create a more dynamic risk management system that will outperform either model individually.

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Appendix

Macro programme used to simulate random portfolios

(General) testVaRmodel

```
Sub testVaRmodel()

n = InputBox("How many simulations do you want?")

For i = 1 To n

Sheets("HistoricVaR").Activate
ActiveSheet.Calculate
Sheets("Data").Range("A1").Offset(i, 1).Value = i
Sheets("Data").Range("A1").Offset(i, 2).Value = Sheets("HistoricVaR").Range("aa7").Value
Sheets("Data").Range("A1").Offset(i, 3).Value = Sheets("HistoricVaR").Range("aa8").Value
Sheets("Data").Range("A1").Offset(i, 4).Value = Sheets("HistoricVaR").Range("aa9").Value
Sheets("Data").Range("A1").Offset(i, 5).Value = Sheets("HistoricVaR").Range("aa10").Value
Sheets("Data").Range("A1").Offset(i, 6).Value = Sheets("HistoricVaR").Range("aa11").Value
Sheets("Data").Range("A1").Offset(i, 7).Value = Sheets("HistoricVaR").Range("aa12").Value
Sheets("Data").Range("A1").Offset(i, 8).Value = Sheets("HistoricVaR").Range("aa13").Value

Next i

MsgBox "Done"
```

Table 1: Single Simulation VaR & ETL (95%)

Overall VaR and Expected Tail Loss	
No. of Days	2265.00
Expected Exceednaces	113.25
Observed Exceedances	118.00
Stndard Deviation	10.37
Z statistic	0.46
Critical Z	1.96
Decision	VaR Model Works
t statistic	0.041672766
Decision	ETL Model Works

Table 2: Single Model VaR & ETL (95% - 1000 Simulations)

VaR & ETL (95%)	
No. of Simulations	1000
Average Exceedances	122.633
No. of VaR failures (%)	3.90%
No of ETL failures	0.05%

Table 4: Single Simulation VaR & ETL (99%)

Overall VaR and Expected Tail Loss	
No. of Days	2265.00
Expected Exceednaces	22.65
Observed Exceedances	36.00
Standard Deviation	4.74
Z statistic	2.82
Critical Z	2.33
Decision on VaR Model	VaR Model Fails
t statistic	0.015659532
Decision on ETL Model	ETL Model Works

Table 5: Single model VaR & ETL (99% - 1000 Simulations)

VaR & ETL (99%)	
No. of Simulations	1000
Average Exceedances	32.524
No. of VaR failures (%)	40.50%
No of ETL failures	0.50%

Figure 2: VaR & ETL Movement (95%)

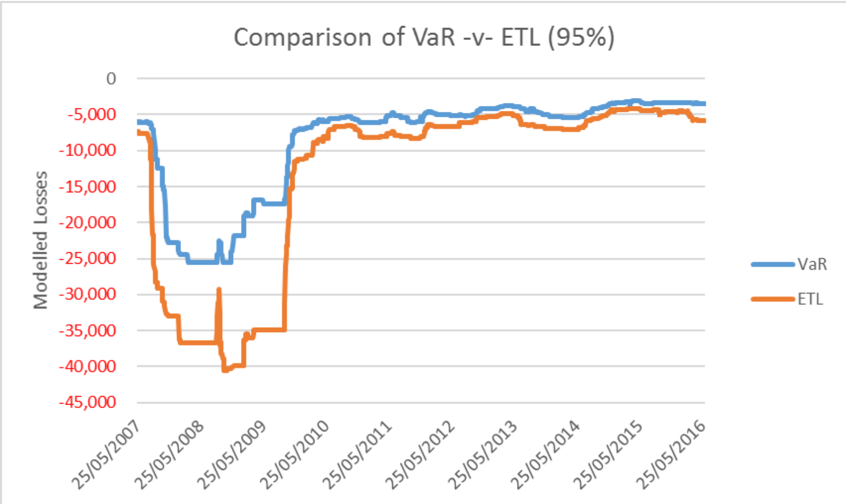

Figure 3: VaR & ETL Movement (99%)