The validity of the Capital Asset Pricing Model, in the Modern Irish Market

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Abstract

Capital Asset Pricing Model (CAPM) is a renowned financial model, that explains the risk associated with stocks expected returns and is based on Henry Markowitz, mean-variance portfolio model developed in the 1950's. Since modern finance has evolved CAPM has received criticism on its restrictive assumptions and dependency on one factor, the systematic risk, beta.

The study will analyse the validity of CAPM in the ISEQ 20 Index, for a sample of 20 stocks daily returns during the most volatile and uncertain period of modern global financial history, 20019-2021. The research will be conducted on both portfolios and individual securities in order understand the working of CAPM's single risk factor, beta. The methodology adopted, will aim to clarify the linear positive risk-return relationship, through a series of a linear regressions, followed by a t-test of the intercept and slope.

Capital Asset Pricing Model, is a valid indicator of the systematic risk, however, the analysis of CAPM in this study is inconsistent and could not find complete support of the CAPM model. However, some of the pricing models features could not be rejected.

The outcome of the study can be seen as a stimulus for further researches in this field, given the poor academic attention at the Capital Asset Pricing Model, in the Irish context and during the global financial crisis.

Keywords: Capital Asset Pricing Model, Risk-Return Relationship, Linear-Regression, Covid-19, Global Minimum Variance Portfolio, Efficient Frontier.

Declaration

I Mark O'Neill declare this dissertation which I have submitted to the National College of Ireland, in completion the MSc Finance (part-time) course is my own work, unless stated and referenced clearly. No work has been taken from any other sources without being correctly referenced.

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List of Abbreviations

CAPM: Capital Asset Pricing Model

ICAPM: Intertemporal Capital Asset Pricing Model

APT: Arbitrage Pricing Theory

MPT: Modern Portfolio Theory

SML: Security Market Line

CML: Capital Market Line

GMVP: Global Minimum Variance Portfolio

LSE: London Stock Exchange

KLSE: Kuala Lumpar Stock Exchange

OLS: Ordinary Least Squares

Chapter 1 – Introduction

William Sharpe (1964), Jack Treynor (1962), John Lintner (1965) and Jan Mossin (1966) derived the Capital Asset Pricing Model (CAPM) in the early 60's, to outline stock return on investments they should receive, based off the risk they are exposed to in the market. CAPM in today's world is used a theoretical foundation depicting the relationship between systematic risk and return within the market (Fama and French, 2004). The model defines the performance of risk and return on stocks but more specifically, it outlines the required return on stocks is only explained by the systematic risk, beta. (Srinivasan, 1988)

Since the model's introduction there has been numerous criticisms on the restrictive assumptions and how the model does not hold, especially through extreme volatility (Fama and French, 2004). The critiques surrounding CAPM is due to unrealistic assumptions and setting of a perfect world within the financial markets. This perfect world see's all investors as rational individuals, where all information is available, both public and private, there is no trading restrictions, fee's or regulations imposed and risk-free assets permit the unlimited borrowing and lending.

A number of attempts have been made to rectify this perfect world phenomenon, to find a better risk vs return indicator for real life scenarios to overcome this theoretical short comings. The most famous of these consist of but are not subjected to the following: Merton (1973) who's analysis constructed the Intertemporal CAPM (ICAPM). ICAPM saw investors are interested not only in investing in the market and redeem their returns immediately but to reinvest these returns also. Fama and French (1992,1996) introduced a multi-factor model depicting the risk and expected return relationship to be determined by more than just the systematic risk defined by beta.

Despite the large amount of empirical evidence and attempts to improve CAPM, minimal attention has been given to its efficiency to predict asset pricing. This research will analyse the risk-return relationship of CAPM in its original form. As per CAPM theory the model should provide a positive and linear relationship, as the systematic risk should be the only factor that affects the stock returns. As such the objective of this research is to approach testing this model in a positivistic way.

The methodology applied follows the study and process undertaken by Fama and Macbeth (1973), who analyse CAPM in its original form focuses on beta, the explanation of the systematic risk-return relationship. This analysis will focus on a two-step linear regression (applied to individual assets and portfolios), consisting of multiple t-tests on beta and alpha variables, to validate CAPM within the ISEQ 20 Index.

Chapter 2 – Literature Review

Capital Asset Pricing Model has been considered a benchmark in modern day finance and has been widely used since the early 1960's. The CAPM has been used to determine returns on investments based on the level of risk exposure implemented into one's portfolio. This is achieved by depicting the relationship between the systematic risk and the expected return within the market. Systematic Risk is defined as the risk inherent to all market assets, which also known undiversifiable risk (Womack, K & Zhang, Y, 2003). Since the introduction of CAPM there has been numerous critiques on the model's restrictive assumptions and inability to withstand periods of extensive volatility (Fama and French, 2004). This has led to various versions of the model being built (Arbitrage Pricing Theory, ICAPM, Fama-French Three Factor Model), by relaxing these assumptions to gain a broader more realistic approach.

The original CAPM is based on Henry Markowitz mean-variance efficient portfolio model and is derived by Sharpe (1964), Lintner (1965), Mossin (1966). CAPM is based on hypothetical perfect markets in a perfect world and receives scrutiny over its restrictive assumptions. However, the model is used as a foundation for educational purposes and for further studies to develop the model (Fama and French, 2004). Even though CAPM is still relevant in today's financial world nearly 60 years later there is relatively minimal evidence of testing the model in its original format studying the relationship between risk and return. Many attempts have been made to adapt the model to suit the ever changing world and compare it with CAPM.

In 1976 an economist Stephen Ross, developed an alternative model to CAPM, introducing the world to the Arbitrage Pricing Theory (APT). APT shows each assets payoff to be the result of the weighted average of the remaining portfolio. Ross (1976) idea behind the APT model was to take the single Factor CAPM model imposing multiple factors that will affect the risk in one's portfolio, such as GDP, Inflation rates etc. Shanken, J (1982) has described the APT model as "the natural successor of CAPM". Miljan Lekovic and Tanja Stanisic (2018) compared CAPM to the deemed "natural successor" APT, through a series of tests on the expected returns of the model. The idea behind these test is that both CAPM and APT models would be exposed to the same systematic risk conditions. The difference in the two models being CAPM implies only the existence of one risk factor the β variable and the APT model multiple systematic risk factors such as GDP and inflation mentioned previously. Lekovic and Stanisic (2018) in their study of CAPM versus APT, valued each asset securities ensuring the expected return of each security corresponds to the systematic risk assumed. This is outlined by the β variable denoted as a measure of market risk in the CAPM model, and by the sensitivity of the return of securities to a large number of risk factors in the APT model. The volume of risk associated in investors portfolio's requires some compensation in return according to the CAPM model. Lekovic and Stanisic (2018) found this similar for the APT model but for the different types of systematic risk associated with APT. In Lekovic and Stanisic (2018) study it was identified how both CAPM and APT "include the absence of undervalued and overvalued securities, that is, the presence of entirely properly valued securities, which is in line with the Efficient Market Hypothesis (EMH) as a common starting point for these models" (Lekovic, M and Stanisic, T, 2018) . Through Lekovic and Stanisic (2018) comparison of CAPM and APT, they have concluded that "although more suited to reality, the APT model is not a dominant applied model in practice" with the CAPM having a practical advantage, due to its "clearly defined systematic risk factor" (Lekovic, M and Stanisic, T, 2018). This conclusion was highly unexpected as the APT as "a more flexible and more liberal model" that "avoids many restrictive assumptions of the CAPM" (Lekovic, M and Stanisic, T, 2018). Even as CAPM provides a practical advantage the study's conclusion outlines superiority of either models cannot be confirmed. Both models have enlightened anomalies that inhibit the accurate evaluation of assets and the variability explanation of the price of securities. The views presented by Lekovic and Stanisic (2018) have shown the practicality of CAPM and APT in both the real world and theoretical. The studies have enlightened how the evolution of CAPM may provide a solution to its extreme restrictions and application in everyday life.

Fama and French approximately 20 years later in the 1990's convey the idea that CAPM is an incomplete model and expands on its ideology, creating the three factor model. The three factor model takes hold of CAPM's market risk factor, introduces size and value risk factors. Kenneth Lam (2005) delved into the research surrounding both the Fama-French three factor model and CAPM to understand if the latter is a better pricing model. A sequence of time series and cross sectional tests, where conducted on both models. The conclusion of the study is somewhat inconclusive as the results where dependent on external factors such as time period chosen, type of tests used, number of portfolio's considered in each test (i.e. F-Test rejects the three Factor Model, while the Fama-Macbeths T-test supports CAPM). The study shows how the three factor model has more explanatory power on the risk variable, over its predecessor, however there is insufficient concrete evidence shown to say the three factor model is better than CAPM (Lam, K, 2005).

In its original format CAPM's relationship should provide a linear and positive result, with the risk in the model being determined by the single risk factor, β. The validation of the model will be conducted through the most uncertain period of modern financial history between 2019 to today. The focus of this period is through the extreme volatility and economic uncertainty brought on by the Covid-19 Global pandemic. This pandemic itself resembles some similarities to the Spanish Flu which occurred nearly one hundred years prior.

The purpose of this study is to validate the capital asset pricing model within extreme volatility and understand the effect of the global pandemic has on the model. This theme and magnitude of volatility of course is not broadly discussed in studies and literature. This chapter will examine the literature review behind the model, by analysing its foundations, assumptions and evolution through time. From here CAPM will be evaluated on its efficiency based off of empirical evidence focusing on the relationship between the systematic risk and return. This Chapter will be structured into four different sections which will be outlined and discussed as follows:

2.1 – ***CAPM's Assumptions -****, foundations and logic that make up the model will be discussed at length. The model's assumptions will be discussed considering investment opportunities and the efficient frontier. The foundation of the model will be discussed by delving into the past and dissecting the Mean Variance Efficient Portfolio Theory by Henry Markowitz in 1952.*

2.2 - CAPM how it works – *This section will outline the makings of the pricing model, why it is used what bears impact on its results and its limitations.*

2.3 – ***The Evolution of CAPM*** *will be critically analysed through the introduction of the model, its evolvement through the years and the development of several versions which implement more moderate and realistic assumptions in line with everyday life.*

2.4 – ***CAPM's Efficiency, Empirical Evidence and Tests –*** *Discuss recent and early tests*

2.5 – ***The Research Question*** *section will outline the purpose of the study and define the research question, goals, and objectives of the study.*

2.1– CAPM's Assumptions

CAPM originally stemmed from Henry Markowitz's Mean-Variance Efficient Portfolio Theory also known as Modern Portfolio Theory (MPT) was derived in 1950's (Markowitz, 1952) . MPT determines the minimum level of risk exposed to an investor's portfolio for an expected return and is based on two factors; Risk and return (i.e. the mean and variance). Markowitz's portfolio theory allowed investors to outline the maximum return for the minimum risk in their portfolios. MPT helps investors to make smart decisions, by assisting in the mitigation of risk through the process of portfolio diversification. Markowitz's (1952) has defined portfolio diversification as a combination of portfolio assets that are less than perfectly positively correlated, in an effort to mitigate the variance (portfolio risk) without sacrificing the portfolios return (Markowitz, 1952). This method of selection and view on the portfolio as a whole "eliminates the idiosyncratic risk inherent in individual securities". (CFI, 2021). Investors are innately risk adverse, and rational individuals would always prefer the highest return and lowest risk investments (Lintner, 1969). Nevertheless each individual is subject to their own preferences and tolerances of the amount of risk willing to take on. This began the evolution of the portfolio theory. Markowitz (1952) established the efficient frontier. The efficient frontier effectively depicts the optimal weightings of each asset in one's portfolio outlining the highest expected return for a specified amount of risk. The efficient frontier is a concept outlined by Markowitz (1952) that reflects portfolios that generates the expected return for various levels of risk.

Since the introduction of Markowitz's theory, the influence of systematic risk of various financial dealings has been at the forefront studies. Concerns surrounding risk in financial transactions may alter capital asset price forecasts in volatile periods. The most uncertain and volatile periods can reshape market, investor and consumers behaviour. The loss of confidence by investors within the market, will alter rational decisions and behaviour causing additional financial loss and economic stress. This in turn will be felt on an individual level, but a global one as well (Lintner, 1965).

This has led to the development of MPT and the creation of CAPM. Sharpe (1964), Treynor (1962), Lintner (1965) and Mossin (1966), further developed the work of Henry Markowitz's MPT, deriving the CAPM. Approximately a decade later CAPM upholds Markowitz theory of what determines the price associated to assets. The perfect world defined by "CAPM is based on the idea that not all risks should affect asset prices", as investments within a well-diversified portfolio in a perfect world, will be free from unsystematic risk (i.e. risk associated to individual assets) (Perold, A, 2004). This perfect world CAPM depicts is surrounded by numerous assumptions for efficient portfolios which seem to most as strict, restrictive and quite frankly unrealistic (Fama and French, 2004). These assumptions are outlined as follows by Black, F (1972); Blitz et al. (2014); Fama and French 2004 as follows: *Maximize economic utilities; Investors are rational & risk-averse always tending towards maximizing their returns based on a given amount of risk; Portfolios are diversified across a range of investments and hedged against risk; Investors are unable to influence prices based on their transactions and decisions; Investors can lend and borrow unlimited amounts under the risk-free rate of interest, with no constraints; Trading without transaction or taxation costs and regulations and limits i.e. a Perfect Market; Assume all information is available at the same time to all investors, both public and private sectors alike.* From these assumptions CAPM is expressed as $\boldsymbol{E(R_i) = R_f + \beta_i\left(E(R_m) - R_f\right)}$ and denoted as $\boldsymbol{E(R_i)}$ is the expected return of the portfolio outlined by the change in the asset price which can also be derived as $\boldsymbol{ln\frac{p1}{po},}$ Where: $\boldsymbol{P_1}$ is the new price; $\boldsymbol{P_0}$ is the old price; $\boldsymbol{R_f}$ is the risk-free rate an investor would expect to receive from a risk free investment; $\boldsymbol{\beta_i}$ represents the systematic risk and is the measure of volatility within the portfolio. $\boldsymbol{\beta_i}$ is the slope coefficient and depicts the relationship between the risk and expected return for assets; $\boldsymbol{E(R_m)}$ is the expected return on the Market investors would expect to receive**;** $\boldsymbol{\left(E(R_m) - R_f\right)}$ – The Difference between $\boldsymbol{E(R_m)}$ and $\boldsymbol{R_f}$ create the Market Risk Premium. This is the difference between the portfolio's expected return on a market portfolio and the risk free rate.

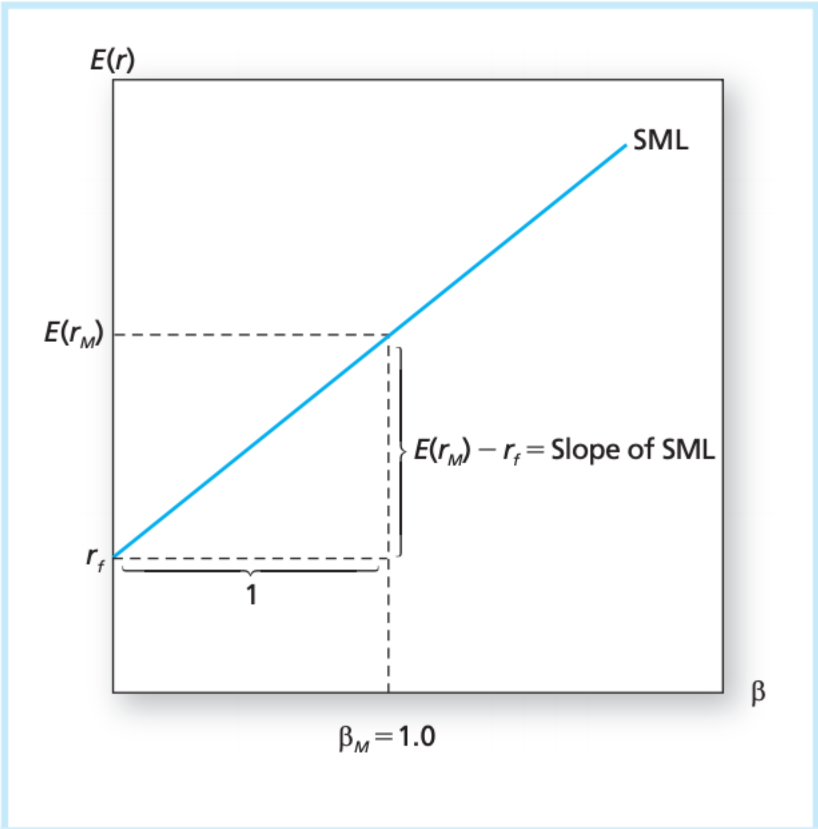
The implementation of CAPM assumptions simplifies the modern portfolio's theory by making it more practical by introducing a clear and concise systematic risk. In its simplest form CAPM links investments to the market as a whole, whereas MPT's outlook is towards the correlation between all investments in the market. The pricing model has been continuously criticised for having unrealistic assumptions. Lintner (1969) referred to the assumptions as being quiet restrictive at times and has shown in studies that by removing the assumption all investors are rational and risk averse does not change the CAPM price, however by removing the unlimited borrowing and lending assumption would cause a drastic change in the models results. Lintner outlines that all investors prefer to maximize their returns, whilst minimizing the risk. (Lintner, 1969). Mullins Jr (1982) does outline these problems when he's exploring CAPM, outlining the model does not to take in to account Behavioral Finance as not everyone takes the rational approach when making an investment. Mullins Jr (1982) also outlines that β, the measure of volatility is not "stable through time" and are "estimated from past data are subject to statistical estimation error". Whilst this view bares some stature, Mullins has not taken into account that the past has a tendency to repeat itself, whilst not on the same magnitude i.e. Recessions, Economic Boom Periods, Global Pandemics (Spanish Flue, 1918 & Corona Virus, 2019/20). This reasoning suggests historical data will provide a good prediction for upcoming economical events. In today's world these assumption might seem even more stricter with the introduction of new rules and regulations such as GDPR and various tax and transaction fees. These assumptions aid CAPM's ideology as it is considered for a one time period, however it is ambiguous on the length of this period is and in a real world scenario investments go through a various different time length . Investors are unable to borrow and lend unlimited amounts due to stricter financial regulations imposed post 2008 global financial crisis. The model also overlooks transaction and taxation cost imposed on trading, hence why it is known and taught as a foundation to build on for an educational perspective. (Fama and French, 2004) . All information is also not readily available to the entire public and private sector, this is also a given as institutions are not allowed to distribute various traders or client information. This was also reinforced by the introduction more restrictive regulations with the introduction of GDPR in 2018. The assumption stating all portfolios are to be diversified and hedged against risk maybe applied in reality, but only since the great recession with position limits being imposed to financial institutions. Prior to the global crash the majority of Irish banks and institutions globally had their investments, mortgages and various financial products linked to the real estate, property sector. This diminished the diversification of portfolio's and their returns and the revolutionary pricing models ideology for a perfect market, inhabitant by rational and risk averse investors. It is clear from modern history we do not live in a perfect risk-free world. The efficiency of the model has been heavily criticized since its introduction, as it is based on "unrealistic assumptions, including complete agreement and either unrestricted risk-free borrowing and lending or unrestricted short selling of risky assets". (Fama and French, 2004) This of course was highlighted in studies where it has been proven to "combine risky securities with risk free assets like treasury bills or bonds", with asset prices are characterised and adjusted appropriately by the risk premium and β (Ayrapetova T, 2012).

CAPM's foundation is based of Harry Markowitz (1952) efficient frontier the cornerstone of MPT. The efficient frontier graphically depicts a set of optimal portfolios that offer the highest expected return for a defined exposure of risk. Expected returns are dependent on the asset combinations that make up the portfolio. One key aspect of Markowitz (1952) efficient frontier is the benefit of portfolio diversification, which shown through the unique curvature of the model. Harry Markowitz (1952) reveals how diversification of portfolio's improves the portfolios risk-reward on investments. The efficient frontier, risk return relationship is not linear as like CAPM, as such adding more risk does not always mean more return. Similar to CAPM, MPT and the efficient frontier is bound by assumptions. These assumptions consist of rational investors, who are risk averse; Investors have unlimited borrowing and lending at a risk free rate; there is not enough investors to influence market prices. Portfolio combinations which lie below the efficient frontier are considered sub-optimal, as they do not offer enough return for the level of risk the portfolio is exposed to. The optimal portfolio does not simply include securities with the highest potential returns or low-risk securities. The optimal portfolio aims to balance securities with the greatest potential returns with an acceptable degree of risk or securities with the lowest degree of risk for a given level of potential return. The points on the plot of risk versus expected returns where optimal portfolios lie are known as the efficient frontier. (Bodie, Kane & Marcus, 2013)

Similar to its predecessor (MPT), CAPM provides an estimate on the relationship between a portfolio's risk which is outlined as $\boldsymbol{\beta_i}$ and its expected return, producing a positive linear relationship (**figure 1.1**). The CAPM theory provides insights about what kind of risk is related to a portfolio returns (Perold, A, 2004), with studies depicting the greater the risk, the greater the return (Tobin, J, 1958). It is paramount that we remember CAPM's relationship between its systematic risk and expected return is outlined and expressed by one single factor, $\boldsymbol{\beta_i}$. This relationship is defined by modern financial theory which rests on two main assumptions. These assumptions being that each asset in the market is competitive and efficient, with all information being available and distributed evenly to both public and private sectors alike. (Mullins Jr, 1984).

The risk and expected return relationship depicted by CAPM is visualised through the security market line (SML), which identifies different levels of systematic risk of marketable assets, against the expected return of the market at any given time. "The security market line provides a benchmark for the evaluation of investment performance" and is valid for both efficient portfolios and individual assets (Bodie, Kane & Marcus, 2013). The SML, graphs individual asset risk premiums as a function of asset risk. The relevant measure of risk for individual assets the contribution of the asset to the portfolio variance, which we measure by the asset's beta. When plotted graphically (**figure 1.1**) the x-axis represents the systematic risk outlined by $\boldsymbol{\beta_i}$ and the y axis representing the expect return. The concept of $\boldsymbol{\beta_i}$ is central to both CAPM and the SML, which is a measure of systematic risk that is unable to be mitigated through diversification. The use of the SML by financial professionals to determine the expected return in relation to the level of risk exposure, to evaluate the inclusion of an asset in their portfolios. If a single asset is above the SML the stock is considered undervalued, as the asset offers a greater return against the systematic risk. Similarly if a single asset is plotted below the SML it would be considered overvalued in price, as the expected return does not overcome the systematic risk. SML is commonly used to compare similar assets which offer relatively the same returns, to distinguish which offer the least amount of systematic risk compared to the expected return. The SML is also used to compare assets of equal risk to determine which asset provide the highest expected return against a specific level of risk (Bodie, Kane & Marcus, 2013).

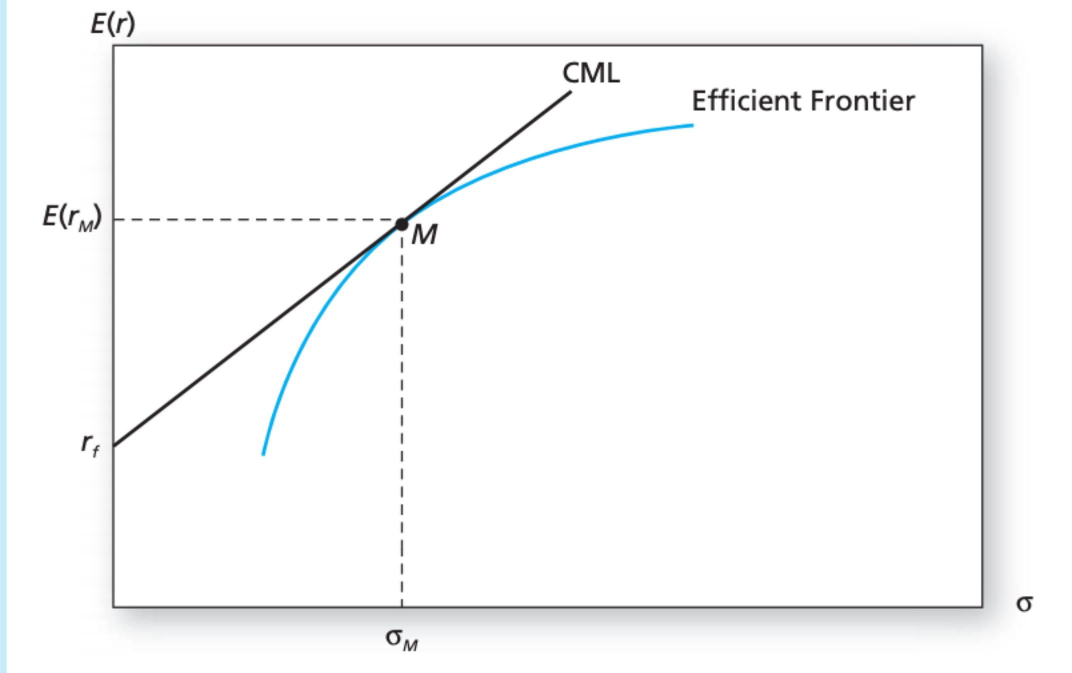
Figure 1.1 – Security Market Line - The expected return–beta relationship



Source: *Bodie, Marcus and Kane, 10th edition, 2013*

The capital market line (CML) graphs (**figure 1.2**) the risk premiums of efficient portfolios as a function of the portfolio standard deviation (Bodie, Kane & Marcus, 2013). The CML theory shows all the portfolios which combines the risk free rate of return and the market portfolio of risky assets. Through the application of CAPM and its restrictive assumptions outlined above, all investors will act rationally maximising their return choosing their investment positions on the CML, by borrowing and/or lending at the risk free rate for a chosen level of risk. Portfolios positioned on the CML optimize performance and the risk-return relationship. CML is quite different to the popular efficient frontier pioneered by Harry Markowitz (1952) as includes the risk-free rate introduced James Tobin (1958). However, the intercept of the CML and the efficient frontier represents the most efficient and optimal portfolio. Tobin's (1958) separation theorem on the optimal market portfolio combination and risk-free rates is dependent on individual investors risk-aversion policies. Risk averse investors will chose to hold their risk-free position at the optimal point (the intercept of the CML and efficient frontier), preferring a low variance (risk) to higher returns. However, investors who are less risk averse may choose to move their positions up the CML, thus increasing portfolios expected returns and the portfolios variance (risk).

Figure 1.2 – Capital Market Line & The Efficient Frontier



Source: *Bodie, Marcus and Kane, 10th edition, 2013*

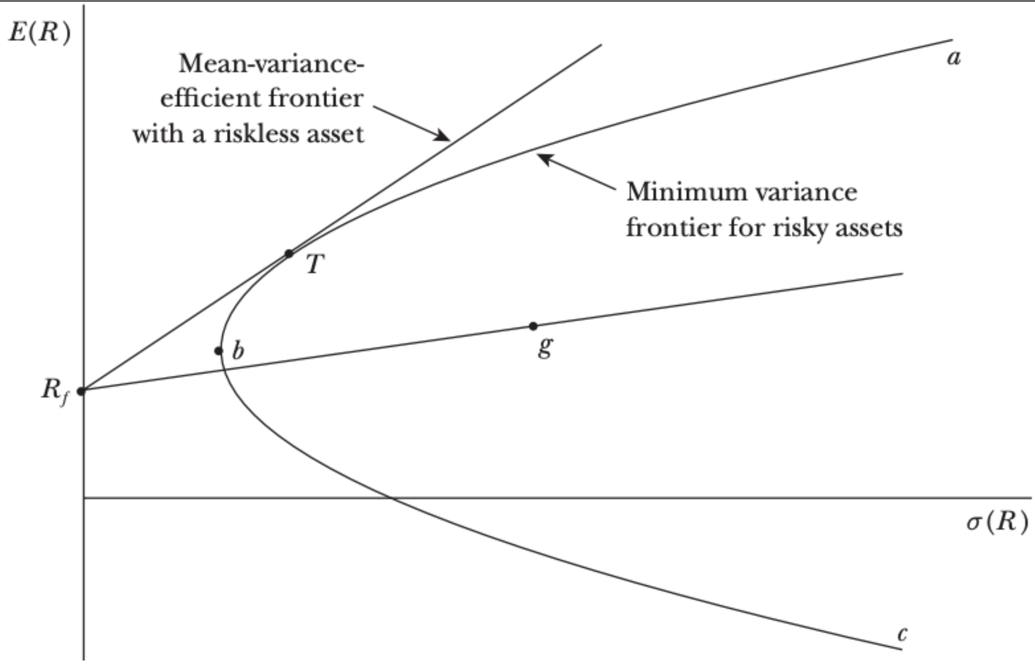
2.2- CAPM: How it Works

The CAPM is an idealized portrayal of how financial markets price securities, which determines expected returns on capital investments. The pricing model provides a methodology for quantifying risk and translating that risk into estimates of expected return on equity (Mullin, Jr, 1982). Investors naturally seek a rate of return to compensate for the level of risk exposure in their portfolio. CAPM calculates the risk exposure and the expected return based on this risk. The pricing model begins with the idea that individual investments contain two types of risk; systematic and idiosyncratic. Systematic risk outlines market risk which is unable to be diversified, such as interest rate fluctuations, economical depressions (i.e. recessions) and wars. Idiosyncratic risk on the other side, is risk that is related to individual assets and can be diversified (Sharpe, W, 1970).

CAPM expands on MPT through the assumptions outlined in section **2.1** previously by Black (1972) and is considered for only one single period of time, meaning that all investors make the same decisions and investments over the same time horizon. This due to the systematic risk defined entirely by one factor, $\boldsymbol{\beta_i}$ However the length of time is ambiguous. This time period investors are able to make investments with the means of borrowing and lending unlimited quantities of money at a risk free rate, with no possibility of losing.

The CAPM story can be portrayed in **figure 1.3**. The x-axis depicts the portfolios risk and is measured by the standard deviation of portfolio's return and the y-axis depicts the portfolio's expected return. The abc curve outlines the minimum variance frontier, with the point *b* being the global minimum variance portfolio (GMVP) and the segment of *b* to *a* portraying the efficient frontier. The GMVP at point *b* outlines the lowest possible portfolio volatility for a number of underlying assets, with the highest return (Kempf, et al, 2006). The efficient frontier from point *b* to *a* is the set of optimal portfolios that provide the highest expected return for a defined level of risk. If there is no risk free borrowing or lending this sub area is considered "mean-variance-efficient" as these portfolio's maximise the expected return, given their level of exposure to risk (Fama and French, 2004). The portfolio's that lie beneath point *b* are considered as sub-optimal as they do not provide sufficient return for the level of risk exposure (Ganti, A, 2021).

Figure 1.3 – Investment Opportunities



Source: *Fama and French, 2004*

The CAPM model however includes the assumption of risk-free borrowing and lending. By including this in the portfolio Fama and French have concluded this turns the efficient set into a straight line. The loaning of funds at the risk free rate of interest is depicted in figure **1.3**, the point *Rf*, a portfolio with a risk free rate of return and zero variance level. This combination of risk-free lending and positive investments plots the straight line graph between the points *Rf* to *g,* with the profits from the borrowing used to increase investments with the portfolio *g.* (Fama and French, 2004)

The assumptions CAPM relies on are far too restrictive and are associated with a hypothetical perfect world. A world which we do not live in. Extensive research in the removal of these assumptions "does not change the structure of the capital asset prices in significant ways" and that the reduction of risk is due to the diversification of one's portfolio. (Lintner, 1969). It is evident the assumptions provide limitations for the model and has been heavily criticized in numerous literatures. This has led to the development of further, more realistic theories attempting to relax CAPM's assumptions, which will be further analyzed in the next section.

2.3 – The Evolution of CAPM:

CAPM was derived by Sharpe (1964), Lintner (1965), Mossin (1966) on hypothetical perfect markets, which does not relate to real world scenarios. The model was designed as the successor for the MPT, developing it theories and assumptions to suit real life scenarios. Numerous studies have adopted CAPM's ideology and attempted to relax and alter the restrictive assumptions since its introduction, in an attempt to create a more reasonable model.

CAPM model has been dubbed as theoretically incomplete, with the model's assumptions depicted as limitations that are quite restrictive at times. By removing the assumption of investors performing rationally and risk averse "does not change the structure of the capital asset prices in significant ways" (Lintner,1969). The reduction of risk within a portfolio would be the result of increased portfolio diversification, which is not dependent on the systematic risk defined by β (Lintner,1969). Lintner (1969) suggests by removing other assumptions like unlimited borrowing/ lending would cause a significant change in the models results as behavioural factors would come into play (Lintner,1969). The problems with CAPM and its restrictive assumptions, is that the irrational nature of investors studied through "behavioural finance" is not taken into account, as not everyone takes the rational approach (Mullins Jr, 1982). The risk factor Beta, that determines the measure of volatility and the theoretical value market risk premium (i.e. expected return minus the risk-free rate) is determined to not be "stable through time" and "estimates from past data are subject to statistical estimation error", as the risk-free rate is deemed a constant through time (Mullins Jr, 1982). Various research has exploited the CAPM, cutting to the heart of the single Beta risk factor, which is outlined as not inclusive of all relevant factors and variables that effect asset pricing far beyond than the market Beta. (Ross, Stephen A. 1976; Fama and French, 2004).

Fama and French tested the CAPM using cross-sectional and time series regression tests on the linear relationship between asset returns and the systematic risk determined the relationship to be linear and indicating the market proxy portfolio is efficient with different versions of CAPM (Fama and French, 2004). However, issues overshadowing the model in today's era, is more than one factor determines the returns on investments aside from systematic risk (Fama and French, 2004). This has led to an array of theories being developed to succeed CAPM, as the model was deemed a "theoretical failure", "a tour de force" that is based off unrealistic assumptions (Fama and French, 2004).

Through time various economists disapproved of the CAPM model, stating the single risk factored Beta is not sufficient, and other factors are to be considered whilst pricing assets. (Ross, Stephen A. 1976; Fama and French, 2004). Fama and French (2004) research conducted in the 1970's uncovered findings that “cut to the heart” of the CAPM model. Fama and French statement is viable as the research undertook in the late 1970’s unearthed variables that effect asset pricing far beyond than the market $\boldsymbol{\beta}$. (Fama & French (2004).

In 1996 Fama and French introduced the multi factor model, developing CAPM’s theory further, as more than one factor explains the expected return in a portfolio. CAPM omits particular characteristics which bares impact on asset returns, such as transaction costs, risk exposure, the availability of information and the fluctuation of interest rates (Fama and French, 2004). Further research conducted by Black (1972), Friend and Blume (1970) have found further variables beyond the systematic risk β, inclusive of firm and stock size, price ratios such as price earnings, book to market ratios and cashflow price ratios. These studies since started the fire for further study developments and the creation of new theories and models that attempt to relax the restrictive assumptions defined by CAPM.

The Fama and French’s multifactor model adds size risk and value risk factors to the market risk factors identified in the CAPM model, explaining inconsistencies in a portfolio’s returns. These anomalies seem to dematerialize by the introduction of 3 main characteristics, that was not accounted for in the systematic risk variable, $\boldsymbol{\beta}$ (Fama and French, 1992, 1996). The additional factors that will cause an effect on asset returns based on this multi-factor model is as follows: Size effect portrayed on the difference between returns on portfolios based off positions within large and small stocks; Book to market values discrepancies outlined by a portfolio’s return of both high and low book to market stocks; Excess return on the market portfolio deduced by the difference between the return on the market and risk-free rate (Fama and French, 1992, 1996).

Fama and French’s multi factor model is considered the most ground breaking, as it not only provided solutions to CAPMs restrictive assumptions but also reasoning as to why the model was in outdated and incomplete. This multi factor model challenges CAPM’s single factor ideology $\boldsymbol{\beta}$, deeming it unjust and in fact determined by more than one characteristic. (Fama and French, 1992)

Similar to Fama and French’s theories, came the introduction of the Arbitrage Pricing Theory (APT) in 1976 by S. Ross and was initially depicted as “the natural successor of CAPM”. Shanken, J (1982) Following the same approach as Fama and French, Ross took the single Factor CAPM model and imposed multiple factors that will affect the risk associated in portfolios, such as GDP, Inflation rates etc. APT also embodies the idea that all investors are not the same and do not always act rationally. (Ross, S, 1976; Fama and French, 2004).

The APT model is also derived as $E(ri) = rf + \beta i1 * RP1 + \beta i2 * RP2 + ... + \beta kn * RPn$, **Rf** = Risk Free Rate of Return; **β =** The Sensitivity of Each Factor; **RP=** Risk Premium of each specified factor (Ross, S, 1976)

APT is seen as an "important intuition" taking in the "the co-variability of an assets returns" with the return on other assets, which can be predicted using the linear relationship between the assets expect return and various macroeconomic variable that capture systematic risk. This is extremely important in the eyes of risk averse investors who holds a "well-diversified portfolio" containing numerous assets. (Shanken, J, 1982). APT is not unrealistic and does not assume a perfect market. APT assumes the market misprices securities, being over valued or undervalued at times, before being corrected to a fair market value. The macroeconomic factors have proven reliable as price predictors include unexpected changes in inflation, gross domestic product, corporate bond spreads and shifts in the yield curve (Hayes, 2020). APT assumptions allows Investors to build diversified portfolios mitigating the unsystematic risk, with no arbitrage opportunities among diversified portfolios DeFusco et al. (2015). Several empirical studies have portrayed the APT model is the first model that tries to eliminate the assumptions and deficiencies in CAPM, adapting to real life scenarios (Zainul Kisman1, Shintabelle Restiyanita M, 2015).

In 1973 Merton developed the Intertemporal CAPM (ICAPM), which was commonly known as the dynamic CAPM. ICAPM relaxes the CAPM assumption that investors only wanted to maximize their cash returns. The adapted model focused on how investors might also reinvest their returns back into the asset. By doing this investors also reduce transaction cost and defer taxation cost. ICAPM is ideal for investor who participate in the market for multiple years and not a single period of time like CAPM. Merton (1973) made observations on how investors participate within the market. ICAPM's assumptions on time periods are the opposite to its predecessor (CAPM) as investors may participate in the market for several years. The investors according to Mertons (1973) model are more concerned with economic shocks, fluctuations and even inflation and deflation rates. This economic concern has see's the hedging of risky positions and market exposure ensuring financial safety for investments within ICAPM as not just one period in time, but for a multiple (Merton, 1973, Shih et al., 2014).

Since the introduction of the CAPM there has been a large focus and effort made to improving the model. Relaxing its assumptions on various different levels, Taxation on dividends (Brennan,1970), the removal of restricted borrowing and short-selling (Black, 1972), and introduction of multi factor models which determines a fair asset price. Despite the numerous attempts of improving the model CAPM is still at the forefront of modern and educational finance, in its simplest and original form.

2.4 - CAPM's Efficiency and Empirical Evidence

This study concentrates solely on CAPM's efficiency and its relationship between the systematic risk, $\boldsymbol{\beta}$ and the portfolios return. Empirical evidence suggest CAPM fails due to its restrictive assumptions of the efficient market and investors that may skew results due to empirical error. (Roll, 1977; Fama and French 1992). CAPM comes with various limitations that do not adhere to real life scenarios. The Estimation of the $\boldsymbol{\beta}$ has been ousted to provide a statistical error that is non recoverable. (Fama and French, 2004) The Model also assumes efficient markets and rationale investors, that incur no transaction costs or fee's (Fama and French, 2004) Recent studies outline more than one factor which determines the return on stock returns. (Jensen, Black and Scholes, 1972) The majority of empirical evidence conducted on the model is focused on bettering the model and altering its original form, as shown above in section **1.3.** There is no doubt CAPM assumptions pander to a perfect world, however few studies focus on the $\boldsymbol{\beta}$, risk and return relationship, within the Irish Market.

Fama and Macbeth (1973) regression procedure is the most commonly used in testing asset pricing models. Tests for CAPM are based on three inferences relative to the relationship between the expected return and the systematic risk, beta. The first inference is that expected returns are linearly related to the beta variable, implying no other variable has explanatory power on this relationship. The second inference is the Slope relationship is defined by the market return less the risk free rate. The final inference is the beta premium is denoted as a positive variable, indicating average assets with high beta value produce higher returns. (Fama & French, 2004)

2.4.1 CAPM a Valid Risk Return Indicator

There is countless studies which outline CAPM is dead. (Yantos and Santos, 2009 & Lai and Stohs, 2015). Shortcomings have been highlighted by Fama and French (1992), through the use of cross sectional techniques proving size, price earnings, book-market ratios provide an explanation on asset returns distinguished by the market beta. Chan and Chui (1996) who covered the 1971 to 1990 time period outlined the beta to be unable to explain UK stock returns and suggested the CAPM has weakened in recent years. Bruner et al. (1998) outlines that 85% of corporations surveyed in the fortune 500 use the CAPM or an extension of the model. A recent survey has outlined that nearly 392 Chief Financial Officers in fortune 500 corporation's will apply CAPM, by using the average stock returns, β and including extra risk factors such as business cycle risk, interest rate risk, exchange rate risk, inflation risk and other macroeconomic factors (Chen, Roll and Ross, 1986).

Numerous Studies and research have been conducted on CAPM, causing a "cut to the heart", diminishing the single risk factored Beta isn't sufficient, and other factors are to be considered. (Ross, Stephen A. 1976. Fama and French, 2004). This is a viable statement, as the research conducted in the late 1970's "uncovered" variables that effect asset pricing far beyond than the market Beta. (Fama & French (2004)). It is evident from the minimal literature available that little attention has been focused on the original model.

Sharpe and Cooper (1972) used a strategy of purchasing portfolios of stocks with different beta values. This study consisted of all the stocks listed on the New York Stock Exchange (NYSE) over a 36 year period between 1931 and 1967. In this test the use of the previous 60 months of data was used to estimate the beta values. From these beta values 10 portfolios were built starting from the highest beta valued stocks going to the lowest valued stocks. This strategy was implemented each year of the chosen period. The results concluded a positive relationship between returns and beta, indicating that past beta values offered significant explanation to future returns.

Black et al. (1973) tested CAPM using a similar test to Sharpe and Cooper (1972) with monthly stock prices of the entire stocks quoted on the equity market from 1926 to 1966 for the 40 year period and also four sub-periods. In order to mitigate errors in the beta values estimation, all stocks were grouped together into 10 different portfolio's. This structure saw the first portfolio being made up of 10% of stocks with the highest beta value and 10% with the lowest beta value being included in the last portfolio. The results of the study concluded a positive relationship between the expected returns and corresponding beta values. Black et. al (1973) has outlined that by using systematic risk, beta of a portfolio built up of numerous stocks in the regression, they in fact mitigated measurement errors on individual stocks.

London Stock Exchange has been the market for recent test on CAPM, mainly using a two pass regression or a cross- sectional technique to examine the relationship between beta and stock returns. Strong and Xu (1997) chose to use the two pass regression technique from the 1960's to 1992. This study concluded in successful test results, finding a positive relationship between beta and the portfolio's expected return. However, this result is only successful when the beta is considered the only explanatory variable for return, thus becoming obsolete when one tries to include other variable factors, forming a multifactor model. Other ratio tools, such as book to market ratio where found useful in this study, to explain changes to stock returns within the London stock exchange.

Empirical Analysis has been conducted on the CAPM model to examine the beta variable's strength, size and book to market value in order to explain returns of a large number of stocks, chosen randomly. This examination saw 300 stocks chosen at random, for just over 20 years. This time period was from the beginning of June 1980 till the end of June 2000. Morelli (2007) used monthly adjusted price data, obtained from the London Share Price Database. This study examined the role of the beta as predicted by CAPM, the firms size and book to market value as outlined by the Fama and French Model. Morelli (2007) applied 3-month UK Treasury Bill Rate as the risk free interest rate, using a value weighted average of all the selected 300 firms as a proxy for the market portfolio. In this study the beta variable test was unsuccessful and insignificant in explaining stock returns for the sample population and time period.

The Cross-sectional regression technique also found some success within the London stock Exchange Market. Hung et al. (2004) conducted a cross-sectional regression test in this market for a 25 year period (1975-2000) and discovered the beta to be a significant variable to explain the returns within this market, when once again considered the only explanatory variable, but also including more variables in the model.

Three CAPM inefficiencies where highlighted in 2013, when Bornholt (2013) built a portfolio consisting of 48 various industries within the U.S markets, over a 46 year period from 1963 to 2009. Bornholt's (2013) study found $\boldsymbol{\beta}$ variable, where portfolios within low β stocks consisted of higher average returns and portfolios with high $\boldsymbol{\beta}$ stocks, returned a lower average return. A similar result occurred in the book to market ratio's in the portfolio's built. Similar to the $\boldsymbol{\beta}$ variable test, this observation saw firms with a high book to market ratio to have a higher expected return, whilst those that have a lower ratio have a lower average return. When testing the CAPM a pattern unfolded, displaying stocks with high average returns in one period to show even higher returns in the next period. (Jegadeesh and Titman, 2001).

In more recent studies it has been found the traditional CAPM cannot be rejected, however it also is ambiguous as to whether it can also be accepted as well. The application of CAPM in mainland Europe found the systematic risk to be an important factor in explaining the expected returns on assets. However, the study also finds the small population size of 18 stocks over a much smaller period of 14 years bares an impact on the test results within the Lisbon Stock Exchange. (Ferreira and Monte, 2015)

2.4.2 CAPM: Risk-Return Relationship

CAPM, built itself on Markowitz (1952) portfolio theory, providing a template for investors without knowing the expected return of each stock, to identify efficient portfolio's inclusive of risky securities. CAPM assumes the capital market to be efficient, a perfect world, with share prices reflecting all information within the market. The linear relationship imposes that the expected returns on stocks is determined by the systematic risk, beta. (Nwani, 2013) Sharpe (1964), Lintner (1965), Mossin (1966) and Stephen Ross (1976) (APT) have identified the relationship between to variables to compute a positive linear relationship on portfolio returns.

Nwani (2013), Fama and Macbeth (1973) outlined CAPM as: $\boldsymbol{E(R_i) = R_f + \beta_i(E(R_m) - R_f)}$. This equation defines the linear relationship between the expected return and systematic risk to be equal to the risk-free rate ($\boldsymbol{R_f}$) plus the risk premium.

Fama and Macbeth (1973) tested CAPM finding positive and linear relationship results between beta and stock returns for the entire period between 1926 – 1985. Fama and Macbeth (1973) used monthly stock prices of stocks listed on the New York Stock Exchange (NYSE), applying the famous two ordinary least square regression technique, also known a two-step regression test, which validates CAPM. In finding their successful results they performed the test on the chosen period as a whole and into 9 specific sub periods, which was further divided in to portfolio's.

Clare et al. (1998) performed another successful positive and linear relationship test, by using the adjusted stock returns of 100 listed firm on the London Stock Exchange, between 1980 and 1993. This information was used to determine a positive linear relationship between beta and the British stock returns over this 13 year period. Their study concluded in the beta variable providing an explanation for expected returns in the London Stock Exchange, depicting a positive linear relationship between beta and the expected returns on assets. This study also shined a light on other factors which resulted in minimal explanations on the risk return relationship, such as the book to market ratio, leverage or Earnings per Share.

There has been arguments presented in favour of CAPM, to be an appropriate model to define the relationship between systematic risk and expected returns in relation to the semi-strong Efficient Market Hypothesis, within the Nigerian Stock Exchange. (Obrimah, Alabi and Ugo-Harry, 2015) . Even during times of disparity during the great recession of 2008, CAPM proved its worthiness predicting asset prices efficiently during the study of the risk return relationship of the Dhaka Stock Exchange. (Hasan et al., 2011)

Similar studies have found a positive relationship between risk and return in more developed economies, in central Europe. However most take the traditional CAPM and alter it slightly. This occurred in Sauer and Murphy's (1992) examination of the total returns of 140 stocks in the Frankfurt Stock Exchange, over a 20 year period between 1968 and 1988. Their findings returned a positive relationship for the risk return model. When they altered the CAPM model to the CCAPM, which is the Consumption CAPM that considers multiple times periods it outperformed the original model concluding in a strong correlation between the risk and return.

Early tests on the linear regression have proven positive for CAPM, however there is no prior research conducted over periods of extreme volatility within a developed economy (Black, Jensen, Scholes,1972), Fama and Macbeth, 1973). Bouchaddekh, Bouri and Kefi (2014) study of the original CAPM, shows validation in predicting asset prices, when applying friction factors outside to the standard model. These friction factors imposed in the model are illiquidity, transaction and information costs. By imposing these anomalies the model is said to be efficient in explaining the stock prices and the additional factors imposed are positively related to expected returns.

CAPM's failure in history has been illustrated by the a non-linear relationship between the systematic risk, $\boldsymbol{\beta}$, and the portfolio's expected return. CAPM's failures in a more recent study saw use of the Fama and French (1996) three factors method, validating the US market showing positive linear results, but for the Sri Lankan Market the results proved CAPM to be inefficient. (Dayaratne, 2010). Ali and Ali (2009) followed the Fama and Macbeth (1973) methodology Dhaka Stock Exchange from

1998 to 2008, suggested that other factors will need to be considered in order to explain the relationship between risk and expected returns as their results formed "extremely weak support of the CAPM". (Ali and Ali, 2009) This evidence has shown the CAPM to be inconsistent in its reliability as it appears to not be validated as often in less developed economies, compared to the developed world economies.

Poterba and Summers (1988) have also found the length of time the tests are conducted over bares some stature depicting is a positive autocorrelation in risk-return relationship over short-term, compared to its negative relationships over the longer time periods. (Poterba and Summers, 1988)

Analysis on emerging economies and markets has been conducted by Lee, Cheng and Chong (2016), who applied the two step regression (Ordinary Least Squares and Cross sectional regression) techniques. The testing of CAPM was conducted in the emerging market in the Kuala Lumpur Stock Exchange. By using weekly data of 60 stocks, over a period of 4 years the study portrays CAPM to be a good indicator of stock prices. The results portrayed a positive linear relationship between β and the portfolio's expected return. This study has defined CAPM to "estimate the behaviour and the systematic risk of the stocks in Malaysia before investing in stock market", in order to minimize downside risk as they understand the stock trend of the company and hence invest rationally" (Lee, Cheng ad Chong, 2016)

2.5 The Research Question

CAPM has been at the fore front of modern financial analysis for decades, still being applied in today's world, receiving various accolades and critiques on the model's limitations throughout the years. The pricing model expresses the risk-return relationship, is portrayed as a positive and linear relationship and is explained and expressed by one single factor, the risk denoted as, $\boldsymbol{\beta}$.

CAPM's critiques focus on one factor in the model, "Beta", the systematic risk and how the model is only subjective to fluctuations in this variable. The limitations identified over the years has led to the evolution of the single factor Pricing Model, to the birth of APT. APT model is CAPM's successor, taking the single risk factor variable of Beta, imposing multiple variables which will affect the risk and return in a portfolio, such as GDP and inflation rates for example.

This study will delve into the most uncertain and volatile period in modern finance from 2019 the onset of Covid-19 and the duration of the global pandemic through to 2021. The study of CAPM will be in its original format as derived by by Sharpe (1964), Lintner (1965) and Mossin (1966), analysing the risk return relationship though a series of linear regressions and t-tests.

This study will focus on a more recent time period, which has seen a significant change in the way the world has operated. This research will delve into historical data pre & post global pandemic, from 2019 to 2020. This time period saw it all, the beginning of US presidency change office to Donald Trump and then to Joe Biden; The initial effects of Brexit on the European Union. The inclusion of this historical time period for America and Great Britain event is due to the old saying that "When America sneezes, the world catches a cold", due to its dominant role in global economics. (Forbes, 2019)

Due to little empirical evidence within the financial markets and the impact these various historical events left on the market, the main objective of this research will be **to Validate the Capital Asset Pricing Model within Extreme Volatility**" the research question I will be attempting to answer will be:

Is the Capital Asset Pricing model a valid indicator of Risk vs Return, within the Modern Irish Market?

The Sub-objective of this research to aid my conclusion would be to be to outline the following question:

Is there a positive linear relationship between Risk vs Return to validate CAPM, within today's Irish Market?

When Analysing the risk-return relationship in CAPM this research may support further research to develop a new model, taking into consideration more variables that may help define the risk vs return in the Irish and global markets

Chapter 3 – Methodology

This chapter consists of the methodology used to validate CAPM in the ISEQ 20 Index, focusing on the relationship between the systematic risk outlined by $\boldsymbol{\beta}$ and the expected portfolio returns through the outlook and use of the SML test.

Sharpe (1964), Treynor (1962), Lintner (1965) and Mossin (1966), CAPM's creators define the risk return relationship to be linear, with the portfolio's returns to be explained by one factor, $\boldsymbol{\beta}$. A quantitative analysis test approach, initially conducted by Fama and Macbeth (1973) will be undertaken to validate the risk return relationship which adopts the SML test. This methodology has been mirrored and applied in various different scenarios and economies as previously mentioned.

The results testing CAPM found both success and failure in their own rights as Lee, cheng and Chong (2016) applied CAPM to 60 stocks within KLSE emerging markets, over the period of four years between 2010 to 2014. The analysis consisted of weekly data, which saw authors apply Fama and Macbeth (1973) methodology of a two-step regression process, both cross-sectional and ordinary least squares (OLS). Lee, cheng and Chong (2016) tested CAPM by organising there data into portfolios. These portfolios were meticulously selected, based on the individual companies $\boldsymbol{\beta}$ value from the first pass regression, with the highest $\boldsymbol{\beta}$ stocks consisting in the first portfolio and the lowest $\boldsymbol{\beta}$ stocks in the last portfolio. This approach was selected by the authors to minimise diversification and minimize standard error as per Hasan et al. (2011) approach. From the construction of the portfolios Lee, Cheng and Chong (2016) conducted the SML test of CAPM, where they confirmed the linear relationship between $\boldsymbol{\beta}$ and the expected return. The result portrayed some success, by applying portfolio diversification through the portfolios reduced the unsystematic risk associated with CAPM. Lee, Cheng and Chong (2016) concluded by adopting this diversification approach to the market will "build up the investors' confidence towards the investment decision" (Lee, cheng and Chong, 2016).

Nwani (2013) tested CAPM within the LSE, once again adopting Fama and Macbeth (1973) methodology, using monthly data. Nwani (2013) used 100 randomly selected stocks listed on the LSE for the analysis of CAPM, over an 18 year period. For this analysis the author used the FTSE All Share index of the LSE as the proxy for the market portfolio. Similar to Lee, Cheng and Chong (2016), the first pass regression was conducted on the individual stocks to calculate $\boldsymbol{\beta}$. All stocks where then ranked in order from lowest $\boldsymbol{\beta}$ value to highest, where portfolios where created with the highest $\boldsymbol{\beta}$ value stocks in one portfolio and the lowest $\boldsymbol{\beta}$ value stocks the final portfolio. This process is the same as Lee, Cheng and Chong (2016) and is used to mitigate standard error and provide diversification for the unsystematic risk. In the second pass regression $\boldsymbol{\beta}$ and the average excess returns of all equally weighted portfolios are estimated in order to perform the SML test. Nwani (2013) take $\boldsymbol{\beta}$ as an explanatory variable to explain the variation in stock returns. The conclusion of the analysis portrayed the stock returns to be not be significantly sensitive to $\boldsymbol{\beta}$ (systematic risk). This result may be down to some choice factors when constructing the model as Daves, Ehrhardt and Kunkel (2000) tested various returns intervals proving daily returns should always be used as it provides the greatest precision of the β estimate, minimising $\boldsymbol{\beta}$ standard error.

LSE has seen success with testing CAPM, through the Fama-Macbeth (1973) methodology. Hung et. Al (2004), used stock prices converted to monthly returns using natural logarithmic in the analysis of CAPM. The first pass regression as derived by Fama-Macbeth (1973) was undertaken to sort the data into portfolios base on their $\boldsymbol{\beta}$ value, reducing $\boldsymbol{\beta}$'s "error associated with mis-estimation". Portfolios were then constructed from highest $\boldsymbol{\beta}$ stock value to lowest, as "portfolio returns will be less noisy than the individual stock returns and will reduce the unexplained variance in cross sectional prediction". Each portfolio was then analysed through the second-pass regression derived by Fama-Macbeth (1973). The conclusion portrayed by Hung et. Al (2004), suggests CAPM may hold, suggesting the market $\boldsymbol{\beta}$ remains significant and even stronger when other factors such as firm size is included.

The objective of the study is to explore the efficiency of CAPM in its original format as derived Sharpe (1964), Treynor (1962), Lintner (1965) and Mossin (1966), testing the risk-return relationship as proposed by Fama-Macbeth (1973), which is to be positive linear result as per the theoretical assumptions to which it was built. This research will take on a positivism approach when applying the model, based only on factual evidence.

3.1 Data Collection

The data collection will consist of historical daily closing prices of all companies from Euronext Dublin Market within the ISEQ 20 index. These stocks are domiciled in the Irish economy, trading through the Euronext Dublin market since 2019, as of the takeover in 2018. Companies selected for this analysis will be diversified in the Euronext Dublin, as Black, Jensen and Scholes (1972) have identified investors to be active in all markets equally and rationally. The reasoning behind this choice is the Euronext Dublin Market is a well-diversified market, ranging from real estate, banking to insurance, textiles and commodities. The ISEQ 20 Index is used as a proxy for the real market portfolio. This has been chosen due to one of the problems outlined by Roll (1977), who argues one of the downfalls of the pricing model is discovering a proxy choice similar to real life scenarios. By choosing the ISEQ All-Share 20 Index, mitigates this limitation when testing the pricing model as the index represents the real Irish Stock Market.

Daily returns over a two-year period will be used in the analysis of CAPM. This choice is based on Daves, Ehrhardt and Kunkel (2000) analysis on the different return intervals (i.e. Daily, Weekly, Monthly and Annually) which concluded for analysis purposes and for financial managers "should always select daily returns because daily returns result in the smallest standard error of beta or greatest precision of the beta estimate" (Daves, Ehrhardt and Kunkel, 2000). The price data gathered from the Euronext database has been transformed into return data using: $\mathbf{ln}(\frac{\boldsymbol{P_{i,t}}}{\boldsymbol{P_{i,t-1}}})$; where $\boldsymbol{P_{i,t}}$, is the price of the stock $\boldsymbol{i}$ at time $\boldsymbol{t}$, and $\boldsymbol{P_{i,t-1}}$ is the price of the stock $\boldsymbol{i}$ at time $\boldsymbol{t-1}$ (previous trading day in our sample) (Beninga, 2014) . The use log return of stock closing prices follows Fama and French (1986) study of the serial correlation of stock returns where the results presented evidence of expected alterations in large data studies. Lognormal returns enables the data to be normalised. Black and Scholes (1973) find this approach of asset pricing to be the most optimal, as stock prices follow a random walk, thus the distribution of possible stock prices of any finite interval is lognormal. (Black and Scholes, 1973)

The time period to which the research is being conducted is situated from the change in structure of the ISEQ Market to the inclusion into the Euronext Markets in 2018/2019 and to the beginning of the Global Pandemic that shock the world, Covid-19. Even as this time period is quite small Daves, Ehrhardt and Kunkel (2000) have found beta estimations over longer time periods to be biased and of minimal use to the financial managers in real life scenarios, as such the approximately two-year period chosen should capture the beta estimation values, with a maximum reduction in the standard error (Daves, Ehrhardt and Kunkel, 2000).

The returns data is comprised of 493 observations on each of the 20 companies that trade daily on the Euronext Dublin Market and of which have been in the market from inception to today's date. The daily closing prices have been gathered from the Euronext database (www.euronext.com). All 20 stocks of the ISEQ 20 have been included in the analysis, since the inception of the ISEQ on the Euronext market.

The analysis of CAPM will be based on historic empirical evidence and facts, with the results and conclusion to depicted by the use of descriptive statistics and various t-tests and hypothesis. The methodology adopted for this analysis is the Fama-Macbeth (1973) two-step regression technique.

3.2 Formulas

CAPM Equation is denoted as a price regression model with the equation expressed as $E(R_i) = R_f + \beta_i(E(R_m) - R_f)$ which was dissected in **section 2.1.** Fama and Macbeth (1973) implemented a two-step procedure known as First Pass (time-series) and Second Pass (cross-sectional) regression, which has become the most common method of testing CAPM.

CAPM will be analysed through the use of Excel tools, as such Benningas (2014) adaptation of the first and second pass regression. The expressions are donated as follows - First Pass Regression: $r_{it} = \alpha_i + \beta_i r_{spt} + \varepsilon_{it}$ with α expressing the intercept; β denoted as the slope; ε_{it} representing the standard error. The Second Pass Regression as depicted by Benninga (2014) examines the validity of CAPM by regressing the mean returns on the β is expressed as $E(r_i) = y_o + y_1\beta_i + \varepsilon_i$ with y_o representing the intercept and y_1 depicting the slope of the regression.

Markowitz (1952) defines his idea of the efficient frontier from a mathematical concept that reflects the combinations or portfolios that generates the maximum expected return for various levels of risk. Markowitz (1952) efficient frontier is depicted clearly in figures **1.2** and **1.3**. Fama-French (2004) outline (**fig. 1.3**) “If there is no risk-free borrowing or lending, only portfolios above b along abc are mean-variance-efficient, since these portfolios also maximize expected return, given their return variances”. Fischer Black (1972) outline CAPM’s development without risk-free borrowing or lending investors will select portfolios along the efficient frontier, depicted clearly in figure **1.2** and between the points *a* to *b* in figure **1.3.**

The Efficient Frontier and the CML is computed from the results of the first pass regression and data returns (section **3.1**). Benninga (2014) methodology is adopted for constructing the model and testing it. The efficient Frontier is built using the expression of the line representing the risk-return combinations between portfolio x and the risk free asset: $E(r_p) = r_f + \frac{E(r_x) - r_f}{\sigma(r_x)}\sigma(r_x)$ (Benninga, 2014).

In order to identify two efficient portfolios two different *c* values will be used to solve for the vector *Z*= $(S^{-1}(E(r) - c))$ to minimize the variance and used to find envelope portfolios x and y, expressed as $x_i = \frac{z_i}{\sum_{j=1}^{N} Zj}$These formulas are denoted as follows : $S^{-1} - inverse\ of\ the\ Var - Cov\ Matrix$; $E(r) - Vector\ of\ the\ Expected\ Returns$; **C** – constant value (Benninga, 2014).

The efficient Frontier is constructed using the combination of these two envelope portfolios mean returns: $E(r_p) = aE(r_x)$+(1-*a*)$E(r_y)$; a – the proportion invested in portfolio x; $E(r_p)$ – expected return of the new portfolio p and standard deviation returns: $\sigma_p = \sqrt{a^2\sigma_x^2 + (1-a)^2\sigma_y^2 + 2a(1-a)Cov(x,y)}$

In finding the CML, Benninga (2014) methodology will once again be adopted for consistency. The formulas used to create the efficient frontier will be adjusted accordingly. The risky portfolio can be found by deriving the equation $S^{-1}(E(r) - r_f)$. and letting *M* be the efficient portfolio $M_i = \frac{z_i}{\sum_{j=1}^{N} Zj}$. Similar to the construction of the efficient frontier a combination of portfolio *M* and the risk-free asset r_f mean and standard deviation returns will be calculated. $E(r_p) = \alpha r_f + (1 - \alpha)\ E(r_{M_i})$; $\sigma_p = \sqrt{a^2\sigma_{r_f}^2 + (1-a)^2\sigma_M^2 + 2a(1-a)Cov(r_f, r_y)}$. Each portfolio on the CML is a linear combination of the Market Portfolio (***M***) and the risk free asset (r_f), where each line line connecting r_f to ***M*** is called the CML. Benninga (2014) outlines all combinations for $\alpha \geq 0$, is known as the CML (portrayed in figure **1.2**).

3.3 The Risk Free Rate

The risk-free rate of return is the interest rate an investor expects to earn money on an investment that carries zero risk (CFI, 2021). For this analysis both the LIBOR and Overnight Index Swap (OIS) rates have been used as the proxy. LIBOR and OIS zero-curves have been used for the risk-free rate used in this analysis is the average of the 30-year zero rates downloaded from Markit, taken between 23rd of July 2019 - 25th of June 2021, the exact dates in which the study is placed. As per John Hull and Alan White (2013) “the OIS zero-curve can be bootstrapped similarly to the LIBOR/swap zero curve”. The maturity used for the risk-free rates was the full 30-year maturity (for the full life cycle). As the maturity is longer than the natural OIS rate (often three months or less) subtracting the spread of the OIS zero-curve from the LIBOR zero-curve, which allows it to be “spliced seamlessly onto the end of the OIS zero curve” to create a risk-free interest rate. (Hull, J and White, A, 2013).

It is well known that there is no perfect risk-free rate, however the OIS rate is deemed the best proxy available (Hull, J and White, A, 2013). Hull and White (2013), outline by using the OIS rate as the proxy due to it clearly separating the following three aspects of the valuation of a derivatives portfolio; **1.** The calculation of the no-default value of the portfolio; **2.** The impact of the credit risk of the dealer and the counterparty; and **3.** The impact of a non-economic the interest rate paid on cash collateral.

3.4 Linear Regression

In respect to the analysis conducted a two-step linear regression has been undertaken and a t-test implemented on the intercept and slope in order to assess if the $\boldsymbol{\beta}$ variable is the sole explanation of the risk-return relationship or if there is other influences and that this relationship is linear. Fama and Macbeth (1973) introduced a two-step process for testing CAPM, where the equations have been derived in **section 2.2**. The first step is known as First Pass Regression (time-series), with the final step being the Second Pass (cross-sectional) regression. This linear regression and hypothesis test will be conducted over a two year time span from the inception of the ISEQ 20 on the Euronext Market till the most recent date, 25th of June 2021.

3.4.1 *First Pass Regression*

The first pass regression sees a number of calculations being computed $(\boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\alpha} \,\&\, \boldsymbol{R-squared})$ from the individual Asset returns (calculated as $\ln(\frac{P_{i,t}}{P_{i,t-1}})$) against the returns of the Market Proxy (ISEQ 20). The first pass regression, which is also known as a time series regression is used to estimate the expression $\boldsymbol{r_{it} = \alpha_i + \beta_i r_{spt} + \varepsilon_{it}}$ for each individual asset, which was dissected in section **2.2**. This estimation included the calculations of the average returns, the slope **(**$\boldsymbol{\beta}$**),** the intercept ($\boldsymbol{\alpha}$) and R-squared (statistical significance of each individual asset) against the market proxy (ISEQ 20 daily returns). Two t-tests have been completed based on Benninga (2014) methodology on testing the SML for the significance of the $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$.

The most crucial part of the first pass regression is the estimation of $\boldsymbol{\beta}$. $\boldsymbol{\beta}$ is the systematic risk and measure of sensitivity of the assets return and represents the slope relationship between the return on assets and the return on the market (Fama and French, 2004). $\boldsymbol{\beta}$ is derived as follows by Fama-French (2004): $\boldsymbol{\beta_i} = \frac{COV(r_i, r_m)}{VAR(r_m)}$, denoted as $\boldsymbol{r_i}$ represents the return on asset $\boldsymbol{i}$ and $\boldsymbol{r_m}$ the return of the market portfolio m. The derivation of $\boldsymbol{\beta}$ described above closely follows the approach taken by Lee, Cheng and Chong (2016) and Nwani (2013), who adopted the methodology as depicted by Fama-Macbeth (1973)

3.4.2 *Second Pass Regression (SML Test)*

After the first pass regression is completed and the estimation of the $\boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\alpha}$ values, the second pass regression is conducted . Benninga (2014) outlines "the SML postulates that the mean return of each security should be linearly related to its beta". This regression will see each individual stock's expected returns regressed against their respected $\boldsymbol{\beta}$ values in the index to estimate the Security Market Line (SML): $\boldsymbol{E(r_i) = y_o + y_1\beta_i + \varepsilon_i}$ (Fama and Macbeth, 1973, Benninga, 2014), which has been depicted in section **2.2.** As per Fama-Macbeth (1973) and Benninga (2014) if the CAPM hold, the second-pass regression should be the SML. i.e. ($\boldsymbol{y_o = r_f}$ and $\boldsymbol{y_1 = E(R_m) - R_f}$). T-tests where then conducted to identify the significance of the y_o and y_1 results to see if they are statistically significant and different from zero, to propose a linear relationship as depicted by Fama and Macbeth (1973) in their original hypothesis.

3.5 Portfolio Construction and SML Test

To minimize the loss of information in the risk-return (SML) test, by using portfolios over individual securities, constructing portfolios on the basis of ranked $\boldsymbol{\beta}$'s for individual securities. Fama-Macbeth (1973) outline to inhibit a regression phenomenon of over/underestimation of the true $\boldsymbol{\beta}$ value, portfolios are to be constructed on the ranking of the individuals assets $\boldsymbol{\beta}$ value as estimated in the first pass regression outlined in **section 2.4.1.** All assets are then ranked in order from lowest $\boldsymbol{\beta}$ stock value to highest, as per Fama-Macbeth (1973) original test on CAPM. Lee, Cheng and Chong (2016) and Nwani (2013) did similar tests using this methodology, however instead of using the returns data, they have used the excess returns of the assets. In this study of CAPM the methodology documented by Fama-Macbeth (1973) is closely followed and as such the daily returns of each asset and the market will be used for this analysis, as it provides a true representation of how each individual asset and the markets performance.

Due to the limited number of stocks available in the ISEQ 20 index each portfolio is constructed equally of 5 stocks per portfolio. Each portfolio constructed is selected according to the β value derived above in descending order from lowest to highest. This technique follows Fama and Macbeths (1973) and Nwani's (2013) direction. This achieved 4 diversified portfolios. The reasoning for this grouping is in the reasoning provided by Fama and French (2004). They state that the use of "cross-section regressions of average returns on betas reduces the critical errors in variables problem". The mitigation of this risk sees portfolios being constructed based on the beta value derived in the first pass regression procedure. The first portfolio "contains securities with the lowest betas, and so on, up to the last portfolio with the highest beta asset" (Fama and French, 2004). Blume (1970), Friend and Blume (1970) and Black, Jensen and Scholes (1972) in their respective tests use portfolios, rather than individual assets.

Once the portfolios were constructed a sample Var-Cov matrix's for each portfolio, where constructed based off the assets returns within the respected portfolios, originally calculated in the data collection section (**3.1**). The methodology for the construction of sample Var-Cov matrix has been adopted from Benninga (2014). Benninga (2014) portrays the construction of the matrix as follows: mean return of asset i; $\bar{r}_i = \frac{1}{M}\sum_{T=1}^{M} r_{it}, i = 1, \ldots, N.$ The covariance of the return of asset i and asset j is calculated as; $\sigma_{ij} = Cov(i,j) = \frac{1}{M-1}\sum_{T=1}^{M}(r_{it} - \bar{r}_i)(r_{jt} - \bar{r}_j), i,j = 1, \ldots, N.$ The sample Var-Cov matrix is constructed efficiently using the matrix of excess returns portrayed as;

$$A = \begin{pmatrix} r_{11} - \bar{r}_1 & \cdots & r_{N1} - \bar{r}_N \\ r_{12} - \bar{r}_1 & \cdots & r_{N2} - \bar{r}_N \\ \vdots & \ddots & \vdots \\ r_{1M} - \bar{r}_1 & \cdots & r_{NM} - \bar{r}_N \end{pmatrix};$$

The columns of matrix A depicted by Benninga (2014) represents the mean asset returns being subtracted from the individual asset returns. Benninga (2014) denotes the transpose of this matrix as;

$$A^T = \begin{pmatrix} r_{11} - \bar{r}_1 & r_{12} - \bar{r}_1 & \cdots & r_{1M} - \bar{r}_1 \\ \vdots & \vdots & \ddots & \vdots \\ r_{N1} - \bar{r}_N & r_{N2} - \bar{r}_N & \cdots & r_{NM} - \bar{r}_N \end{pmatrix}.$$

Through the multiplication of $(A^T x\, A)$ and dividing the result by *M-1,* would result in the sample Var-Cov matrix denoted as $S = (\sigma_{ij}) = \frac{A^T.A}{M-1}$

Once the Var-Cov matrix have been constructed, the mean, variance and standard deviation has been calculated as per Roll (1978) and Benninga (2014) portfolio test of the SML. For an unbiased result independent of Roll (1978) study the portfolio proportions have been randomly selected, using various excel functions, with the entire portfolio, summing to 1.

From these calculations the first pass and second pass regression depicted in section **3.4** is ran for the various portfolios. As previously outlined in section **3.4.2,** the cross-sectional regression depicts each portfolios expected return against their respected $\boldsymbol{\beta}$ values in the index to estimate the SML: $\boldsymbol{E(r_i) = y_o + y_1\beta_i + \varepsilon_i}$ (Fama and Macbeth, 1973, Benninga, 2014). The same criteria will hold through as for the individual asset tests (outlined in in section **3.4.2)** to identify if CAPM holds. T-tests have been conducted to outline any statistical significance of $\boldsymbol{y_o}$ and $\boldsymbol{y_1}$ results to propose a linear relationship as depicted by Fama and Macbeth (1973) hypothesis.

3.6 Efficient Frontier and the CML

Fama and French (2004) in their theoretical study of CAPM, best describes the pricing model as a frontier (fig. 1.3) portraying portfolio investments, based on the risk and return. The minimum variance frontier outlines combinations of the expected return and risk for portfolios inclusive of risky assets, thus determining a given level of risk will be associated with one's return.

In the construction of the efficient frontier and the CML 2 propositions outlined by Benninga (2014) will be used depicting the portfolio's returns with a given level of risk.

Proposition 1: This proposition states that if x is an envelope portfolio, then a constant c and a vector z exists also**,** $\boldsymbol{Sz = E(r) - c}$ and $x_i = \frac{z_i}{\sum_{j=1}^{N} zj}$

Let c be a constant

$$x = \frac{s^{-1}(E(r) - C)}{Sum(S^{-1}(E(r) - c))}$$

Proposition 2: The establishment of two envelope portfolio's is based off of Black's (1972) finding that any two envelope portfolios are enough to establish the whole envelope (i.e. Portfolio x & Portfolio *y*).

The data analysis consists of 20 assets Variance- Covariance matrix (The same steps conducted in section 3.**5** for the whole data set) is computed based off the daily returns and average returns against the market proxy, which was computed within the first pass regression **(Section 3.4.1)**

The first step in calculating the efficient frontier is to uncover two efficient portfolios. By Benninga (2014) first proposition $(\boldsymbol{S^{-1}}(\boldsymbol{E}(\boldsymbol{r}) - \boldsymbol{c}))$ solves for $\boldsymbol{z}$. In order to find two efficient portfolios, two different $\boldsymbol{c}$ values will be used. These of course will be randomly selected and independent of each other. For each $\boldsymbol{c}$ value the vector $\boldsymbol{z}$ is solved, which subsequently aids the finding of the efficient portfolio $\boldsymbol{x_i}$ *and* $\boldsymbol{y_i}$ *through the formula* $\frac{\boldsymbol{z_i}}{\sum_{j=1}^{N} \boldsymbol{z_j}}$.

By recalling proposition 2 by Benninga (2014) the calculation of the efficient portfolios $\boldsymbol{x_i}$ *and* $\boldsymbol{y_i}$, the whole efficient frontier can now be calculated. In order to create the efficient frontier a combination of the two portfolios will need to be created into one portfolio called denoted as $\boldsymbol{p}$, i.e. proportion $\boldsymbol{a}$ invested within the portfolio $\boldsymbol{x_i}$ and a proportion of $(\boldsymbol{1} - \boldsymbol{a})$ invested in $\boldsymbol{y_i}$. For this analysis a proportion of **30%** was invested in portfolio Portfolio $\boldsymbol{x_i}$ as such a proportion of **70%.** $\boldsymbol{p}$ statistics (mean: $\boldsymbol{E}(\boldsymbol{r_p}) = \boldsymbol{aE}(\boldsymbol{r_x})$***+(1-a)***$\boldsymbol{E}(\boldsymbol{r_y})$ and standard deviation: $\boldsymbol{\sigma_p} = \sqrt{\boldsymbol{a^2\sigma_x^2} + (\boldsymbol{1}-\boldsymbol{a})^2\boldsymbol{\sigma_y^2} + \boldsymbol{2a}(\boldsymbol{1}-\boldsymbol{a})\boldsymbol{Cov}(\boldsymbol{x},\boldsymbol{y})}$) are then calculated in order to graph the efficient frontier which represents the risk-return relationship as originally defined by Markowitz (1952). The Frontier is then graphed from a range of **-300%** to **+300%** (randomly selected) based on the standard deviation and mean derived for portfolio $\boldsymbol{p}$.

In finding the CML, Benninga (2014) methodology will once again be adopted for consistency. The formulas used to create the efficient frontier will be adjusted accordingly. The CML "is a graphical representation of all the portfolios that optimally combine risk and return. CML is a theoretical concept that gives optimal combinations of a risk-free asset and the market portfolio" Thakur, M, 2021). In order to calculate the CML the risk-free asset defined by OIS and LIBOR rates in section **3.3,** this asset is assumed under Benninga (2014) methodology to have an expected return of $\boldsymbol{r_f}$. Similar to the construction of the portfolios $\boldsymbol{x_i}$ *and* $\boldsymbol{y_i}$ to form the efficient frontier, a portfolio denoted as $\boldsymbol{M_i}$ will be constructed by solving for the vector $\boldsymbol{z}$, expressed as $(\boldsymbol{S^{-1}}(\boldsymbol{E}(\boldsymbol{r}) - \boldsymbol{r_f}))$. Once the newly formed vector $\boldsymbol{z}$ is the portfolio $\boldsymbol{M}$ can be computed similar to the portfolios $\boldsymbol{x_i}$ *and* $\boldsymbol{y_i}$ and is expressed as; $\boldsymbol{M_i} = \frac{\boldsymbol{z_i}}{\sum_{j=1}^{N} \boldsymbol{z_j}}$.

Similar to the combination of $\boldsymbol{x_i}$ *and* $\boldsymbol{y_i}$ to create the portfolio $\boldsymbol{p}$ statistics, consisting of the portfolio mean returns ($\boldsymbol{E}(\boldsymbol{r_p}) = \boldsymbol{\alpha r_f} + (\boldsymbol{1} - \boldsymbol{\alpha})\, \boldsymbol{E}(\boldsymbol{r_{M_i}})$) and standard deviation returns ($\boldsymbol{\sigma_p} = \sqrt{\boldsymbol{a^2\sigma_{r_f}^2} + (\boldsymbol{1}-\boldsymbol{a})^2\boldsymbol{\sigma_{M_i}^2} + \boldsymbol{2a}(\boldsymbol{1}-\boldsymbol{a})\boldsymbol{Cov}(\boldsymbol{r_f},\boldsymbol{r_y})}$), will be computed through the combination of portfolio $\boldsymbol{M_i}$ and $\boldsymbol{r_f}$. The range of combinations a depicted by Benninga (2014) for $\boldsymbol{\alpha} \geq \mathbf{0}$ is known as the CML (graphed in fig. **1.2**).

3.7 Hypothesis Test

Revisiting the research question of **is the Capital Asset Pricing model a valid indicator of Risk vs Return, within the Modern Irish Market?** with the Sub-objective of this research to aid my conclusion to determine **Is there a positive linear relationship between Risk vs Return to validate CAPM, within today's Irish Market?**

The main test for this analysis will consist of the SML test as conducted by Fama-Macbeth (1973). Testing the SML on the various levels of systematic/market risk, of the market securities within the ISEQ 20, plotted against the expected return of the entire market at the chosen time frame. This test will follow the original hypothesis depicted by the Fama-Macbeth (1973) which is expressed as, :
H_0: $\boldsymbol{\gamma_0}$=$\boldsymbol{r_f}$ & $\boldsymbol{\gamma_1}$=$\boldsymbol{E(r_m) - r_f}$; **$H_1$:** $\boldsymbol{\gamma_0} \neq \boldsymbol{r_f}$ **&** $\boldsymbol{\gamma_1} \neq \boldsymbol{E(r_m) - r_f}$.

Broken down **H_0** states $\boldsymbol{\gamma_0}$ the intercept of the second pass regression (Section **3.4.2**) will be equal to the $\boldsymbol{r_f}$ the risk-free rate (as a risk-free asset exist in this analysis; section **3.3**) and $\boldsymbol{\gamma_1}$ is equal to the average monthly returns of the ISEQ 20 stocks subtracted by the risk-free asset (section **3.3**), known as the market risk premium as originally defined by Sharpe (1964), Treynor (1962), Lintner (1965) and Mossin (1966) . CAPM's positive linear risk relationship is satisfied and **H_0** will be accepted if the $\boldsymbol{\gamma_0}$ (intercept) is equal to $\boldsymbol{r_f}$ and the $\boldsymbol{\gamma_1}$ (slope) is equal to the Market risk premium $(\boldsymbol{E(r_m) - r_f})$ and that the models beta value is positive, meaning that the expected return on the market portfolio exceeds the expected return on assets (Fama and French, 2004).

The alternative **H_1** as expressed by Fama-Macbeth (1973) as $\boldsymbol{\gamma_0}$ (intercept) is not equal to $\boldsymbol{r_f}$ (risk-free rate; section **3.3**) and $\boldsymbol{\gamma_1}$ (slope) is not equal to the Market risk premium $(\boldsymbol{E(r_m) - r_f})$.

To evaluate **the results from** the first and second pass regressions a two tailed t- test will be constructed. A two tailed test is chosen for this analysis as it is seen as more reliable and accurate than the one tailed t-test and p-value criteria (Georgiev,G, 2018). During the empirical analysis the t-value statistic will be given for every estimate. The critical values for the t-statistic are **-1. 96** and **1. 96.** If the t-value is greater than **1.96** or smaller than **-1. 96** the results are statistically significant with 95%

confidence level. The critical value of p-statistic for the **95%** confidence interval is **0.05** which mean that if the p-value $<$ **0.05** then the $\mathbf{H_0}$ is rejected as the data favours $\mathbf{H_1}$.

The CAPM's positive linear risk-return relationship is satisfied and $\mathbf{H_0}$ will be accepted if the $\boldsymbol{\gamma_0}$ (intercept) is equal to $\boldsymbol{r_f}$ and the Slope $\boldsymbol{\gamma_1}$ is equal to the Market risk premium $(\boldsymbol{E(r_m)} - \boldsymbol{r_f})$ and also if the models the models beta value is positive producing a positive linear relationship between the systematic risk and expected return of stocks**.** If this is not the case and the alternative hypothesis is accepted it could be a telling that other factors will be required to explain the portfolio's returns

3.8 Limitations

CAPM itself brings some limitation when applied to real life scenarios. The main limitation is down to the assumptions built around the perfect world discussed in **section 2.1**. Past empirical research (Roll, 1977; Fama and French, 1992) have outlined the failure of CAPM is due to its strict and unrealistic assumptions, which make the model difficult to validate in a real life scenario. The limitations of the model built is subjected to the size of the sample data gathered, the number of portfolios constructed and the time frame selected. The implementation of the test over the global Covid-19 pandemic an anomaly event in the world economy may have distorted the results. Empirical studies suggest there is more than one factor which influences the stock returns (Jensen, Black and Scholes, 1976; Fama and French, 1992), however it is arguable that, there is no consensus about the validation of this relationship.

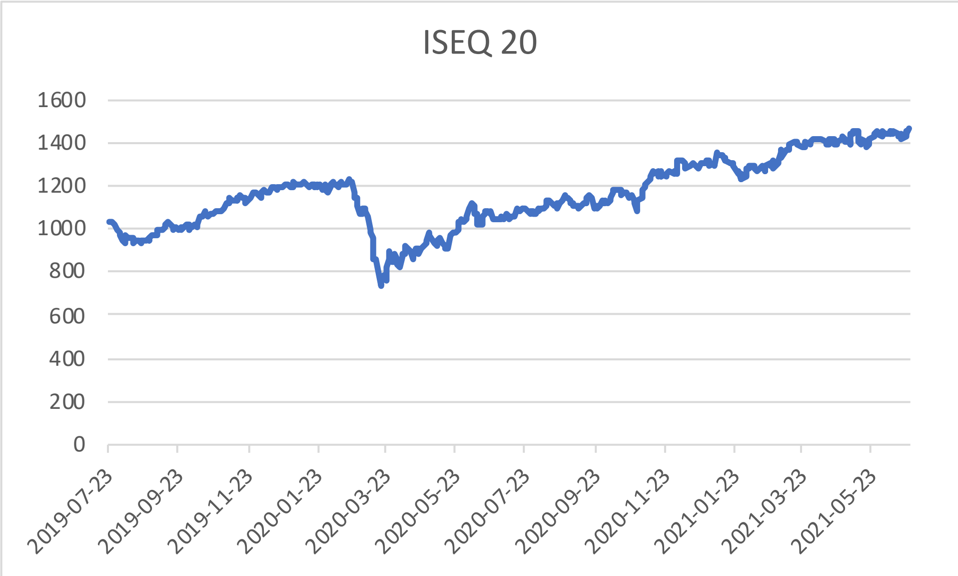
Chapter – 4 Analysis & Findings

The Analysis of the study in the validation of CAPM and the understanding of the risk-return relationship, within the Irish Market was conducted through a very historic part of the global economy, COVID – 19. This major impact saw the mean returns of nearly 35% of the ISEQ 20 Index stocks return a negative result. Recalling the research question for this analysis **is the Capital Asset Pricing model a valid indicator of Risk vs Return, within the Modern Irish Market?** with the Sub-objective of this research to aid my conclusion to determine **Is there a positive linear relationship between Risk vs Return to validate CAPM, within today's Irish Market?**

The tests on CAPM are based on three implications of the relationship between expected return and market beta implied by the model as outlined by Fama and French (2004) **1**. Expected returns on all assets are linearly related to their betas, and no other variable has marginal explanatory power. **2**. The beta (slope) premium is positive. This states the expected return on the market portfolio exceeds the expected return on assets whose returns are uncorrelated with the market return. **3.** In relation Sharpe-Lintner version of the model, assets uncorrelated with the market have expected returns equal to the risk-free interest rate, and the beta premium is the expected market return minus the risk-free rate

The trend of closing prices of the ISEQ 20 Index was used to calculate the stock returns, which is the foundation of the descriptive statistics and the analysis of CAPM as a whole. **Figure 1.4** portrays the trend of the closing prices from **23.07.21** to **25.06.2021.** It is evidently shown when the global pandemic first hit the Irish economy with a dramatic downfall in the ISEQ 20 Index from late **February 2020.** Since March 2020 ISEQ 20 Index has made a remarkable recovery over the next 12 to 18 months, coming back stronger than before, with the re-opening of Irish and Global economies with the aid of vaccine rollouts to the population.

Figure 1.4 : *Time-Series of ISEQ 20 Index Closing Prices from 23.07.21 to 25.06.2021*



Source: *Excel*

4.1 Descriptive Statistics:

Descriptive statistics enables the data set to be analysed, described and summarised, such as the data's central tendency and dispersion. The Central tendency indicated the central point to which all the data gathers, outlining values which are common such as the mean (average value), mode (most occurring value) and median (Middle value). Dispersion outlines the size of the distribution, indicating how dispersed the variables are from the central tendency.

The 20 stocks that make up the ISEQ 20 Index provide a diversified investment as they are from various different sectors within the Irish market from banking, insurance, residential and commercial real estate to bookmaking, airlines and nutrition. The companies that comprise of the ISEQ 20 Index are as follows: **AIB; Bank of Ireland; CAIRN HOMES PLC; CRH PLC ORD; DALATA HOTEL GP.; FBD HOLDINGS PLC; FLUTTER ENTERTAIN; GLANBIA PLC; GLENVEAGH PROP.PLC; Greencoat renewables; HIBERNIA REIT PLC; IRISH CONT. GP.; IRISH RES. PROP.; KERRY GROUP PLC; KINGSPAN GROUP PLC; ORIGAN ENTERPRISES PLC; RYANAIR HOLD. PLC; SMURFIT KAPPA GP; TOTAL PRODUCE PLC; UNIPHAR PLC.**

From the stock returns data calculated by the $\mathbf{ln}(\frac{P_{i,t}}{P_{i,t-1}})$ (Section **2.1**), provided a total 493 observations. The central tendency is depicted in **Table 1** by 493 observations through the mean range from **+0.002143 (UNIPHAR PLC)** to **-0.000873 (AIB);** median ranges from **+0.01542** to **-0.002547;**

the mode of the data set surprisingly returned a zero value for 90% of the companies within the ISEQ 20 Index. The mode outlines the most occurring return as the result is zero for **90%** of the ISEQ 20 Index, reveals the stocks volatility has been mitigated through the use of daily returns as advised by Daves, Ehrhardt and Kunkel (2000). Two companies Ryanair Holdings PLC returned a mode value of **+0.039221** and SMURFIT KAPPA GP with a +**0.004651.** Both companies operate in highly hit industries such as the airline (Ryanair Holdings PLC) and packaging (SMURFIT KAPPA GP) industry which was hit phenomenally through the global Covid-19 pandemic and Brexit. Due to the restriction on traveling imposed globally for this time period and the implications caused on by both Brexit and Covid-19 on importing and exporting goods from Ireland.

The Dispersion of the data set is portrayed in **Table 1** by the standard deviation and range from **+0.043910** to **+0.16294.** The standard deviation in descriptive statistics measures the dispersion of the data set to its mean value (average return). The analysis indicates banking, airline and hospitality sectors to be the most volatile over this time period, with renewable energy and nutritional sectors proving the least affected by the global pandemic. Kurtosis is a statistical measure used to depict the distribution of the data. Kurtosis measures extreme values in either tail of the distribution. Kenton, W (2020) (https://www.investopedia.com/terms/k/kurtosis.asp) outlines a high kurtosis on the return distribution suggests investors may experience extremely high positive or negative returns. From the descriptive statistics outlined in **Table 1,** which the Kurtosis values range from **+1.728682** to **+46.975818** which 90% lie above the designated normality value of 3, suggesting the data to be visualised as a thin bell with high peaks and is known as a leptokurtic distribution. Similar to Kurtosis, Skewness helps depicting the shape and direction of data set. A skewness result of zero, would indicate a normal distribution. It is evident from **Table 1** the data set is negatively skewed as **75%** of the stocks have a negative skewness result, which ranges from **+3.390771** to **-0.850029**

Table 1: *Descriptive Statistics of the ISEQ 20 Market*

	ISEQ 20	*AIB*	*BOI*	*CAIRN HOMES PLC*	*CRH PLC ORD*	*DALATA HOTEL GP.*	*FBD HOLDINGS PLC*	*FLUTTER ENTERTAIN*	*GLANBIA PLC*	*GLENVEAGH PROP.PLC*
Mean	0.000706	-0.000873	0.000116	0.000019	0.000806	-0.000280	-0.000085	0.001483	-0.000104	0.000468
Standard Error	0.000740	0.001978	0.001767	0.001176	0.001090	0.001687	0.000945	0.001169	0.001141	0.001096
Median	0.000905	-0.002547	-0.000492	0.000000	0.000836	-0.001188	0.000000	0.001180	0.000000	0.000000
Mode	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
Standard Deviation	0.016438	0.043910	0.039238	0.026115	0.024201	0.037468	0.020991	0.025963	0.025331	0.024331
Sample Variance	0.000270	0.001928	0.001540	0.000682	0.000586	0.001404	0.000441	0.000674	0.000642	0.000592
Kurtosis	7.478101	5.929596	2.885603	3.452094	8.051788	11.087127	11.024315	8.712642	6.301428	8.386869
Skewness	-0.850029	-0.267587	-0.209749	-0.402231	-0.539653	1.022387	-0.782734	-0.180031	-0.834171	-0.354633
Range	0.182916	0.494055	0.314695	0.239451	0.268897	0.434583	0.267800	0.302804	0.255706	0.289038
Minimum	-0.107759	-0.273916	-0.170958	-0.111311	-0.153700	-0.169362	-0.172185	-0.164808	-0.169528	-0.158748
Maximum	0.075157	0.220139	0.143737	0.128141	0.115197	0.265221	0.095614	0.137996	0.086178	0.130290
Sum	0.348163	-0.430338	0.057235	0.009479	0.397402	-0.137843	-0.041995	0.731069	-0.051220	0.230484
Count	493	493	493	493	493	493	493	493	493	493

	Greencoat renewables	*HIBERNIA REIT PLC*	*IRISH CONT. GP.*	*IRISH RES. PROP.*	*KERRY GROUP PLC*	*KINGSPAN GROUP PLC*	*ORIGAN ENTERPRISES PLC*	*RYANAIR HOLD. PLC*	*SMURFIT KAPPA GP*	*TOTAL PRODUCE PLC*	*UNIPHAR PLC*
Mean	0.000108	-0.000364	0.000144	-0.000266	0.000195	0.001131	-0.000774	0.001013	0.000979	0.001076	0.002143
Standard Error	0.000734	0.000917	0.001351	0.000858	0.000759	0.001104	0.001146	0.001358	0.001030	0.001031	0.001056
Median	0.000000	0.000000	-0.001117	0.000000	0.000000	0.001542	0.000000	0.000309	0.001187	0.000000	0.000000
Mode	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.039221	0.004651	0.000000	0.000000
Standard Deviation	0.016294	0.020361	0.029998	0.019042	0.016848	0.024517	0.025445	0.030158	0.022879	0.022893	0.023438
Sample Variance	0.000266	0.000415	0.000900	0.000363	0.000284	0.000601	0.000647	0.000910	0.000523	0.000524	0.000549
Kurtosis	14.305675	8.389515	28.505868	5.210491	4.590329	4.245670	1.728682	3.962094	3.393171	46.975818	3.560121
Skewness	0.390015	-0.264336	1.935079	-0.570792	-0.190600	-0.274205	-0.035090	0.220525	0.008197	3.390771	-0.015615
Range	0.228974	0.239605	0.506384	0.195239	0.182182	0.236357	0.192178	0.290988	0.227102	0.400924	0.224859
Minimum	-0.105361	-0.139262	-0.191891	-0.090450	-0.093989	-0.128871	-0.086818	-0.144610	-0.098468	-0.122137	-0.106719
Maximum	0.123614	0.100343	0.314493	0.104789	0.088193	0.107486	0.105361	0.146378	0.128634	0.278787	0.118140
Sum	0.053110	-0.179693	0.070983	-0.131256	0.096331	0.557591	-0.381437	0.499238	0.482716	0.530704	1.056648
Count	493	493	493	493	493	493	493	493	493	493	493

Source: *Excel*

4.2 - First Pass Regression:

The initial step in testing CAPM as per the Fama-Macbeth (1973) process will be to estimate the $\boldsymbol{\beta and\ \alpha}$ values for the individual stocks. In **Table 2** the results of the first pass regression are summarised below, revealing the statistical relationship between each asset and the market index for the entire sample period.

The range of $\boldsymbol{\beta}$ values outlined in **Table 2** produce a range of values. $\boldsymbol{\beta}$ is a measure of systematic risk, comparing a security's volatility relative to the volatility market as a whole. $\boldsymbol{\beta}$ is used to describe the relationship between the systematic risk and stock returns. It gauges the tendency of the return of a security to move in parallel with the return of the stock market as a whole. Every time the performance of the stock market moves in a certain direction, this will be explained by the Individual assets to a certain extent based on their $\boldsymbol{\beta}$ value. Not surprisingly ISEQ 20 Index returned a $\boldsymbol{\beta}$ value of **100%** (Benninga, 2014). The $\boldsymbol{\beta}$ values for each stock within index range from **+28.1113%** to **-12.5578%.** $\boldsymbol{\beta > 0}$ are said to be more volatile than the market and contains more systematic risk compared to the market as a whole; with $\boldsymbol{\beta < 0}$ negatively correlated to the market. The $\boldsymbol{\beta}$ value of **+28.1113%** sees a positive correlation in relation to the market index, as the index performance moves, the performance of stock moves with it. $\boldsymbol{\beta}$ value of **-12.5578%** sees a negative correlation to the market index, as the index performance moves the performance of stock moves against it, in the opposite direction. The $\boldsymbol{\beta}$ t-statistic results at 95% confidence level for the first pass regression are promising as **55%** of the results are significantly different from zero.

$\boldsymbol{\alpha}$ is the vertical intercept, which indicates how much better (**+**$\boldsymbol{\alpha}$) or worse (**-**$\boldsymbol{\alpha}$) the individual stock has performed against the market. Similar to Benninga (2014) results $\boldsymbol{\alpha}$ returns a zero value. The ISEQ 20 index stocks $\boldsymbol{\alpha}$ values range from **+0.2027%** to **-0.1071%.** The $\boldsymbol{\alpha}$ values in **Table 2** depict a weak performance in this time period as **50%** of the stock in this period resulting in negative values, underperforming for what CAPM predicts. The t-statistic results at 95% confidence level on the intercept $\boldsymbol{\alpha}$ provide an extremely promising result.**100%** of the Stocks within ISEQ 20 Index are shown to not be statistically significant from zero.

In finance, **R-Square** measures how well CAPM predicts the actual performance of an investment, how closely the performance of a stock can be attributed to the performance of market Index (ISEQ 20 Index). As per Benninga (2014) results it is not surprising that a value of **100%** was achieved for the Market Index computation. For the Individual stocks however, the R-Square results show a very weak positive correlation ranging from **4.7227%** to **0.0117%.** This outlines that the stock's performance against the market proxy (ISEQ 20), explaining the performance of the stocks to explained by its risk exposure of range **4.7227%** to **0.0117%. The R-Square** of the regression portrayed **Table 2** shows that between **4.7227%** to **0.0117%** of the individuals stock is accounted for in the variability of the ISEQ 20 Index. As per Benninga (2014) and CAPM theory approximately **4.7227%** to **0.0117%** of the variation within the individual assets is accounted for in the ISEQ 20 Index. This provides some positives as up to **95.2773%** of the individual stock returns can be diversified. This result reinforces CAPM's theory as the intercept of the first pass regression should be zero.

Table 2. *First Pass Regression Results*

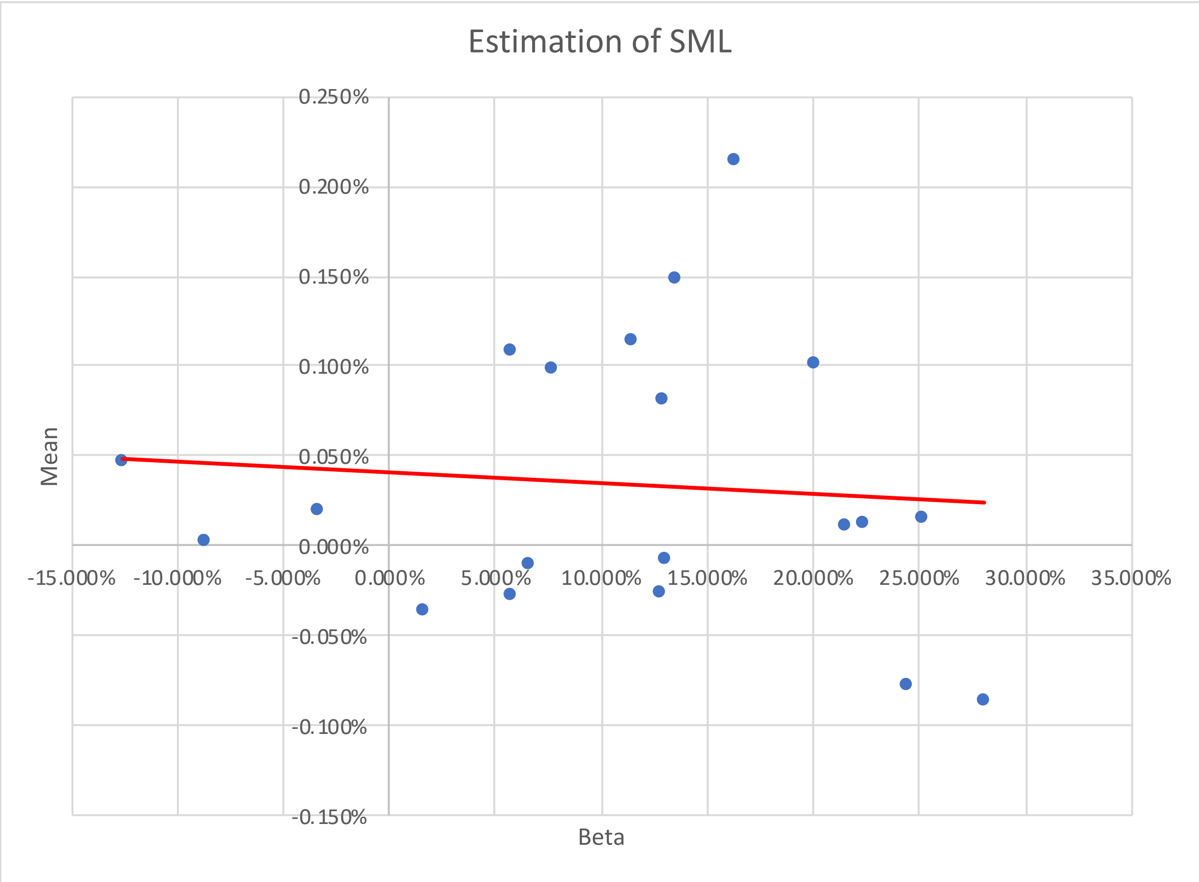
FIRST PASS REGRESSION	*Mean*	*Beta*	*Alpha*	*R-squared*	*t-statistic, Beta*	*t-statistic, Alpha*
ISEQ 20	0.0706%	100.0000%	0.0000%	100.0000%		
AIB	-0.0873%	28.1113%	-0.1071%	1.1075%	2.3449	-0.5438
BOI	0.0116%	22.4700%	-0.0043%	0.8861%	2.0952	-0.0004
CAIRN HOMES PLC	0.0019%	-8.6929%	0.0081%	0.2994%	-1.2143	0.0011
CRH PLC ORD	0.0806%	12.9270%	0.0715%	0.7710%	1.9532	0.0108
DALATA HOTEL GP.	-0.0280%	5.7891%	-0.0320%	0.0645%	0.5630	-0.0031
FBD HOLDINGS PLC	-0.0085%	13.0271%	-0.0177%	1.0407%	2.2723	-0.0031
FLUTTER ENTERTAIN	0.1483%	13.5207%	0.1387%	0.7328%	1.9038	0.0195
GLANBIA PLC	-0.0104%	6.6853%	-0.0151%	0.1882%	0.9622	-0.0022
GLENVEAGH PROP.PLC	0.0468%	-12.5578%	0.0556%	0.7198%	-1.8868	0.0084
Greencoat renewables	0.0108%	21.5419%	-0.0044%	4.7227%	4.9334	-0.0010
HIBERNIA REIT PLC	-0.0364%	1.6484%	-0.0376%	0.0177%	0.2949	-0.0067
IRISH CONT. GP.	0.0144%	25.2352%	-0.0034%	1.9122%	3.0939	-0.0004
IRISH RES. PROP.	-0.0266%	12.8800%	-0.0357%	1.2362%	2.4791	-0.0069
KERRY GROUP PLC	0.0195%	-3.3669%	0.0219%	0.1079%	-0.7283	0.0047
KINGSPAN GROUP PLC	0.1131%	11.4893%	0.1050%	0.5934%	1.7120	0.0156
ORIGAN ENTERPRISES PLC	-0.0774%	24.4965%	-0.0947%	2.5044%	3.5514	-0.0137
RYANAIR HOLD. PLC	0.1013%	20.1090%	0.0871%	1.2013%	2.4434	0.0106
SMURFIT KAPPA GP	0.0979%	7.6817%	0.0925%	0.3046%	1.2248	0.0147
TOTAL PRODUCE PLC	0.1076%	5.8100%	0.1035%	0.1740%	0.9252	0.0165
UNIPHAR PLC	0.2143%	16.4003%	0.2027%	1.3230%	2.5657	0.0317

Source: *Excel*

4.3 Second Pass Regression:

The next step is to perform the second pass regression as outlined in section **3.4.2.** The second pass regression will see the average returns of the stocks regressed against their estimated $\boldsymbol{\beta}$**.** The results of the regression is presented in **Table 3.,** with a two-tailed t-test being conducted at a 95% confidence interval.

figure 1.5: *Estimation of SML: 20 stocks listed on the ISEQ 20 from 2019-2021*



Source: *Excel*

Figure 1.5 is a graphical representation of the estimated SML on the ISEQ 20 Index. The intercept is definitely not zero, however it is relatively close. Bodie, Marcus and Kane (2013) outline how the SML is valid for both individual assets and portfolios alike. The SML line depicted in **fig 1.5** does not show a positive upward slope, instead it shows a negative downward slope, suggesting the expected returns are negatively correlated and proportionally related to $\boldsymbol{\beta}$.

Table 3: *Second Pass Regression Statistic*

SUMMARY OUTPUT	
Entire Sample	
Regression Statistics	
Multiple R	0.0876
R Square	0.0077
Adjusted R Square	-0.0474
Standard Error	0.0008
Observations	20

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Gamma 0	0.0004	0.0003	1.6266	0.1212
Gamma 1	-0.0006	0.0016	-0.3733	0.7133

Source: ***Excel***

To recall the hypothesis of this test from section **3.7; H$_0$:** $\boldsymbol{\gamma_0}$=$\boldsymbol{r_f}$ & $\boldsymbol{\gamma_1}$=$\boldsymbol{E(r_m) - r_f}$**; H$_1$:** $\boldsymbol{\gamma_0}$≠ $\boldsymbol{r_f}$ **&** $\boldsymbol{\gamma_1}$≠ $\boldsymbol{E(r_m) - r_f}$**.** By performing the second pass regression as per Fama-Macbeth (1973) $\boldsymbol{\gamma_0}$ has returned a coefficient of **0.004,** an extremely close to zero value. $\boldsymbol{\gamma_1}$ however, supports the graphical representation of the our SML estimation, which does not support CAPM theory that expected returns are positively and proportionally related to $\boldsymbol{\beta}$ as depicted by Fama and French (2004) and Bodie, Marcus and Kane (2013) in **fig. 1.1.**

The **R-square** value of **0.0077** is considerably low suggesting it has more idiosyncratic risk and only a small portion of variation in the individual stocks returns can be explained by the $\boldsymbol{\beta}$. This is similar to the **Multiple R** result of **0.0876** which indicates a weak positive relationship between the expected returns and $\boldsymbol{\beta}$**.**

For this analysis to hold $\boldsymbol{\gamma_0}$ **,**the intercept needs to be equal to $\boldsymbol{r_f}$ the risk-free rate (**section 3.3**), for this analysis is **0.0023**. Logically speaking these figures do not match and show a discrepancy of **0.0019.** However the t-test conducted under 95% confidence interval proves statistically significant with the γ_0 p-value of **0.1212 > 0.05,** we must accept the first criteria of the **H_0.**

The second criteria in order to confirm CAPM holds within todays Irish market is γ_1 **,** the slope coefficient is equal to $E(r_m) - r_f,$ the market risk premium. Similar to the test on γ_0 these variable do not match and show a small discrepancy of **0.0010.** However the t-test conducted under 95% confidence interval proves statistically significant with the γ_1 p-value of **0.7133 > 0.05**, we must accept the second criteria of the **H_0**. The **H_0** for the analysis on individual securities has proven successful and consistent for CAPM to hold.

It is evident from the results a weak linear relationship between the systematic risk and return is existent within the sample data. However a linear relationship has been formed and deemed statistically significant within the confidence interval of 95%. The second implication of the model states the beta (Slope coefficient) is positive. It is clearly depicted in Table **3** and figure **1.5**, the slope determined by beta is negative. The final implication of CAPM, suggests that the market have expected returns equal to the risk-free interest rate, and the beta premium is the expected market return minus the risk-free rate. However due to the weak negative linear relationship distinguished between the systematic risk and expected return the study fails to reject **H_0** due to insufficient evidence.

4.4 Portfolio Results

A similar test on Portfolios was run again on the SML in order to test the validity of CAPM. As discussed in section **3.5**. The Stocks were ranked from smallest to largest $\boldsymbol{\beta}$ **(Table 4),** to minimize the loss of information in the risk-return test. This ranking is based on the beta value derived in the first pass regression procedure

Table 4: *ISEQ 20 individual asset Beta values ranked from smallest to largest*

Beta smallest to largest	
Asset Name	**Beta**
GLENVEAGH PROP.PLC	-12.2943%
CAIRN HOMES PLC	-8.6917%
KERRY GROUP PLC	-3.3637%
HIBERNIA REIT PLC	1.6376%
DALATA HOTEL GP.	5.7235%
TOTAL PRODUCE PLC	5.8652%
GLANBIA PLC	6.7475%
SMURFIT KAPPA GP	7.5811%
KINGSPAN GROUP PLC	11.4268%
CRH PLC ORD	12.9048%
IRISH RES. PROP.	12.9206%
FBD HOLDINGS PLC	13.0271%
FLUTTER ENTERTAIN	13.5398%
UNIPHAR PLC	16.3763%
RYANAIR HOLD. PLC	19.8903%
Greencoat renewables	21.5124%
BOI	22.2863%
ORIGAN ENTERPRISES I	24.5008%
IRISH CONT. GP.	25.3306%
AIB	27.9705%

Source: *Excel*

The 4 portfolios have been evenly created, consisting of 5 stocks each **(Table 5)**. The choice of constructing the portfolios with 5 stocks is for diversification, to eliminate the idiosyncratic risk, specific for each stock, reducing the noise and standard error within the $\boldsymbol{\beta}$ values (Markowitz, 1953; Sharpe, 1964; Lintner, 1965).

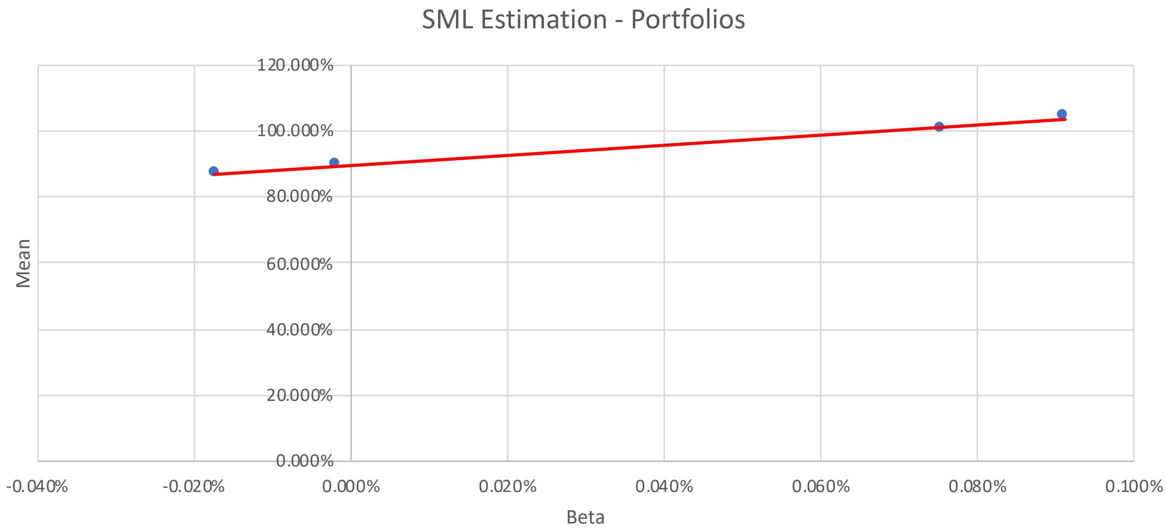
Table 5: *Portfolios Created as per there Beta Ranking*

Portfolio 1 - Lowest beta stocks			
Asset Name	**Alpha**	**Beta**	**Mean**
GLENVEAGH PROP.PLC	0.056%	-12.558%	0.047%
CAIRN HOMES PLC	0.008%	-8.693%	0.002%
KERRY GROUP PLC	0.022%	-3.367%	0.020%
HIBERNIA REIT PLC	-0.038%	1.648%	-0.036%
DALATA HOTEL GP.	-0.032%	5.789%	-0.028%
Portfolio 2 - 2nd Lowest Beta stocks			
Asset Name	**Alpha**	**Beta**	**Mean**
TOTAL PRODUCE PLC	0.104%	5.810%	0.108%
GLANBIA PLC	-0.015%	6.685%	-0.010%
SMURFIT KAPPA GP	0.092%	7.682%	0.098%
KINGSPAN GROUP PLC	0.105%	11.489%	0.113%
CRH PLC ORD	0.071%	12.927%	0.081%
Portfolio 3 - Middle Performing Beta stocks			
Asset Name	**Alpha**	**Beta**	**Mean**
IRISH RES. PROP.	-0.036%	12.880%	-0.027%
FBD HOLDINGS PLC	-0.018%	13.027%	-0.009%
FLUTTER ENTERTAIN	0.139%	13.521%	0.148%
UNIPHAR PLC	0.203%	16.400%	0.214%
RYANAIR HOLD. PLC	0.087%	20.109%	0.101%
Portfolio 4 - Highest beta stocks			
Asset Name	**Alpha**	**Beta**	**Mean**
Greencoat renewables	-0.004%	21.542%	0.011%
BOI	-0.004%	22.470%	0.012%
ORIGAN ENTERPRISES PLC	-0.095%	24.496%	-0.077%
IRISH CONT. GP.	-0.003%	25.235%	0.014%
AIB	-0.107%	28.111%	-0.087%

Source: *Excel*

Table 5 depicts the portfolios constructed from the first pass regression results. The construction of portfolios was completed by arranging all the smallest $\boldsymbol{\beta}$ values in one portfolio all the way to the highest $\boldsymbol{\beta}$ values.

Figure 1.6: *SML Estimation - Portfolios*



Source: *Excel*

Figure 1.6 is a graphical representation of the estimated SML for the portfolios created within the ISEQ 20 Index. The intercept is definitely not zero and considerably further away than the results of the individual securities. As previously stated in section **1.1,** Bodie, Marcus and Kane (2013) outline how the SML is valid for both individual assets and portfolios alike. The SML line depicted in **fig 1.6** shows a positive upward slope, suggesting the expected returns are positively correlated and proportionally related to $\boldsymbol{\beta}$**,** supporting the CAPM theory and is in fact similar to **fig. 1.1** as depicted by Bodie, Marcus and Kane (2013).

Table 6: *Second Pass Regression Statistics: Portfolio Analysis*

SUMMARY OUTPUT	
Portfolio Analysis	
Regression Statistics	
Multiple R	0.99624
R Square	0.99249
Adjusted R Square	0.98873
Standard Error	0.00006
Observations	4

	Coefficients	***Standard Error***	***t Stat***	***P-value***
Gamma 0	-0.0058	0.0004	-15.2314	0.0043
Gamma 1	0.0064	0.0004	16.2533	0.0038

Source: *Excel*

Once again recalling the hypothesis of this analysis (section **3.7**) **H₀:** $\gamma_0 = r_f$ & $\gamma_1 = E(r_m) - r_f$; **H₁:** $\gamma_0 \neq r_f$ & $\gamma_1 \neq E(r_m) - r_f$. By performing the second pass regression as per Fama-Macbeth (1973) γ_0 has returned a coefficient of **-0.0058,** a negative but again close to zero value. γ_1 supports the graphical representation of our SML estimation with a value of **0.0064**. This supports CAPM theory as outlined by Bodie, Marcus and Kane (2013) that expected returns are positively and proportionally related to β as depicted in **fig. 1.1.**

The **R-square** value of **0.99249** is considerably high, indicating the stock performance moves relatively in line with the index. This is similar to the **Multiple R** result of **0.99624** which indicates a strong positive relationship between the expected returns and β**.** The dramatic increase in correlation is due to diversification, mitigation of risk and loss of information of the portfolios as outlined in section **3.5**.

However**,** in order CAPM to be validated γ_0 **,**the intercept needs to be equal to r_f the risk-free rate (**section 3.3**), for this analysis is **-0.0058**. Mathematically these figures do not match and show a discrepancy of **0.00810.** The t-test conducted under 95% confidence interval proves statistically insignificant with the γ_0 p-value of **0.0043 < 0.05,** as such we must reject the first criteria of the **H_0.**

The final criteria in order to confirm CAPM holds within todays Irish market for well diversified portfolios γ_1 (the slope) is equal to $E(r_m) - r_f$ (the market risk premium). Similar to the test on γ_1 for the individual securities these variable do not match and show a much larger discrepancy of **0.00806.** The t-test conducted under 95% confidence interval proves statistically insignificant with the γ_1 p-value of **0.0038 < 0.05**, we must reject the second criteria of the **H_0** as well. The **H_0** for the analysis on portfolios has proven unsuccessful and not enough information for CAPM to hold.

4.5 Efficient Frontier

The efficient frontier as depicted by Markowitz (1952) set of portfolios representing the boundary of a set of feasible portfolios that have the maximum return for a given level of risk. Fama and French (2004) best describes the risk-return relationship with CAPM through the portrayal of the efficient frontier as it represents the maximum return for a given level of risk.

By plotting the Efficient Frontier, enables the analysis of investment opportunities and make more informed decisions. The Efficient Frontier helps identify and depict a combination of assets with the optimal level of expected return for any given risk level. Portfolios on the curve (**fig. 1.7**) are most efficient. Subsequent portfolios either have lower expected returns for the same risk level or higher risk levels for similar expected returns.

The construction of two optimal portfolios from our data set is based on Black (1972) finding and Benninga (2014) proposition 2 (section **3.6**) that any two envelope portfolios are enough to establish the whole envelope. The combination of two optimal portfolios discovered the whole efficient frontier. From the efficient frontier the GMVP results provides the greatest return, for the lowest possible portfolio volatility saw an average return of **0.03%** on investment. The GMVP standard deviation of **0.85%** highlights the level of volatility (risk) associated with the portfolio, which is relatively low.

Table 7: *GMVP Statistics for the combination of Xi & Yi; Portfolio M Statistics*

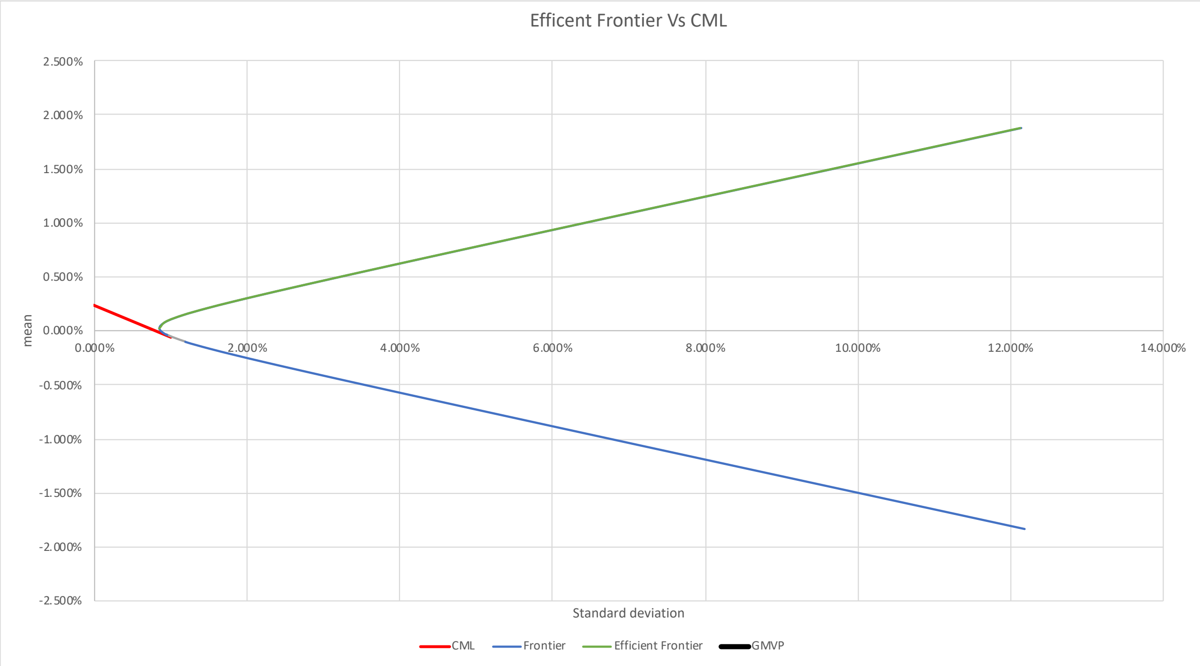
GMVP	
Mean	0.03%
Variance	0.01%
St. Deviation	0.85%

Portfolios M statistics	
Mean	-0.0547%
Variance	0.0101%
St. Deviatior	1.0033%

Source: *Excel*

The CML represents portfolios that optimally combine risk and return. Portfolio *M* (**Table 7**) as per Benninga (2014) represents the market portfolio and includes all risky assets included in any optimal portfolio. The CML plots the risk premiums of efficient portfolios as function of the portfolio standard deviation (fig.**1.7**) (Bodie, Marcus and Kane, 2013). CML draws its foundation from Capital Market theory and the CAPM (i.e. Frictionless Market; No limits on short selling; rational investors). Theoretically as one moves up the CML the risk of the portfolio increases and so does the expected return (fig. **1.2**). The CML combines the market portfolio the risk-free asset, which is the y-intercept. However, as depicted in fig. **1.7** the CML has a negative slope and as one moves away from the risk-free investment joining the frontier, just under the efficient frontier, it takes on more risk, but no reward.

Figure 1.7: *Efficient Frontier Vs CML*



Source: *Excel*

Chapter 5 – Discussion

The study finds CAPM and $\boldsymbol{\beta}$ is a good indicator of risk and expected return relationship under the Fama-Macbeths (1973) analysis of the individual securities. However, when group into diversified portfolios as suggested by Fama and French (2004); Nwani (2013); Roll (1978) it be proved to be insignificant. In diversified portfolios, CAPM and $\boldsymbol{\beta}$ showed great explanatory power for systematic risk and return under the diversified portfolios with an **R-squared** value of **0.99249.** By constructing portfolios based on their $\boldsymbol{\beta}$ value, reduces loss of information and mitigates the standard error of the regression. In comparison the regression posed on individual stocks against the market resulted in a very weak explanatory result of **0.0077.** The individual asset **R-Square** result was correct in stating a large quantity of the systematic risk defined by $\boldsymbol{\beta}$ can be diversified, mitigating systematic risk for investors. The enhanced explanatory power shown in the diversified portfolios supports Fama-Macbeth (1973); Nwani (2013) and Lee, Cheng and Chong (2016) results as not only did the R-Square value increase dramatically, but the standard error associated with the regression was mitigated by **0.00074.**

The analysis of the two linear regressions (Individual Asset and Portfolios) provided contrasting results. CAPM developed by Sharpe (1964), Lintner (1965) and Mossin (1966) outline the relationship between the systematic risk and returns to be positive and linear to be explained by one factor $\boldsymbol{\beta}$**.** The first pass regression saw some promising and similar results to Czekeirda (2007), from the t-stats produced for both the $\boldsymbol{\beta}$ **(slope)** and $\boldsymbol{\alpha}$ **(intercept),** with $\boldsymbol{\beta}$ being statistically significant explain the systematic risk and expected return relationship and $\boldsymbol{\alpha}$ **(**$\boldsymbol{\alpha}$ **=0.004)** variable being not statistically significant from zero, as the intercept of the estimated regression line should be zero in the first pass regression, for CAPM to hold (Fama and French, 2004)

The second pass regression on the individual stocks, found interesting, but contrasting results again. The criteria for CAPM is validated under the Fama-Macbeth (1973) hypothesis by testing it through the SML, supports Sharpe and Cooper (1972) study on the NYSE and Strong and Xu (1997) study on the LSE, who's result indicates past $\boldsymbol{\beta}$ values provides an explanation of expected returns to an extent. It is clearly depicted through the **R-square** value of **0.0077**, **Multiple R** of **0.0876** and $\boldsymbol{\beta}$ result of **-0.0006** is extremely low suggesting that only a small portion of variation in the individual stocks returns can be explained by the $\boldsymbol{\beta}$. However, the $\boldsymbol{\beta}$ has proven itself statistically significant in each regression test and holds CAPM theory on the explaining expected returns firmly through the

Covid-19 pandemic. The negative result of the $\boldsymbol{\beta}$ variable may be a result of the impact caused by the pandemic, suggesting the systematic risk to be negatively correlated due to the economic downturn and uncertainty within the market over this time period. The results of this relationship may have proven more consistent with the models theory by using a different time period, with less dramatic fluctuations within the market. The Irish market is relatively small in comparison to other countries, such the United Kingdom and the United states of America as such the effects of the global pandemic would impact the individual stocks more.

Prior research does not support the negative linear relationship received on the SML, as CAPM in its original format to validate should produce a positive and linear relationship between the systematic risk and expected return (Bodie, Marcus and Kane, 2013). This suggests the expected returns are negatively correlated and proportionally related to $\boldsymbol{\beta}$**.** This change of slope direction may be influenced by external factors such as, the anomaly of the Covid-19 pandemic to which the test was based. The contrasting result does support Ferreira and Monte (2015) and Hung et. al (2004) who find the traditional CAPM, cannot be rejected, in today's era, suggesting the market **β** remains significant, however they are unable to confirm if it can be accepted. It is evident from the analysis $\boldsymbol{\beta}$ is a formidable explanatory variable for the systematic risk and expected return relationship as 45% of the $\boldsymbol{\beta}$ t-statistics prove statistical significance in explaining stock returns from the first pass regression. The $\boldsymbol{\alpha}$ variable showing extremely positive results with 100% of the stocks not being statistically different from zero meeting its criteria on a 95% confidence interval level. The results of the second pass regression prove to hold for CAPM and the one factor $\boldsymbol{\beta}$ model, however the low R-Square and Multiple R results suggests another factor may be useful in providing a broader understanding of the risk-return relationship.

The performance of CAPM within well diversified portfolios as described by Fama-Macbeth (1973), saw the reverse of the individual of the results of the individual assets. The two-step regression test was in fact failed by the diversified portfolios. However, the construction of portfolio's based on their $\boldsymbol{\beta}$ ***values***, supports Fama and Macbeth (1973) Black et al. (1973) and Nwani (2013) tests as it significantly increased the correlation and explanatory power of the $\boldsymbol{\beta}$ variable towards expected return, returning strong positive results of **R-square** value of **0.99249** and **Multiple R** value of **0.99624.** By diversify and grouping the portfolios on the basis of the $\boldsymbol{\beta}$ value helped reduce loss of information, noise from the $\boldsymbol{\beta}$ variables, the idiosyncratic risk and measurements errors exposed to individual stocks. Similar to Black et. al (1973), the two-pass regression test provided a positive relationship between the expected returns of diversified portfolios and corresponding beta values.

The analysis of CAPM within diversified portfolios supports Lee, Cheng and Chong (2016) study, who applied CAPM to 60 stocks within KLSE emerging markets as per the Fama and Macbeth (1973) methodology. Their findings with diversified portfolios constructed on the ranking of $\boldsymbol{\beta}$ values minimized standard error, reducing the unsystematic risk associated with CAPM. By grouping the portfolios based on $\boldsymbol{\beta}$ rankings from smallest to largest, where able to find a linear relationship between $\boldsymbol{\beta}$ and the expected return. However, even though CAPM did not find success in the KLSE market or the Irish market, a weak positive linear relationship was defined with $\boldsymbol{\beta}$ and the expected returns. The results of this relationship may have been stronger by using a larger market index, which would consist of more stocks and essentially more portfolio to be analysed.

The efficient frontier is a set of optimal portfolios that offer the highest expected return for a given level of risk. Any portfolio that lies below the efficient frontier are considered sub-optimal as they do not provide enough return for the level of risk exposure. The analysis of the efficient frontier provided a visual representation of the CAPM story from a theoretical perspective (Fama and French, 2004). The analysis of the efficient frontier supports Black (1972) finding, that any two efficient portfolios are enough to find the entire efficient frontier. By graphing the efficient frontier aids the analysis of make informed decisions on investment opportunities.

The CML analysis did not support Bodie, Marcus and Kane (2013) theory as the slope returned a negative result. This could be down to one of two factors; **1.** The risk-free rate is greater than the portfolios return; or **2.** The portfolio's return is expected to be negative. As such this result may be similar to the SML and the timing of the study due to the economic shock Covid-19 has caused on Irish and global economies.

Chapter 6 – Conclusion and Recommendations

After performing the empirical analysis on the ISEQ 20 Index to test the performance of the CAPM as a valid indicator of the systematic risk and expected return is inconclusive and further studies in this area is required. CAPM has been supported by the data set to an extent, through the analysis of individual securities. Following Fama and Macbeth (1973) methodology, this study tested the validity of CAPM and the strength of the explanatory variable $\boldsymbol{\beta}$. The test undertaken to validate CAPM within extreme volatility is the SML test as depicted by Fama and Macbeth (1973), in order to explain the variation in daily stock returns listed on all 20 stocks included in the ISEQ 20 market over a sample period of 23rd of July 2019 to 25th of June 2021. The results from the first pass and second-pass regression analysis show $\boldsymbol{\beta}$ to be a significant explanatory variable in CAPM. $\boldsymbol{\beta}$ has proven to be statistically significant in all but one area, from the criteria defined by both Fama and Macbeth (1973) and Fama and French (2004). CAPM showed significant evidence to show $\boldsymbol{\beta}$ explains the variations in stock returns over the chosen period, when analysed individually.

$\boldsymbol{\beta}$ proved itself significant in passing the majority of tests at a 95% significance level. However, the results of the linearity test does not produce a positive linear relationship as the model theoretically proposes. This study concludes the results of this study to be inconclusive due to the weak negative relationship proposed by the $\boldsymbol{\beta}$ value. CAPM as a whole cannot be supported completely. Consistency within CAPM's results was not clearly explained in this study. For the regression tests on individual assets proved to hold, it did not however provide a positive linear relationship as Fama and French (2004) outlined. The opposite occurred within the portfolio regression test, who held a weak positive linear relationship, but failed to pass the criteria standards held for the SML test on CAPM.

One must bear in mind that the version of the CAPM tested in this model has quite strong assumptions of a hypothetical perfect world. Perhaps modifying some of these assumptions would present more accurate and consistent results. This analysis may be used for future studies exploring additional risk factors, which may impact stocks expected returns through both the investment of individual asset and diversified portfolios. The limited number of stocks may have also affected the result for both individual assets and portfolios. The inclusion of a larger date range and a larger market with more stocks may have proven a more consistent result to validate CAPM within the Irish market.

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