# Long -Term Value at Risk:

**Modeling and testing VaR on a Defined-benefit Pension Scheme**

-By

**Bansi Purshottam Mavani**

**18196985**

A dissertation submitted in partial fulfilment of the requirement for the degree of

**Master of Science in Finance**

at

**National College of Ireland**

Submitted to National College of Ireland,

August 2020

# ABSTRACT

Long-Term Value at Risk: Modeling and testing VaR model on a defined-benefit pension scheme

-Bansi Mavani

Value at Risk is identified amongst the significant developments in the field of risk measurement and management. Value at Risk (VaR) can be regarded as a metric that enables us to calculate and quantify the degree of risk of financial nature associated with a given investment or a portfolio of investments over a specific period.

The objective of this study is to compute long horizon Value at Risk on a defined-benefit pension scheme. Such pension schemes are regarded as defined in the context of the benefit formula which is well in advance defined as well as known. To accomplish the stated objective the author undertakes the building of a model for computing VaR with Monte Carlo simulation using a non-parametric bootstrapping method. The method employed for computation is not a very common one and therefore the author aims to explore its validity. An important aspect of any constructed model is check for its validity and accuracy because unless the accuracy of the model is proved the computation cannot be relied on. Backtesting of a model is performed as a test for its accuracy. To achieve the aim of building and backtesting the VaR model use of Microsoft excel along with virtual basic for application (VBA) and R programming is made. This study addresses more of an industrial problem and thereby incorporates the floors and options to various risk factors involved in the computation. The time horizon and the confidence level are the two important parameters in VaR computation. The time horizon considered for VaR computation is long-term and at 99% confidence level whereas the backtesting is performed with 95%, 97.5%, and 99% VaR. The results of the study conclude the constructed VaR model as valid and accurate for practical implementation based on the backtesting results.


**Keywords:** Value at Risk, Long-term horizon, Defined-benefit Pension Scheme, Monte Carlo Simulation, non-parametric bootstrapping, Backtesting.

## DECLARATION

# Submission of Thesis and Dissertation

**National College of Ireland**
**Research Students Declaration Form**
***(Thesis/Author Declaration Form)***

**Name: Bansi Purshottam Mavani**

**Student Number: 18196985**

**Degree for which thesis is submitted: Master of Science in Finance**

**Title of Thesis: Long -Term Value at Risk: Modeling and testing VaR on a Defined-benefit Pension Scheme**

**Date: 19th August 2020**

## Material submitted for award

A. I declare that this work submitted has been composed by myself.
☑

B. I declare that all verbatim extracts contained in the thesis have been distinguished by quotation marks and the sources of information specifically acknowledged.
☑

C. I agree to my thesis being deposited in the NCI Library online open access repository NORMA.
☑

D. ***Either*** *I declare that no material contained in the thesis has been used in any other submission for an academic award.

***Or*** *I declare that the following material contained in the thesis formed part of a submission for the award of **Master of Science in Finance**
☑

*(State the award and the awarding body and list the material below)*

**Signature: Bansi Purshottam Mavani** **Date: 19th August 2020**

ACKNOWLEDGMENT

I wish to acknowledge all those who contributed towards my journey of successfully undertaking and completing this study. Throughout the process, I have been provided with a lot of guidance, inspiration, and encouragement and I am deeply grateful for the same.

First of all, I would like to thank the supervisor of the Thesis, Professor Joe Naughton, for his immeasurable support throughout my research. His advice, critique, and encouragement made the completion of this study far easier. His contribution added value to my knowledge and helped me develop academically.

A special thanks to the risk manager from an Irish corporation who remains unnamed for his valuable time and insights on the industry.

I also appreciate the support I received from my father and the continuous motivation from my mother and brother. Last, I wish to express gratitude to a friend, Afnan, who has been during the emotional turbulence of this long academic journey.

# TABLE OF CONTENTS

# LIST OF TABLES



# LIST OF FIGURES

## LIST OF APPENDICES

# 1 INTRODUCTION

## 1.1 INTRODUCTION TO VALUE AT RISK (VAR)

What is the maximum loss that can be incurred on a specific investment? Any individual or an institution that has undertaken an investment or is considering an investment has had the above question in mind when the said investment is into a risky asset at any given point of time (Damodaran, 2007). Precision modeling of risk is crucial for correct prediction and control of risk (Alexander, 2008).

VaR has become widely popular as the risk measure in finance over the last decade. Value at Risk is regarded as a method used for quantifying the risk of financial nature and it does this by attempting to forecast future risk (Micocci, Gregoriou and Masala, 2010). Glyn (2002) said that VaR can be termed as market risk's probabilistic measure. In today's time, this method is gaining popularity and is also being adopted by non-financial institutions and regulators and so its use is not limited only to financial institutions. Portfolio Value at Risk can be defined as the maximum portfolio loss which will be suffered at a certain confidence level over a specified time horizon (Dowd, Blake and Cairns, 2004). In simple words, it is the maximum loss that will be incurred on a portfolio with a certain confidence level, say 95% or 99%, over a given time horizon that could 1-day, 1-month, or 1-year. The time horizon and the confidence level have a direct impact on the VaR value.

Major areas in finance deal with risk over a relatively shorter time horizon and its applicability over a long-term horizon to say, insurance and pension are quite limited (Dowd *et al.*, 2004). Over the past few years application of VaR has grown in the context of actuarial background and various techniques using different VaR models have been applied to evaluate long-term VaR in the insurance sector. On the other hand, still, the attempts to VaR's applicability to pension schemes are comparatively less and are still being explored. This study works around the concept of a defined-benefit pension scheme which refers to a pension scheme where the amount received on retirement is based on the number of years of service and the remuneration earned by an individual.

There are 3 methods of calculating VaR which include: first, Parametric method of VaR called Variance Covariance (Jorion, 2007), second being non-parametric method commonly regarded as historical simulation (Sharma, 2012), and the last is the semi-parametric method of VaR, namely Monte Carlo simulation (Hull, 2006). The study states that the semi-parametric model of VaR, which is monte Carlo simulation, provides the best results in comparison to the variance-covariance which enables modeling the greater risk and on the other end historical simulation enables modeling the smaller risk (Danielsson & Vries, 2000).

Despite its popularity, VaR has received criticism as a risk measure by various authors and has therefore been quite controversial. Past literature goes to an extent of naming Value at Risk as a flawed risk model where the author says VaR is similar to an airbag which is functional at all time except in a situation of a car accident (Einhorn and Brown, 2008). This reflects correctly that not only is constructing a VaR model for estimating future risk crucial but alongside checking for the VaR model's validity is also or rather even more important. The process of backtesting enables checking for VaR models' accuracy and validity.

## 1.2 Research Objective

The motivation behind this research is to model and backtest the long horizon VaR associated with a defined-benefit pension scheme. The defined benefit pension plan is a kind of pension scheme wherein a promise for designated retirement payout is made by the employer, the formulae for computation of the amount is predetermined, the payment may be a lump-sum or variation of that amount, and the same is based on the history of earnings, the period of service and age of the employee instead of simply being calculated on the individual investment return. The author builds a VaR model to estimate the change in the value of a pension scheme.

A quite important aspect governing the study is that though this study has academic interest more specifically it is research from industry and aims at solving an industrial problem. The author justifies the relevance and worth of the research by addressing a practical problem.

There is no single study that comes close to this research in every aspect considering the research question, the VaR model in the study, and lastly, the methodology and underlying assumptions for model building and backtesting.

This paper identifies the gap and extends the previous research in three aspects:

1. First, application of Monte Carlo Simulation using non-parametric bootstrapping to long-horizon VaR model, unlike past research where other VaR models have been built to avoid the complexity involved in Monte Carlo Simulation with non-parametric bootstrapping (Mancini and Trojani, 2011; Jorion, 2007).
2. Secondly, the Application of Monte Carlo Simulation using non-parametric bootstrapping replacing the standard practice of using the GARCH model when computing long horizon VaR (Berrar, 2019; Wong and So, 2003; Mancini and Trojani, 2011).
3. Thirdly, application of the above-stated methodology to defined-benefit pension scheme differing from the previous studying where similar methods have been applied in the context of a defined-contribution pension scheme or Insurance to compute long horizon VaR (Abbasi and Guillen, 2013; Basu and Drew, 2007).

## 1.3 Outline of the Study

This structure of this thesis is in the following manner:

Chapter 1 is the **Introduction** to the study.

Chapter 2 is the **Literature Review** which briefly states the research carried out by previous authors and the findings from their work are compared and critically analyzed.

Chapter 3 states the **Research question** which forms the basis of the study as well as guides the research.

Chapter 4 concentrates on the **Methodology** which states the logical methods and procedures that the author has adopted to achieve the research objective. This section of the study will cover in-depth the process of building and testing the Value at Risk model.

Chapter 5 focuses on the **Findings/Results** by stating and analyzing the outcomes obtained from the building and testing of the model in the preceding chapter.

Chapter 6 is the **Discussion** where the author aims at contrasting the findings of the study in light of previous research and also talks about the practical implications and future scope.

Chapter 7 is the **Conclusion** of the study where the researcher concludes the research while suggesting the avenues for further research.

# 2 LITERATURE REVIEW

## 2.1 INTRODUCTION

This section of the study focuses on the literature around the subject area of study and begins by providing a synopsis and background of VaR. Followed by it is the definition of VaR. The author then presents the relevance of the forecast horizon and the confidence level, followed by the pension fund. Next, the VaR model – Monte Carlo Simulation along with the model building and shortcoming of it is stated. Lastly, Backtesting for validating the model along with different techniques applied are discussed.

Value of Risk has gained widespread popularity as a risk measure of financial risk and it has been adopted by sectors like baking and insurance for the purpose relating to regulatory capital requirement (Micocci *et al.*, 2010).

During the initial stages Value at Risk was believed to have been constructed based on corresponding two grounds. One being the Portfolio theory which was part of studies conducted by authors like Markowitz (1952) and Roy (1952). The second study was conducted by Holton (2002) which was about capital adequacy computation. Later JP Morgan was the institute due to which VaR became widely used and recognized amongst the financial institutes and corporates along with risk metrics service (Holton, 2002). VaR has gained popularity because of the fact that it easy to compute and understand (Hull, 2006).

VaR of a portfolio is the cumulative loss at a certain degree of confidence that the portfolio will suffer over a period (Alexander and Baptista, 2008). VaR offers forward-looking risk indicators, using a blend of current status and uncertainty forecast (Jorion, 2007). The literature emphasizes the use of VaR due to its ability to identify a tolerable amount of uncertainty and enables one to reduce the losses that might occur as the result (Micocci *et al.*, 2010, Jorion, 2003; Jorion, 2001). Across the financial industry though VaR is very popular yet this method isn't a standard tool in the case of pension funds (Micocci *et al.*, 2010).

## 2.2 VaR - Definition

Over a specified time horizon, Value at Risk represents the quantile of the estimated distribution of profits and losses (Jorion, 2007). According to Linsmeier and Pearson (1996), Value at Risk is the manner in which the extent of possible losses on a given portfolio can be defined. Value at Risk is a statistical measure of the potential losses on the portfolio resulting from normal market fluctuations (Campbell, 2005).

A simple mathematical representation of computing VaR over a long horizon is based on a basic assumption:

$$\text{N – day VaR} = \text{1-day VaR} * \sqrt{N}$$

The formula is suitable when the change in portfolio value on the subsequent days has the mean of zero and are independent identical normal distributed whereas in cases other than this it is an approximation (Hull, 2015).

## 2.3 Forecast horizon and Confidence Level

The two most important parameter to be considered in VaR computation are: firstly, the significance level which is denoted by α or the confidence level which is simply '1 - α' and second is the risk horizon and that is the period of time over which the Value at Risk is computed and it is generally denoted by '*h*' (Alexander, 2008). The duration over which the potential loss is estimated is regarded as a risk horizon. Albrecht, Bährle and König (1996, p. 12) in their study state that there exists a degressive dependency of Value at Risk on time horizon length. VaR is popular as a risk measure over a shorter duration because of its easy computation but it is not so very simple when it is applied over a longer horizon like in the case of the pension fund, as there may be a need for modifications to the manner of computation. (Micoccci *et al.*, 2010).

The level of confidence or the significance level of Value at Risk relies on the users' approach towards risk which means that if the user is more cautious then the α value will be lower and the level of confidence applied will be higher (Alexander, 2008). VaR shows great dependency on the confidence level meaning that if VaR has a low confidence level then VaR has chances of reaching the peak rapidly and also dropping is the same manner whereas if the level of confidence is high then though the VaR will reach the peak but comparatively at a slower pace and further it has a trend to remain at the highest level of value for quite a long time (Dowd *et al.*, 2004). The suitability of confidence level at 99% or more is said to be accurate when used in accordance to the relevant time horizon and the previous study provide for it to be apt in assessing risk for a long time horizon, and also literature emphasizes on significant changes in VaR values resulting from varied confidence levels applied (Beder, 1995).

## 2.4 Pension Fund and Risk

Over the years pension fund has gained value across the globe. The pension fund acknowledges like many financial institutions, the value of assessing, monitoring, and mitigating their financial risks (Jorion, 2007). Pension offers consistent guaranteed income or a payout for workers following the retirement age (Biadoo Jnr, Andoh and Bokpin, 2019). Past studies confirm the outcome that the timeframe has a direct impact on pension fund asset allocation and in the long term, pension fund holding performs well (Biadoo Jnr *et al.*, 2019; Chen, Sun and Li, 2017).

A defined benefit pension scheme is where the retirement pay is a function of income, namely labor income, in the last years before retirement and in this case the sponsor of the scheme, generally, the government or the corporation, undertakes the risk pertaining to investment and longevity (Berstein and Chumacero, 2010). In economies like the UK, Australia, and North America popular sources of retirement income are through defined benefit pension scheme but recently a gradual switch has also been made towards defined contribution Scheme (Haberman, 1996, Turner and Beller, 1992). With changing time and growing popularity VaR is being applied in the context of pension schemes and the use of VaR has also been encouraged in other sectors like in banks by Basel II and similarly, it has also been adopted in relation to insurance (Solvency II – New European prudential system) (Micocci *et al.*, 2010).

The author highlights that in the context of pension fund the associated liabilities are of long duration and accounting to the same cause, a mismatch between the liabilities and assets is the biggest systematic risk faced in a pension fund (Blake, 1999). Risk models should be reflective of the risk interaction and the same is crucial keeping in mind the various risk factors impact the valuation (Micocci *et al.*, 2010).

Mexico is where the application of VaR to pension fund supervisory framework was first introduced and the same was used for the purpose volatility risk approximation. Such an application received criticism because it lacked the necessary modification for applicability to the pension scheme rather it was a technique simply picked from the banking sector and applied in the same manner (Antolin *et al.* 2009).

The importance of risk management and a risk-adjusted basis for measuring performance has grown because of well know losses in pension funds. There is a variety of risk that financial institutions are faced with but one particular type of risk which is very crucial is the market risk. Market risk refers to the risk of fall in the value of investment accounting to economics changes or due to other changes or events that affect the market factors like interest rates, exchange rate, and stock prices (Micocci *et al.*, 2010). Investment risk from the pension fund emerges from three main causes: the risk that the portfolio may decline in worth, the possibility that perhaps the pension fund's earnings will not go ahead with inflation (negative returns), and the danger that show how the pension fund might not raise enough to cover the expense of providing retirement benefits (The Pension Board, 2018). The literature of researchers pertaining to long-term risk measurement is quite limited. The calculation of VaR is regarded as complex and this is mainly due to the long term volatility forecast element involved in long-horizon computations whereas the short-term horizon calculation is comparatively easy (Dowd *et al.*, 2004). Over the years, risk tolerance is bound to be altered or vary and pension funds should take this into account when making portfolio decisions (Berstein and Chumacero, 2010).

## 2.5 Model building: Monte Carlo Simulation using non-parametric bootstrapping

Monte Carlo Simulation model of Value at Risk is a kind of simulation that generates results by depends on statistical analysis and repeated random sampling. This approach is deemed related to the random experiments which are kind of experiments where it is not possible to know the results in advance (Raychaudhuri, 2008). The method of Monte Carlo simulation depends on a predefined distribution like for example, from a normal distribution the realizations can be pulled, numerically: $dSIS \approx (\mathrm{N}, \sigma^2)$ (Jorion, 2007). Dowd (1998) said that Monte Carlo Simulation is an apt approach when there arises a need for a powerful yet sophisticated VaR model, but when it comes to implementation so far, the most challenging one.

Monte Carlo makes no underlying assumptions regarding the collective distribution of the given data and therefore it is suitable for a portfolio with irregularities (Estrella *et al.*, 1994). Studies have regarded Monte Carlo Simulation as a model that showcases superior performance in comparison to analytical models when taking into consideration a longer holding period with a high level of confidence (Gnamassou, 2010; Jorion, 2007; Reich, 2001; Coronado,2000). Monte Carlo Simulation holds the capacity of including nonlinear instruments like options (Damodaran, 2007).

Drawbacks recognized in using Monte Carlo simulation is that the speed of computation is slow because the portfolios involved are revalued several times (Hull, 2006). Though accuracy is one of the advantages of this method, but this method also has higher time requirements for computation (Jorion, 2007). Authors like Srinivas and shah (2001) and Antonelli and lovino (2002) have suggested improvisation in the VaR computation through Monte Carlo simulation to overcome its disadvantage of higher computational cost and to improve efficiency.

In the Simulation model not always is it possible to get underlying distribution for a variable and one cause for this is lack of sufficient data. In scenarios where for the input parameters not much but only a few historical values are available then the use of bootstrapped Monte Carlo Simulation is made to generate random variates. In such cases, sampling by replacement is opted for where the use of no new random variates is actually generated but instead repeated sampling is done from the originally available set of data and this is, in turn, is termed as generated random variants. In cases when the parametric distribution of data set is unavailable or absent then bootstrapping simulation proves to a highly effective tool (Raychaudhuri, 2008).

Non-Parametric bootstrapping was first proposed as a non-parametric randomization technique that pulls out data from observed distribution for modeling the distribution of a statistic of interest (Efron, 1979). “The bootstrap provides striking verification for the infinite capabilities of modern statistical computation” (Efron, 2012, pp.1293).

A general and simple formula for a non – parametric model is as given below:

$$X_t = m(X_{t-1},\ldots,X_{t-p}) + \sigma(X_{t-1},\ldots,X_{t-p})\varepsilon_t, \quad t = 1,\ldots,n$$

In the above formula, the unknown smooth functions are *m (-)* and *σ (-)* and $\varepsilon_t$ are the independently and identically distribution random variable.

The non-parametric method enables the estimation of the risk factor's distribution and in order to do that, a simple technique is bootstrapping. Based on known historical data the method of bootstrap generates possible values of the various factors (Valášková, Spuchľáková, and Adamko, 2015).

In the bootstrap method, *F* is the unknown distribution function which is replaced by

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I_{(-\infty,x]}(X_i)$$

Another simpler way to understand this is: In Monte Carlo Simulation there is an alternative available of sampling from the historical data with replacement instead of producing random numbers from a hypothetical distribution. Let us consider an instance if we assume to observe a series of $M$ returns $R = \Delta S/S$, $\{R\} = (R_1 \ldots R_M)$, which is nothing but series of let's say, M returns R where the '$(R_{1\ldots\ldots}R_M)$' are assumed to be independently and identically distributed random variable drawn from an unknown distribution. An important element of bootstrapping considered in the study is sampling with replacement (Jorion, 2007). When using Monte Carlo Simulation for computing VaR it is feasible to scale up the short horizon VaR estimated in order to compute a long-term VaR while making a certain necessary assumption (Alexander, 2008).

Past literature highlights the benefits and convenience that bootstrap offers: firstly the method is regarded as quite a user friendly, secondly it takes into account the considerations for correlation across series, thirdly it can overcome the drawback faced in the classical method which makes assumptions for data to be normally distributed whereas bootstrap facilitates fat tails or any deviations from a normal distribution and lastly, it enables faster integration of normal distribution and is suitable when applied to VaR models (Valášková, Spuchľáková and Adamko, 2015; Kollar and Kliestik, 2014; Jorion, 2007). On contrary there are also limitations to the method of bootstrap that have been noted which are, first, poor approximation in contrast to the real one of the bootstrapped distribution when the given sample size is small and second is that the bootstrap method has a high dependence on the assumption stating that the returns are independent (Jorion, 2007).

## 2.6 BACKTESTING

VaR models are indeed effective only when they can forecast the potential future risk explicitly. It is vital to deploy a suitable statistical method for backtesting to ensure that the conclusions derived based on VaR computation are both valid as well as consistent (Nieppola, 2009). The process of backtesting involves a comparison between the actual profits and losses versus the projected VaR simulations. The author rightly terms the process of backtesting as a reality check process (Jorion, 2001).

Brown (2008, pp. 20) said "When someone shows me a VaR number, I don't ask how it is computed, I ask to see the back test."

Backtesting refers to a method of identifying the accuracy of the results obtained through various approaches and backtesting also recognizes any biases involved in computations (Jorion, 2007). There is no one standard approach of Value at Risk that is suitable in all situations. No matter which method has been adopted in the calculation of VaR, the most important check for the same is backtesting. The real profit and loss can be compared to forecasted VaR estimates through backtesting (Halilbegovic and Vehabovic, 2016). The process of backtesting involves taking past performance into consideration in order to identify how accurate was the VaR estimate (Hull, 2006). When backtesting is for the longer horizon with high confidence level the power or the efficiency of the test might reduce because the number of independent observations as well as the exceedances are fewer (Jorion 2007).

Aspects of the model like inapt assumptions, the parameter being incorrect or flawed simulations or models, all of these should be re-examined in cases where the VaR computation is inaccurate (Nieppola, 2009). In scenarios where market risk is involved, backtesting of the model becomes comparatively more complex because the data or the history of data available might be short and when the data available is less the longer horizons computations become difficult (Jorion, 2007).

### 2.6.1 Z-Test

One of the easiest and straight forward methods of backtesting the VaR model is recording the instances of failure or the failure rate. In order to determine the validity and accuracy of the model, the use of binomial distribution can be made. In the case of such test, the user needs to state the null and alternative hypothesis, and based on the stated hypothesis the failing or passing of a given model can be ascertained. These said models are tested at a certain desired confidence level and time horizon. The number of expected exceedances is based on the level of confidence (Alexander 2008; Jorion, 2007; Hull, 2006). The author emphasizes on the probability of 2 types of error in such computation. The first error is where an accurate model is rejected and this type of error is generally known as type I error whereas in the type II error there is a probability of failing to reject an inaccurate model. There is a need to correctly balance the two aforesaid errors against each other (Jorion, 2007).

### 2.6.2 Kupiec Test

Kupiec test is not only one of the popular test but also the widely used technique of backtesting. This test is also known as the proportion of failure or more commonly as POF-test. Kupiec test was first proposed by in the year 1995 by Kupiec (Kupiec, 1995). Campbell (2005) states that acceptance or rejection of the backtest model is based on the computation of the ‘x’ which is the number of exceedance and similar Dowd (2005) also states that the model is accepted only if the number of exceedance is within the stated range or else it is rejected. Accepting a given model is easier when the confidence level is not very high as then the number of exceedances will be more (Alexander, 2008; Jorion 2007).

Though this model is largely used and is popular yet two main drawbacks have been identified. Firstly, when the size of the sample is small the efficiency of the test reduces as it becomes statistically weak and secondly, this test fails to take into consideration the number of times positive results or succession have been observed rather its focal point remains on analyzing the failure rate (Katsenga, 2013; Jorion, 2007).

# 3 RESEARCH QUESTION

**What is the feasibility and validity of the Value at Risk model that is built and backtested with high confidence interval and a long holding period?**

Over the years various modifications to the existing methods have been developed to overcome the shortcoming but even then, the gap remains where research needs to be directed regarding the method of building and backtesting long horizon VaR. Particularly pension fund has opted because the same reflects long term liabilities and past literature (Dowd *et al*., 2003) prove its suitability and along with this, the past researches also state that traditionally, VaR was a tool used for short-horizon risk but again there are situations where risk needs to be measured in a longer horizon through the adaptability of VaR (Giannopoulos, 2002).

The study aims at building and backtesting the VaR model for portfolios with a long holding period (1-year)as well as a high confidence level. The author intends to build a VaR model that will simulate potential changes in the value of a defined-benefit pension scheme of a financial institution for the purposes of capital attribution in the financial institution. The model built will be a Monte Carlo Simulation VaR model which will make use of non-parametric bootstrapping for computation. Bootstrapping forms an important part because the data available for computational purposes is quite limited and to overcome this, sampling with replacement will prove effective. The author considers four market risk factors which are Discount yield, Equity prices, Bund yields, and inflation rate. For the purpose of modeling, the author assigns hypothetical sensitivities to the various market risk factors. The Author computes 1-year VaR at a 99% confidence level. To test the accuracy of the built model, backtesting of the VaR will be done and the results of the same will determine the validity of the model. The backtesting techniques adopted will be a Z-test and Kupiec test. A truncated VaR at 95%, 97.5%, and 99% will be computed for the purpose of backtesting.

As the author is dealing with an industrial problem and to showcase the practical implications, effective floors on discount yields and put options impacting the equity prices are incorporated. Model and backtesting will compare the results with and without such inclusions. The models are built and backtested in Microsoft Excel and RStudio.

# 4 METHODOLOGY

## 4.1 INTRODUCTION

The outlook and methodology that is adopted for the research is of a quantitative nature. The objective behind choosing a quantitative research method is due to the nature of data, the data that is used to fulfil the aim of the research is secondary data. Literature states that quantitative research is the type that focuses on procurement and analysis of data and it uses deductive logic (Rehman,2016; Bryman, 2012; Payne and Payne, 2004). The use of a quantitative research method will facilitate the computation of long-term horizon VaR. Collecting statistical and numerical data is the focal point of any quantitative analysis and in order to analyze and interpret data, the researchers make use of qualitative tools as well as statistical techniques (watkings and Gioia, 2015).

This section of the study focuses on providing a thorough description of the steps involved in the process of building a VaR model along with computation of the VaR and lastly, testing the build VaR model for accuracy.

## 4.2 PENSION SCHEME

A defined benefit pension scheme is different from a regular or a defined contribution pension scheme in terms that the regular pension scheme has no pre-defined liability and the liability matches the asset. For instance, if an individual saves in a regular pension scheme and manages to save a million euro by the time he retires then after his retirement he receives the million euros, in simple words, the amount that has been saved is exactly what is received at the time of retirement. On the other hand, in the case of defined benefit pension scheme the pensioner knows that, let's say, he will get 50% of his salary for the rest of his life which states that irrespective of the assets of the scheme the liabilities of the scheme are defined based on the final salary of the pensioner and how long he lives.

The pension scheme referred to in the study is a defined benefit pension scheme and these pension schemes are final salary pension schemes, for instance, an individual works for the company and at their retirement, they get 50% of their final salary for their life or they get 2/3rd of their final salary for rest of their life so the company has liability and the value of those liabilities and the duration of those liabilities may vary depending on the nature of the scheme.

These schemes have notional liabilities that means they cannot physically be traded in the market but they are cashflows owed by the company of certain value and predefined duration behaving to all purposes like a zero-coupon bond with ascertained maturity in order to meet those liabilities, the company or the pension fund manager will have purchased several assets and in this study, those assets can are a mixture of bonds and equities and so to put it in an equation form it can be said that the net value of the scheme is the surplus or the deficit of the assets minus the liabilities.

A company's defined pension scheme is a function of market rates meaning that as the market rates change the value of the pension scheme changes. The pension scheme comprises of assets and liabilities. Both the assets and liabilities vary with the market rates. In the case of this study, the Pension scheme is equal to the assets minus liabilities. The study is based on the belief that the assets are owned by the scheme and it comprises of equities and bonds (Bunds in this case) whereas Liabilities are the benefits owed by the scheme and they are modeled as a set of cashflows which is simply the money that the scheme is required to payout. Therefore, the value of the liabilities is equal to:

$$= \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \dots\dots\dots\dots\dots\dots\dots\dots + \frac{CF_n}{(1+r)^n}$$

As seen above, $CF_1$ is the first cashflow divided by 1 + interest rate ^ 1, then $CF_2$ is the second cashflow divided by 1 + interest rate ^ 2, and so on until the last cashflow on the year n. The liabilities are modeled as a Zero-Coupon cashflow in the 'n' years' time and n in this study is 12.5 years. The 'r' which is the discount rate to be used should be the yield on a suitable corporate bond. The present value of the liabilities is equal to the amount of the liabilities discounted at a reasonable interest rate and the reasonable interest rate that is used in industry is the Yield on a corporate Bond. In this study, the researcher approximates such a discount rate using a swap rate and a credit default swap index (CDSI). The study makes use of the history of the 'ITRX EUR CDSI GEN 5Y Corp' for the credit default swap index and '20y swap rate' as the swap rates. The ITRX Europe CDS Index represents the 125 European investment-grade corporate issuers therefore it is approximately equal to the average credit spread in this index. The author's approach will be to model the discount yield for the pension liabilities as equal to the swap rate plus the ITRX. The pension liability discount yield will be the main risk factor meaning that changes in this particular risk factor will have a greater effect than any of the other risk factors.

The value of the Pension Scheme is equal to the Net Present Value of the Assets less the Net Present Value of the Liabilities. For the purpose of the study, the initial value of the Pension Scheme is assumed to be just in surplus which means that the value of its assets exactly matches the value of its liabilities. The author makes the assumption that the pension scheme is fully funded from an IAS 19 valuation perspective in so far as the value of the assets is exactly matched by the value of the liabilities.

## 4.3 Data

The data used for the purpose of this research is procured from the pension scheme of a large Irish Corporation. The data acquired is for the period of 12.5 years from January 2007 to June 2020. The risk factors considered along with the sensitivities chosen of the hypothetical pension portfolio in this study are:

1. The major risk factor that is **Discount Yield** reflected as the ITRX EUR CDSI GEN 5Y Corp and 20y swap rate and the hypothetical sensitivity assigned to Discount yield is **€ 2,000,000**.
2. The **Equity Prices** reflected in the Morgan Stanley World Index (MSCI) as being a proxy and the hypothetical sensitivity assigned to Equity Prices are **€ 250,000,000**.
3. The Government **Bond Yield** reflected as Bund Yield that is the German Government Bond Yields and the hypothetical sensitivity assigned to Bund Yield is (**- € 1,500,000)**.
4. The **Inflation Rate** reflected as 'FWISEU55 Index' and the hypothetical sensitivity assigned to the Inflation Rate is (**- € 600,000)**.

The sensitivities reflect the amount of change in the value of the pension scheme as a result of changes in market risk factors. The sensitivities selected by the researcher for the hypothetical pension portfolio are deemed to be reasonable and proportionate as these have been decided upon after an informal discussion with a risk manager from a large Irish corporation who is responsible for managing pension risk. The discussion involved a series of open-ended questions designed to understand the practices in the industry with regard to modeling risk for capital attribution purposes.

| | 20y Euro Swap | | MSCI world Index - Total Return EUR | | EUROSTOXX 50 | | FTSE All World Index | | | | EURUSD | | Bund Yield | | iTraxx 5y | | | iTraxx 10y | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | Date | Last Price | Date | Last Price | Date | Last Price | Date | Last Price | Volume | SMAVG (15) | Date | Last Price | Date | Last Price | Date | ITRX EUR CDSI GEN 5Y Corp | FWISEU5 5 Index | Date | ITRX EUR CDSI GEN 10Y Corp | FWISEU5 5 Index |
| 4 | 7/3/2020 | 0.097 | 7/3/2020 | 299.916 | 7/3/2020 | 3294.38 | 7/3/2020 | 351.13 | 2.2E+11 | 1.5E+12 | 7/3/2020 | 1.1248 | 7/3/2020 | -0.432 | 6/30/2020 | 66.69 | 1.1208 | 6/30/2020 | 106.6 | 1.1208 |
| 5 | 6/30/2020 | 0.0751 | 6/30/2020 | 296.829 | 6/30/2020 | 3234.07 | 6/30/2020 | 346.11 | 1.8E+12 | 1.6E+12 | 6/30/2020 | 1.1234 | 6/30/2020 | -0.454 | 5/29/2020 | 72.26 | 0.9765 | 5/29/2020 | 107.21 | 0.9765 |
| 6 | 5/29/2020 | 0.086 | 5/29/2020 | 291.987 | 5/29/2020 | 3050.2 | 5/29/2020 | 336.1 | 1.4E+12 | 1.5E+12 | 5/29/2020 | 1.1101 | 5/29/2020 | -0.447 | 4/30/2020 | 80.76 | 0.911 | 4/30/2020 | 106.22 | 0.911 |
| 7 | 4/30/2020 | 0.036 | 4/30/2020 | 282.868 | 4/30/2020 | 2927.93 | 4/30/2020 | 322.48 | 2.4E+12 | 1.4E+12 | 4/30/2020 | 1.0955 | 4/30/2020 | -0.586 | 3/31/2020 | 96.62 | 0.9565 | 3/31/2020 | 128.06 | 0.9565 |
| 8 | 3/31/2020 | 0.228 | 3/31/2020 | 254.556 | 3/31/2020 | 2786.9 | 3/31/2020 | 291.84 | 3.3E+12 | 1.3E+12 | 3/31/2020 | 1.1031 | 3/31/2020 | -0.471 | 2/28/2020 | 64.375 | 1.119 | 2/28/2020 | 106.35 | 1.119 |
| 9 | 2/28/2020 | 0.1196 | 2/28/2020 | 293.063 | 2/28/2020 | 3329.49 | 2/28/2020 | 338.41 | 1.8E+12 | 1.1E+12 | 2/28/2020 | 1.1026 | 2/28/2020 | -0.607 | 1/31/2020 | 46.365 | 1.258 | 1/31/2020 | 90.705 | 1.258 |
| 10 | 1/31/2020 | 0.3045 | 1/31/2020 | 317.305 | 1/31/2020 | 3640.91 | 1/31/2020 | 368.72 | 1.7E+12 | 1.1E+12 | 1/31/2020 | 1.1093 | 1/31/2020 | -0.434 | 12/31/2019 | 44.255 | 1.3262 | 12/31/2019 | 84.86 | 1.3262 |
| 11 | 12/31/2019 | 0.6042 | 12/31/2019 | 315.181 | 12/31/2019 | 3745.15 | 12/31/2019 | 373.24 | 1.2E+12 | 1E+12 | 12/31/2019 | 1.1213 | 12/31/2019 | -0.185 | 11/29/2019 | 47.745 | 1.208 | 11/29/2019 | 92.24 | 1.208 |
| 12 | 11/29/2019 | 0.4232 | 11/29/2019 | 311.536 | 11/29/2019 | 3703.58 | 11/29/2019 | 360.82 | 1.5E+12 | 1E+12 | 11/29/2019 | 1.1018 | 11/29/2019 | -0.36 | 10/31/2019 | 51.68 | 1.207 | 10/31/2019 | 96.135 | 1.207 |
| 13 | 10/31/2019 | 0.3949 | 10/31/2019 | 299.55 | 10/31/2019 | 3604.41 | 10/31/2019 | 352.81 | 1.2E+12 | 9.5E+11 | 10/31/2019 | 1.1152 | 10/31/2019 | -0.407 | 9/30/2019 | 54.97 | 1.1688 | 9/30/2019 | 98.91 | 1.1688 |
| 14 | 9/30/2019 | 0.1885 | 9/30/2019 | 298.935 | 9/30/2019 | 3569.45 | 9/30/2019 | 343.53 | 1.6E+12 | 9.1E+11 | 9/30/2019 | 1.0899 | 9/30/2019 | -0.571 | 8/30/2019 | 48.415 | 1.2065 | 8/30/2019 | 91.61 | 1.2065 |
| 15 | 8/30/2019 | 0.052 | 8/30/2019 | 289.769 | 8/30/2019 | 3426.76 | 8/30/2019 | 336.82 | 1.3E+12 | 8.7E+11 | 8/30/2019 | 1.0982 | 8/30/2019 | -0.7 | 7/31/2019 | 50.205 | 1.32 | 7/31/2019 | 93.985 | 1.32 |
| 16 | 7/31/2019 | 0.4575 | 7/31/2019 | 292.592 | 7/31/2019 | 3466.85 | 7/31/2019 | 345.62 | 1.6E+12 | 8.4E+11 | 7/31/2019 | 1.1076 | 7/31/2019 | -0.44 | 6/28/2019 | 52.37 | 1.2132 | 6/28/2019 | 97.94 | 1.2132 |
| 17 | 6/28/2019 | 0.652 | 6/28/2019 | 284.656 | 6/28/2019 | 3473.69 | 6/28/2019 | 345.12 | 1E+12 | 8.1E+11 | 6/28/2019 | 1.1373 | 6/28/2019 | -0.327 | 5/31/2019 | 71.69 | 1.291 | 5/31/2019 | 118.935 | 1.291 |
| 18 | 5/31/2019 | 0.8345 | 5/31/2019 | 272.918 | 5/31/2019 | 3280.43 | 5/31/2019 | 324.73 | 9.2E+11 | 8.1E+11 | 5/31/2019 | 1.1169 | 5/31/2019 | -0.202 | 4/30/2019 | 57.97 | 1.421 | 4/30/2019 | 104.19 | 1.421 |
| 19 | 4/30/2019 | 1.025 | 4/30/2019 | 287.988 | 4/30/2019 | 3514.62 | 4/30/2019 | 346.18 | 6.9E+11 | 8.1E+11 | 4/30/2019 | 1.1215 | 4/30/2019 | 0.013 | 3/29/2019 | 64.765 | 1.3512 | 3/29/2019 | 108.885 | 1.3512 |
| 20 | 3/29/2019 | 0.9825 | 3/29/2019 | 277.594 | 3/29/2019 | 3351.71 | 3/29/2019 | 335.45 | 6.8E+11 | 8.2E+11 | 3/29/2019 | 1.1218 | 3/29/2019 | -0.07 | 2/28/2019 | 61.56 | 1.4788 | 2/28/2019 | 108.645 | 1.4788 |
| 21 | 2/28/2019 | 1.227 | 2/28/2019 | 270.181 | 2/28/2019 | 3298.26 | 2/28/2019 | 332.07 | 6.4E+11 | 8.3E+11 | 2/28/2019 | 1.1371 | 2/28/2019 | 0.183 | 1/31/2019 | 69.995 | 1.505 | 1/31/2019 | 116.665 | 1.505 |
| 22 | 1/31/2019 | 1.171 | 1/31/2019 | 260.294 | 1/31/2019 | 3159.43 | 1/31/2019 | 324.14 | 7.1E+11 | 8.4E+11 | 1/31/2019 | 1.1448 | 1/31/2019 | 0.149 | 12/31/2018 | 87.565 | 1.605 | 12/31/2018 | 130 | 1.605 |
| 23 | 12/31/2018 | 1.324 | 12/31/2018 | 242.412 | 12/31/2018 | 3001.42 | 12/31/2018 | 300.96 | 6.6E+11 | 8.3E+11 | 12/31/2018 | 1.1467 | 12/31/2018 | 0.242 | 11/30/2018 | 81.155 | 1.625 | 11/30/2018 | 123.04 | 1.625 |
| 24 | 11/30/2018 | 1.3965 | 11/30/2018 | 264.887 | 11/30/2018 | 3173.13 | 11/30/2018 | 324.09 | 8.4E+11 | 8.4E+11 | 11/30/2018 | 1.1317 | 11/30/2018 | 0.313 | 10/31/2018 | 73.98 | 1.666 | 10/31/2018 | 116.305 | 1.666 |
| 25 | 10/31/2018 | 1.4555 | 10/31/2018 | 261.727 | 10/31/2018 | 3197.51 | 10/31/2018 | 319.94 | 8.6E+11 | 8.2E+11 | 10/31/2018 | 1.1312 | 10/31/2018 | 0.385 | 9/28/2018 | 68.39 | 1.697 | 9/28/2018 | 113.105 | 1.697 |
| 26 | 9/28/2018 | 1.4805 | 9/28/2018 | 275.549 | 9/28/2018 | 3399.2 | 9/28/2018 | 346.08 | 7.5E+11 | 8E+11 | 9/28/2018 | 1.1604 | 9/28/2018 | 0.47 | 8/31/2018 | 68.63 | 1.713 | 8/31/2018 | 113.69 | 1.713 |
| 27 | 8/31/2018 | 1.3945 | 8/31/2018 | 273.552 | 8/31/2018 | 3392.9 | 8/31/2018 | 345.06 | 8E+11 | 8E+11 | 8/31/2018 | 1.1602 | 8/31/2018 | 0.326 | 7/31/2018 | 60.5 | 1.74 | 7/31/2018 | 105.035 | 1.74 |

*Figure 1: The data procured viewed as seen in the figure.*

| 2 | Date | ITRX EUR CDSI GEN 5Y Corp | FWISEU55 Index | 20y Swap Rate | Bund Yield | FTSE All World | EURUSD | Discount Yield | Equity Market (In Euro) | Bund Yield | Inflation Rate |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 1/31/2007 | 23.315 | 2.2488 | 4.455 | 4.10 | 241.2 | 1.3032 | 4.69 | 185.08 | 4.10 | 2.2488 |
| 4 | 2/28/2007 | 23.07 | 2.2471 | 4.33 | 3.957 | 239.5 | 1.3229 | 4.56 | 181.04 | 3.96 | 2.2471 |
| 5 | 3/30/2007 | 24.41 | 2.205 | 4.4801 | 4.057 | 243.81 | 1.3354 | 4.72 | 182.57 | 4.06 | 2.205 |
| 6 | 4/30/2007 | 22.955 | 2.238 | 4.5305 | 4.154 | 254.34 | 1.3648 | 4.76 | 186.36 | 4.15 | 2.238 |
| 7 | 5/31/2007 | 20.375 | 2.3059 | 4.7775 | 4.422 | 261.36 | 1.3453 | 4.98 | 194.28 | 4.42 | 2.3059 |
| 8 | 6/29/2007 | 25.115 | 2.3547 | 4.967 | 4.574 | 260.36 | 1.3541 | 5.22 | 192.28 | 4.57 | 2.3547 |
| 9 | 7/31/2007 | 50.99 | 2.38 | 4.8085 | 4.347 | 256.32 | 1.3684 | 5.32 | 187.31 | 4.35 | 2.38 |
| 10 | 8/31/2007 | 45.495 | 2.39 | 4.808 | 4.242 | 255.16 | 1.363 | 5.26 | 187.20 | 4.24 | 2.39 |
| 11 | 9/28/2007 | 37.57 | 2.41 | 4.881 | 4.329 | 268.48 | 1.4267 | 5.26 | 188.18 | 4.33 | 2.41 |
| 12 | 10/31/2007 | 37.2 | 2.4685 | 4.8285 | 4.239 | 279.1 | 1.4487 | 5.20 | 192.66 | 4.24 | 2.4685 |
| 13 | 11/30/2007 | 52.12 | 2.4605 | 4.8515 | 4.126 | 266.99 | 1.4633 | 5.37 | 182.46 | 4.13 | 2.4605 |
| 14 | 12/31/2007 | 50.61 | 2.443 | 4.908 | 4.307 | 263.66 | 1.4589 | 5.41 | 180.73 | 4.31 | 2.443 |
| 15 | 1/31/2008 | 78.63 | 2.396 | 4.668 | 3.93 | 241.98 | 1.4861 | 5.45 | 162.83 | 3.93 | 2.396 |
| 16 | 2/29/2008 | 127.365 | 2.4205 | 4.6215 | 3.891 | 242.16 | 1.5179 | 5.90 | 159.54 | 3.89 | 2.4205 |
| 17 | 3/31/2008 | 116.99 | 2.4005 | 4.6965 | 3.899 | 238.02 | 1.5788 | 5.87 | 150.76 | 3.90 | 2.4005 |
| 18 | 4/30/2008 | 72.26 | 2.408 | 4.847 | 4.123 | 250.66 | 1.5622 | 5.57 | 160.45 | 4.12 | 2.408 |
| 19 | 5/30/2008 | 77.14 | 2.5035 | 5.0365 | 4.405 | 253.44 | 1.5554 | 5.81 | 162.94 | 4.41 | 2.5035 |
| 20 | 6/30/2008 | 100.645 | 2.5895 | 5.094 | 4.621 | 231.7 | 1.5755 | 6.10 | 147.06 | 4.62 | 2.5895 |
| 21 | 7/31/2008 | 91.92 | 2.649 | 4.918 | 4.355 | 225.97 | 1.5603 | 5.84 | 144.82 | 4.36 | 2.649 |
| 22 | 8/29/2008 | 99.5 | 2.657 | 4.7605 | 4.176 | 220.55 | 1.4673 | 5.76 | 150.31 | 4.18 | 2.657 |
| 23 | 9/30/2008 | 118.435 | 2.663 | 4.7135 | 4.015 | 192.95 | 1.4092 | 5.90 | 136.92 | 4.02 | 2.663 |
| 24 | 10/31/2008 | 151.22 | 2.289 | 4.5535 | 3.9 | 154.41 | 1.2726 | 6.07 | 121.33 | 3.90 | 2.289 |
| 25 | 11/28/2008 | 170.335 | 2.4325 | 4.0347 | 3.258 | 143.99 | 1.2691 | 5.74 | 113.46 | 3.26 | 2.4325 |
| 26 | 12/31/2008 | 177.09 | 2.607 | 3.883 | 2.951 | 149.17 | 1.3971 | 5.65 | 106.77 | 2.95 | 2.607 |
| 27 | 1/30/2009 | 160.58 | 2.581 | 3.9995 | 3.296 | 136.03 | 1.2813 | 5.61 | 106.17 | 3.30 | 2.581 |
| 28 | 2/27/2009 | 180.5 | 2.445 | 3.8505 | 3.112 | 122.52 | 1.2669 | 5.66 | 96.71 | 3.11 | 2.445 |
| 29 | 3/31/2009 | 172.67 | 2.4485 | 3.859 | 2.994 | 132.52 | 1.325 | 5.59 | 100.02 | 2.99 | 2.4485 |

*Figure 2: The procured data refined for the purpose of the study for the period 2007 to 2020.*

## 4.4 AUTOCORRELATION

This research for the purpose of VaR computation uses non-parametric bootstrapping wherein the monthly changes are required to be independent which if not true then serial correlation exists. If a serial correlation is identified, then there is a need to correct for it. The author aims to check for serial correlation. In the study, serial correlation is autocorrelation where the change in one month is correlated with changes in different months and the researcher tests for the same using R programming language. The test for autocorrelation can be performed using the autocorrelation function 'acf()' in R and in this case giving reference of the data input from excel of the monthly change in the four risk factors – Discount yield, Equity prices, Bund yields, and Inflation rate. The R function for autocorrelation in the R console will look like – 'acf(M)' and 'M' is the matrix containing the market price returns. This function will generate the autocorrelograms and the charts that will be focused on are the ones representing the correlation of a given variable with itself at a certain lag. Lastly, the author does not intend to explicitly model or parameterize the autocorrelation but rather only intend to recognize if autocorrelation exists.

## 4.5 Building Long-term horizon VaR Model

### 4.5.1 Monte Carlo Simulation using non-parametric bootstrapping

Sampling with replacement from historical data is an alternative bootstrapping method used in Monte Carlo simulation for the generation of random numbers from a hypothetical distribution (Jorion, 2007). The non – parametric method is regarded as yet another method of modeling the path of the financial time series.

Formula for non – parametric model:

$$X_t = m(X_{t-1},\ldots,X_{t-p}) + \sigma(X_{t-1},\ldots,X_{t-p})\varepsilon_t, \quad t = 1,\ldots,n$$

In the above formula, the unknown smooth functions are *m (-)* and *σ (-)* and $\varepsilon_t$ are the independently and identically distributed random variable (Valášková, Spuchľáková and Adamko, 2015).

#### 4.5.1.1. VaR computation without any Floor and Options

**Method 1**: Computing VaR in Excel using VBA

1. The four factors that are considered for VaR computation are Equity prices, Discount yields, Bund yields, and Inflation rate.
2. Equity prices available as values of the FTSE All-World Index are in USD and for the purpose of the study, the same are converted to Euro. The values of the FTSE All-World Index are divided by the Euro-USD rate as on the given date to generate the equity prices in euro. Bund yields and inflation rate (iTraxx 5y - FWISEU55 Index) are considered as given.

3. Discount yields are calculated taking into consideration the value of ‘ITRX EUR CDSI GEN 5Y Corp’ and ‘20y swap rate’. The excel function used is ‘=D1+(B1/100)’ where D1 is the 20y swap rate and B1 is the ITRX EUR CDSI GEN 5Y Corp.
4. Once the data is refined for the purpose of computation, the history of monthly changes in the equity price is generated by dividing the price in the current month by the price in the previous month minus one. For instance, the use excel function (=(D2/D1-1)) is made where D2 is say the value in the month of February 2007, and D1 is the value in the month of January 2007. This computes series of monthly percentage changes.
5. Along with changes in equity prices, the computation of the history of changes in the discount yield, Bund yield, and the inflation rate is undertaken by deducting the value of the previous month from the current month and multiplying it by 100. For instance, the excel function (=(D2-D1*100)) is used where D2 is say the value in the month of February 2007, and D1 is the value in the month of January 2007. This computes series of monthly changes in the discount yield, Bund yield, and inflation rate.
6. Now 161 instances of the monthly change in equity prices, bund yields, discount yields, and inflation rates are generated.
7. The next step involves identifying the hypothetical values, or in other words, the hypothetical sensitivities are assigned to Equity prices, Bund yields, Discount yields, and Inflation rate. The hypothetical sensitivities are as follows:

a. The hypothetical sensitivity assigned to the Discount yield is **€ 2,000,000**.
b. The hypothetical sensitivity assigned to Equity Prices is **€ 250,000,000**.
c. The hypothetical sensitivity assigned to Bund Yield is **- € 1,500,000**.
d. The hypothetical sensitivity assigned to the Inflation Rate is **- € 600,000**.

8. Next step involves building a hypothetical history of changes using bootstrapping method.
9. The values are simulated for 12 months. The total instances of monthly changes that have been obtained are 161. So next is the simulation of random months from the given 161 months and to do this use of the excel function ‘=RANDBETWEEN (1,161)’ is made. This function randomly chooses month from 1 to 161 which is the index number assigned to months beginning from January 2007 to June 2020.

10. Once the 12 random month values are generated then using the '=VLOOKUP()' function in excel where reference of the randomly generated month number is provided and using that month as reference excel looks up the corresponding monthly change values of the Discount yield changes, Equity market returns, Bund yield changes, and change in the inflation rate.

| Simulated Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Simulated Annual Change |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Month Chosen | 157 | 130 | 143 | 44 | 92 | 8 | 1 | 91 | 38 | 47 | 11 | 92 | |
| Discount Yield Change | -0.48 | -5.72 | -0.84 | 21.39 | 17.85 | -0.625 | -12.745 | -40.07 | -9.94 | 14.605 | 4.14 | 17.85 | 5.415 |
| Equity Market Return | -0.0766263 | -0.0041436 | -0.0835165 | 0.0181831 | 0.0045250 | 0.0052233 | -0.0218347 | 0.0400952 | 0.0722906 | 0.0409206 | -0.0094940 | 0.0045250 | -0.0098524 |
| Bund Yield Change | -17.3 | 0.4 | -7.1 | 16.2 | 5.7 | 8.7 | -14.3 | -26.5 | -0.9 | 29.3 | 18.1 | 5.7 | 18 |
| Inflation Rate | -13.9 | 3.53 | -2 | -1.1 | -7.55 | 2 | -0.17 | -8.95 | -4.8 | 21.5 | -1.75 | -7.55 | -20.74 |
| Change in Value | 14173434.1 | -15193911 | -10709124.9 | 23685782.6 | 32811240 | -14194178.6 | -9396665.6 | -24996200 | 2422643.51 | -17409847 | -20193505.7 | 32811240 | **-6189093.209** |

*Figure 3: The Values in the Row 2 are the randomly generated months and the Discount yield changes, Equity market returns, Bund yield changes, inflation rate are the values that are looked up based on the random month.*

11. Next, calculation of the overall change in value is performed and in order to do that the values of the Discount yield change, Equity market returns, Bund yield changes, change in inflation rate are multiplied with their respective sensitivities, and then add all together. In other words, say from the above picture we take the 1st month which is month number 132 and we calculate its change in value by '= Discount yield sensitivity * Discount yield change + Equity market sensitivity * Equity market return + Bund yield sensitivity * Bund yield change + Inflation rate sensitivity * Inflation rate'. In numeric term it is '=2000000*(-0.48)+250000000*(-0.0766)+(-1500000)*(-17.3)+(-600000)*(-13.9)' and that generates the final figure of 14,173,434. The same process is repeated for all the 12 months.
12. Once the computation of the values for all the 12 months is done, then the final and the total values are generated by summing all the 12 months computed values using excel function 'Sum ()'. This will generate the annual equity return changes and the annual changes in the Discount yield, Bund yield, Inflation rate.
13. Next, this process is repeated to have 100,000 simulations but doing this manually is slow and time-consuming so in order to overcome this shortcoming we make use of VBA. VBA enables automation of tasks that are repetitive in nature.

14. The following is this code that is used to run the program which is automated bootstrapping process for generating desired number of simulations:

```
Sub simulate ()
n = InputBox ("How many simulations do you want? ")
For i = 1 To n
  ActiveSheet.Calculate
  dValue = ActiveSheet.Range ("delta_value").Value
  dDiscYield = ActiveSheet.Range ("delta_discount_rate").Value
  dEqValue = ActiveSheet.Range ("delta_equity_value ").Value
  dBundYield = ActiveSheet.Range ("delta_bund_yield").Value
  dInflation = ActiveSheet.Range("delta_inflation").Value

  ActiveSheet.Range("ag1").Offset(i, 0).Value = dValue
  ActiveSheet.Range("ah1").Offset(i, 0).Value = dDiscYield
  ActiveSheet.Range("ai1").Offset(i, 0).Value = dEqValue
  ActiveSheet.Range("aj1").Offset(i, 0).Value = dBundYield
  ActiveSheet.Range("ak1").Offset(i, 0).Value = dInflation
Next i
End Sub
```

The first half of the program is providing the input which the researcher wishes to replicate for said number of times and the second half of the program instructs as to where the output is desired in terms of the cell destination.

Once the VBA program is drafted, we then run the macro that brings up the input window that allows us to input the number of simulations that we wish to obtain. For the purpose of this study, we generate 100,000 simulations. A Snapshot of the input window that pops-up is provided in the appendix as appendix 1 and appendix 2.

15. Now, 100,000 simulations of the future values of the portfolio or the changes in the values have been obtained. Next, using the 100,000 simulated value VaR can be

worked out. To obtain the VaR value, the ' =PERCENTILE(AG:AG,0.01)' function in excel is used and the AG is reference provided of the column where the simulated final change in values have been obtained and 0.01 is the 1% significance level or the alpha at 99% confidence level.

**Method 2**: Computing VaR in R

1. The VaR computation is based on four factor approach that is the Discount yield, Equity prices, Bund yields, Inflation rates.
2. Equity prices available as values of FTSE All World Index are in USD and for the purpose of study the same are converted to Euro. The values of FTSE All World Index are divided by the Euro-USD rate as on the given date to generate the equity prices in euro. Bund yields and inflation rate (iTraxx 5y - FWISEU55 Index) are considered as given. Discount yield are calculated taking into consideration the value of 'ITRX EUR CDSI GEN 5Y Corp' and '20y swap rate'. The excel function used is '=D1+(B1/100)' where D1 is the 20y swap rate and B1 is the ITRX EUR CDSI GEN 5Y Corp.
3. 161 instances of monthly data pertaining to equity prices, bund yields, discount yields and inflation rate have now been refined for the purpose of study and saved as an Excel file namely 'MKT data'.
4. Next, the work in RStudio begins where use of the function of 'input data set from Excel' to import the monthly change data file namely – MKT data that contains 162 rows and 5 column when row are the heading and column 1 consist of the date.
5. Next step involves writing the R-command in the R-Script window and the R-command for computation of VaR for this research is done using the following steps:

```
#Sensitivities
D1 = 2e6
D2 = 250e6
D3 = -1.5e6
D4 = -6e5
```

1. First step is laying out the sensitivities:

   D1 is the discount yield sensitivity – **2,000,000**.

D2 is the Equity delta position – **250,000,000**.

D3 is the Bond sensitivity – **(-1,500,000)**.

D4 is Inflation sensitivity – **(-600,000).**

```
numSims = 100000
numMonths = 12
res  = vector()
DY = vector()
EQ=vector()
BY=vector()
INF=vector()
```

2. The total number of Simulation intended to be undertaken are 100,000 which are projected over 12 months. 'Res = Vectors' stores the results of the simulation.
   'DY = vector()', 'EQ=vector()','BY=vector()', 'INF=vector()' records the simulated changes in the Discount yield, Equity market, Bund yield and inflation expectation respectively.

```
M =matrix(0,161,4)
for(i in 1:161){
  M[i,1]=(mkt_data$Disc_Yield[i+1]-mkt_data$Disc_Yield[i])*100
  M[i,2]=mkt_data$Eq_Mkt[i+1]/mkt_data$Eq_Mkt[i]-1
  M[i,3]=(mkt_data$Bund_Yield[i+1]-mkt_data$Bund_Yield[i])*100
  M[i,4]=(mkt_data$Infl_rate[i+1]-mkt_data$Infl_rate[i])*100
}
```

3. Next is constructing the matrix 'M' containing market price returns which is a manner of reading in the data. In total 161 observations, so the matrix 'M' consists of 161 rows and 4 column and into that the history of market price changes are put. The first column consists of the discount yield minus the previous discount yield * 100. The second column will contain the relative return in the equity markets and so it is new price over old price minus 1. The third column is going to be the new Bund yield

minus the old Bund yield * 100. The last column is going to contain the new inflation rate minus the old inflation rate * 100. This process is repeated 100,000 times.

```
for (i in 1:numSims){
  factorMatrix =matrix(0,numMonths,4)
  chosenMonths = -sample(x=1:161,size =12,replace = TRUE)
  for (j in 1:numMonths){
    factorMatrix[j,1]=M[chosenMonths[j],1]
    factorMatrix[j,2]=M[chosenMonths[j],2]
    factorMatrix[j,3]=M[chosenMonths[j],3]
    factorMatrix[j,4]=M[chosenMonths[j],4]
  }
DY[i] = sum(factorMatrix[,1])
EQ[i]= sum(factorMatrix[,2])
BY[i]= sum(factorMatrix[,3])
INF[i]=sum(factorMatrix[,4])
res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i]
}
```

4. Next step is simulating annual changes by bootstrapping the monthly returns for use in the VaR model. A factor matrix is constructed which is a matrix of the 4 factors consisting of 12 rows and 4 columns. R-command is written such that it randomly chooses one month out of the 161 months recorded and performs the same 12 times therefore written as 'size=12'. 'Replace=True' is command instructing to undertake the sampling with replacements. Further, based on the months it selects the various corresponding values of the factors which is a version of R like the Excel's VLOOKUP function.

| | |
|---|---|
| DY[i] = sum(factorMatrix[,1]) | It is the vector containing the simulated Discount Yield change |
| EQ[i]= sum(factorMatrix[,2]) | It is the vector containing the simulated Equity Returns |

| | |
|---|---|
| BY[i]= sum(factorMatrix[,3]) | It is the vector containing the simulated Bund Yield change |
| INF[i]=sum(factorMatrix[,4]) | It is the vector containing the simulated Inflation rate change |
| res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i] | It gives out the result that is simulated change in the value of the pension scheme<br>Discount yield sensitivity * Simulated Discount yield change + Equity market sensitivity * Simulated Equity market return + Bund yield sensitivity * Simulated Bund yield change + Inflation rate sensitivity * Simulated Inflation rate change |

```
hist(res/1e6,100, main = 'Distribution of simulated changes in
value', xlab = 'Change in value of pension scheme (in
million)')print(cat('The VaR is €',quantile(res,0.01)/1e6,
"million.\n"))
```

5. To compute the VaR at 99% confidence level the given R-command is ’print(cat('The VaR is €',quantile(res,0.01)/1e6, "million.\n")) and additionally, R-command to generate the histogram of the simulate change in values is written as ‘hist(res/1e6,100, main = 'Distribution of simulated changes in value', xlab = 'Change in value of pension scheme (in million)')’.
6. Lastly the block run of the R-command is undertaken by selecting the entire text in the R-Script window and run the program by click on “Run” option. The results are obtained in the Console window and the histogram is generated into the window that is to the right of the console window.

```
#Sensitivities
D1 = 2e6
D2 = 250e6
D3 = -1.5e6
D4 = -6e5

numSims = 100000
numMonths = 12
res  = vector()
DY = vector()
EQ=vector()
BY=vector()
INF=vector()

M =matrix(0,161,4)
for(i in 1:161){
 M[i,1]=(mkt_data$Disc_Yield[i+1]-mkt_data$Disc_Yield[i])*100
 M[i,2]=mkt_data$Eq_Mkt[i+1]/mkt_data$Eq_Mkt[i]-1
 M[i,3]=(mkt_data$Bund_Yield[i+1]-mkt_data$Bund_Yield[i])*100
 M[i,4]=(mkt_data$Infl_rate[i+1]-mkt_data$Infl_rate[i])*100
}
for (i in 1:numSims){
 factorMatrix =matrix(0,numMonths,4)
 chosenMonths = sample(x=1:161,size =12,replace = TRUE)
 for (j in 1:numMonths){
  factorMatrix[j,1]=M[chosenMonths[j],1]
  factorMatrix[j,2]=M[chosenMonths[j],2]
  factorMatrix[j,3]=M[chosenMonths[j],3]
  factorMatrix[j,4]=M[chosenMonths[j],4]
 }
 DY[i] = sum(factorMatrix[,1])
 EQ[i]= sum(factorMatrix[,2])
 BY[i]= sum(factorMatrix[,3])
 INF[i]=sum(factorMatrix[,4])
 res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i]
}
hist(res/1e6,100, main = 'Distribution of simulated changes in value', xlab = 'Change in
value of pension scheme (in million)')print(cat('The VaR is €',quantile(res,0.01)/1e6,
"million.\n"))
```

#### 4.5.1.2. VaR computation when incorporating the Floor and Options

The author expands the VaR model to take into account features like options that are commonly used in equity portfolio management and along with this author also includes effective floors on the discount rate and intends to restrict the dynamics of the discount rate so that it does not fall below the specified floor.

The floor of -200 basis points has been incorporated in order to protect the discount yield that is applied to discount the liabilities in the Pension scheme from fall below the specified floor. It is an effective method of ensuring that the simulated discount rate remains at -200 basis points or above. A floor of -200 basis points means that if the simulated discount yields are less than -200 basis points then the simulated discount yield is equal to -200 basis points or else the simulated discount yield is unaltered.

The author incorporates options in the study because this particular research addresses not only an academic problem but also an industrial problem. In reality, big financial corporations do include options and thereby researcher integrates this aspect in the study. The research incorporates Put options for equities which are Put options with a strike price equal to 75% of the prevailing market. This means that the minimum value for the stock market in the research's simulations is 75% which reflects that it can never be below 75% and it is similar to incorporating a floor. If the simulated change in equity market return is less than -25% then the simulated equity market return equals to -25% or else the simulated equity market return remains unaltered.

Ultimately, the author aims at analyzing the influence on the VaR value with the inclusion of floor and options.

**Method 1**: Computing VaR in Excel using VBA

1. The computation process remains the same till step 11 as seen in VaR computation without floor and options. The only change that is made, as seen in the table below, is to the 'Simulated Annual Change' column which is initially the sum total of the respective rows of Discount Yield and Equity Market return. To incorporate the floor of -200 basis points to the discount yield, the formulae entered in Excel is '=MAX(R3,SUM(O14:Z14))' where R3 is the cell with the value of -200 basis points and the sum of 'O14:Z14' is the sum total of the simulated 12-month value of the Discount Yield change. This function gives Excel command to select the greater value between the total of 12 simulated Discount Yield value or -200 basis points.
2. Along with the floor in the Discount yield, the author also incorporates a put option with a strike price of 75% of the current market price. So, equity value is simulated

using the excel function '=MAX(SUM(O15:Z15),R4-1)'.The future equity price is the greater of the simulated stock price or the -0.25.

| Simulated Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Simulated Annual Change |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Month Chosen | 6 | 142 | 39 | 9 | 11 | 35 | 138 | 139 | 86 | 136 | 58 | 113 | |
| Discount Yield Change | 10.025 | 1.275 | -3.97 | -5.62 | 4.14 | -2.06 | -6.29 | 2.38 | 0.69 | 8.46 | 40.53 | -14.71 | 34.85 |
| Equity Market Return | -2.58% | 1.25% | 1.60% | 2.38% | -0.95% | 6.79% | 2.85% | 1.29% | 0.54% | 3.02% | -0.35% | -0.60% | 15.24% |
| Bund Yield Change | -22.7 | -7.2 | -7.5 | -9 | 18.1 | 22.8 | 14.1 | -11.7 | -5.8 | -21.8 | 25.5 | -26.9 | -32.1 |
| Inflation Rate | 2.53 | -4.1 | -4.9 | 5.85 | -1.75 | 9.15 | 1.77 | -2.7 | -1.84 | 0.85 | 14 | -15.7 | 3.16 |
| Change in Value | 46,130,754 | 18,940,910 | 10,249,301 | 4,692,323 | -20,193,506 | -26,824,112 | -27,676,638 | 27,146,658 | 12,544,843 | 56,659,546 | 33,528,279 | 18,843,683 | **154,042,042** |

*Figure 4: Simulated Annual change Value*

3. Next, this process is repeated to generate 100,000 simulation using VBA. The following is this code that is used to run the program which is automated bootstrapping process for generating desired number of simulations:

```
Sub simulate ()
n = InputBox ("How many simulations do you want? ")
For i = 1 To n
   ActiveSheet.Calculate
   dValue = ActiveSheet.Range ("delta_value").Value
   dDiscYield = ActiveSheet.Range ("delta_discount_rate").Value
   dEqValue = ActiveSheet.Range ("delta_equity_value ").Value
   dBundYield = ActiveSheet.Range ("delta_bund_yield").Value
   dInflation = ActiveSheet.Range("delta_inflation").Value

   ActiveSheet.Range("ag1").Offset(i, 0).Value = dValue
   ActiveSheet.Range("ah1").Offset(i, 0).Value = dDiscYield
   ActiveSheet.Range("ai1").Offset(i, 0).Value = dEqValue
   ActiveSheet.Range("aj1").Offset(i, 0).Value = dBundYield
   ActiveSheet.Range("ak1").Offset(i, 0).Value = dInflation
Next i
End Sub
```

The first half of the program is providing the input which the researcher wishes to replicate for said number of times and the second half of the program gives instruction as to where the output is desired in terms of the cell destination.

Once the VBA program is drafted, then the macro is run which brings up the input window that allows the author to input the number of simulation that are wished to obtain. For the purpose of this study 100,000 simulation are generate.

Now, 100,000 simulations of the future values of portfolio or the changes in the values have been obtained. Next, using the 100,000 simulated value VaR can be worked out. To obtain the VaR value, the ‘ =PERCENTILE(AG:AG,0.01)’ function in excel is used and the AG is reference provided of the column where the simulated final change in values have been obtained and 0.01 is the 1% significance level or the alpha at 99% confidence level.

**Method 2**: Computing VaR in R The overall R-command remains the same with slight alterations which are highlighted

```
#Sensitivities
D1 = 2e6
D2 = 250e6
D3 = -1.5e6
D4 = -6e5
floor = -200
strike = 0.75

numSims = 100000
numMonths = 12
res  = vector()
DY = vector()
EQ=vector()
BY=vector()
INF=vector()

M =matrix(0,161,4)
for(i in 1:161){
  M[i,1]=(mkt_data$Disc_Yield[i+1]-mkt_data$Disc_Yield[i])*100
  M[i,2]=mkt_data$Eq_Mkt[i+1]/mkt_data$Eq_Mkt[i]-1
  M[i,3]=(mkt_data$Bund_Yield[i+1]-mkt_data$Bund_Yield[i])*100
  M[i,4]=(mkt_data$Infl_rate[i+1]-mkt_data$Infl_rate[i])*100
}
for (i in 1:numSims){
  factorMatrix =matrix(0,numMonths,4)
  chosenMonths = sample(x=1:161,size =12,replace = TRUE)
  for (j in 1:numMonths){
    factorMatrix[j,1]=M[chosenMonths[j],1]
    factorMatrix[j,2]=M[chosenMonths[j],2]
    factorMatrix[j,3]=M[chosenMonths[j],3]
    factorMatrix[j,4]=M[chosenMonths[j],4]
  }
  DY[i] = max(sum(factorMatrix[,1]),floor)
  EQ[i]= max(sum(factorMatrix[,2]),strike-1)
  BY[i]= sum(factorMatrix[,3])
  INF[i]=sum(factorMatrix[,4])
  res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i]
}
hist(res/1e6,100, main = 'Distribution of simulated changes in value', xlab = 'Change in value of pension scheme (in million)')
print(cat('The VaR is €',quantile(res,0.01)/1e6, "million.\n"))
```

## 4.6 BACKTESTING THE VAR MODEL

The author has successfully constructed the VaR model and therefore the next step is to test the VaR model and in order to do that author uses a backtest type approach. Backtesting is a terminology used by the industry to define a method used for testing the risk models' accuracy (Alexander, 2008). Backtesting is a model validation tool. Backtesting is undertaken for both the VaR model that has been constructed – VaR model without Floor and options and VaR model with Floor and options. 2 methods of backtesting are adopted for determining the effectiveness of both the model. The first method being used is the Z – test and the second being the Kupiec test.

### Assumptions for the Backtest model:

The backtest model that is built makes to underlying assumptions:

1. The risk factors are independent through time.

   The author has already tested for this assumption by testing if there exists any serial autocorrelation among the monthly changes in the returns of risk factors. The results of the test prove there exists no serial correlation.

2. The variables are independent identically distributed random variables, in order words, variables are constant through time. To check for the fulfilment of the assumption the author conducts an F-test in Microsoft Excel to determine if the variances are constant with time or not.

**F –Test: Test to determine if variances are constant through time.**

The following steps are involved in performing the F-test:

1. The data used for computation are the monthly changes of the risk factor which if 161 observations for each risk factor.

2. The use of data analysis function is made for computation and the test is performed at a standard confidence level of 95%. The data for each risk factor is divided into 2 for the purpose of providing the data range and thereby data range doe each factor is 79 and 78 which sums up to 161.

   The formula for computation of F-test: $F = s^2_1 / s^2_2$

3. This process is repeated for each risk factor.

The hypothesis that is being tested is:

$H_0$: $\sigma^2_1 = \sigma^2_2$

$H_a$: $\sigma^2_1 \neq \sigma^2_2$

Lastly, the F value is computed and compared to the F critical value in order to determine if the variables are constant or not.

### 4.6.1 Backtesting the VaR model in Excel

#### 4.6.1.1. Z-Test

**Backtesting VaR model in Excel using VBA – No Floor or Option**

1. The first 12 step remains the same performed in the model building for VaR computation without floor and option. The initial steps remain the same because for the purpose of backtesting as well the process of bootstrapping is undertaken.
2. The next step is formulating to compute the simulated hypothetical P&L. In any given cell, we input the value from where the computation is deemed desirable to begin. In this study the total number of instances are 161 and computation, in this case, begins from 61 corresponding to the year 2012 onwards. From here onwards an expanding model is creating where for instance, for the first computation the random numbers will be selected from the first 60 values and for the next computation from the first 61 values and this goes on expanding until it reaches 161. Also, the model that is built enables to project through time more easily rather than making an assumption about the square root of time.
3. Next, Program is written in VBA as seen in appendix 5 so as to repeat the process for n time. The program gives input of the number of simulation to be considered for computing the VaR values and the Simulated P&L which is 10,000 in this study. The program inputs for truncated VaR for the purpose of backtesting to be computed at 3 different confidence levels – 95%, 97.5%, and 99%. The code provides the reference of the destination cell where the output is desired.
4. Once the output has been derived the next step involves finding the number of exceedances in the obtained result. The use of an excel function of 'if' analysis is made and the formula looks like 'IF(C9<D9,1,0)' where C9 is the hypothetical P&L value derived and D9 is the respective VaR value. This is done 3 time to compute the

exceedances of 95% VaR, 97.% VaR and 99% VaR respectively. The results will be obtained in 0 and 1 where 1 is the number of time VaR exceeds the hypothetical P&L. The sum of 1 is the observes exceedances.

5. Next, the 3 standard deviation values are computed for VaR at 3 different confidence levels. The last step, involves computing the Z value using the following formula:

$$z = \frac{X - \mu}{\sigma}$$

In the above formula: X is the observed exceedances

μ is the expected exceedances

σ is the standard deviation

The hypothesis being test is:

$H_0$: Number of observed exceedance = Number of expected exceedance

$H_a$: Number of observed exceedance ≠ Number of expected exceedance

6. Lastly, the Z value is computed and compared to the Z critical value in order to determine where the model has passed or failed the backtest.

**Backtesting VaR model in Excel using VBA – With Floor on discount yield and Put Option to equity price**

1. The first 2 steps remain the same as performed in the model building for VaR computation with floor and option. The initial steps remain the same because for the purpose of backtesting as well the process of bootstrapping is undertaken and the only difference being from the steps above in the inclusion of effective discount yield on the floor and put option to equity prices.
2. The same steps from 2 to 6 are implemented as applied for the backtesting VaR model without floor and option.

   The hypothesis remains:

   $H_0$: Number of observed exceedance = Number of expected exceedance

   $H_a$: Number of observed exceedance ≠ Number of expected exceedance

   The results determine whether or not the model passes the backtest

### 4.6.1.2. Kupiec Test

**Kupiec test - Backtesting VaR model in Excel using VBA – No Floor or Option**

1. The first 5 steps remain the same as in the Z-test backtest model.
2. The next step involves computing the Kupiec statistic using the following formula:

$$\ln(LR_{uc}) = n_1 \ln\left(\pi_{exp}\right) + n_0 \ln\left(1-\pi_{exp}\right) - n_1 \ln(\pi_{obs}) - n_0 \ln(1-\pi_{obs})$$

   In the above formula: $\pi_{obs}$ is the observed proportion of the exceedances

   $\pi_{exp}$ is the expected proportion of the exceedances

   n is the sample size of the backtest

   $n_1$ is the observed number of exceedances

   $n_0$ is $n - n_1$

   The hypothesis being test is:

   $H_0$: proportion of observed exceedance = proportion of expected exceedance

   $H_a$: proportion of observed exceedance ≠ proportion of expected exceedance

3. The confidence level of the VaR model tested are 95% VaR, 97.5% VaR, and 99% VaR and all the 3 tests at Kupiec confidence level of 99%.
4. Lastly, the VBA program is drafted for computation of the Kupiec statistic and compared to the Kupiec critical value which is computed using '=CHIINV' function in excel, in order to determine where the model has passed or failed the backtest.

**Kupiec test: Backtesting VaR model in Excel using VBA – With Floor on discount yield and Put Option to equity price**

1. The first 2 steps remain the same as performed in the model building for VaR computation with floor and option. The initial steps remain the same because for the purpose of backtesting as well the process of bootstrapping is undertaken and the only difference being from the steps above in the inclusion of effective discount yield on the floor and put option to equity prices.
2. The same steps from 1 to 4 are implemented as applied for the Kupiec test - backtesting VaR model without floor and option.

   The hypothesis remains:

   $H_0$: proportion of observed exceedance = proportion of expected exceedance

$H_a$: proportion of observed exceedance ≠ proportion of expected exceedance

The results determine whether or not the model passes the backtest.

### 4.6.2 Backtesting the VaR model in RStudio.

The Author has drafted 2 R-commands as seen in appendix 6 and 7: first, performs the Z-test along with the Kupiec test in the same program without incorporating any floor or option and the second, performs the Z-test along with the Kupiec test in the same program while incorporating floor on discount yield of -200 basis points and Put option with strike of 75% to the equity prices.

An additional functionality is reflected in R-command which is of Seeding which in simple words means that every time the program is run the use of same random numbers is made and due to which the output is more consistent.

The hypothesis remains the same as stated before:

For Z-test:

The hypothesis being test is:

$H_0$: Number of observed exceedance = Number of expected exceedance

$H_a$: Number of observed exceedance ≠ Number of expected exceedance

For Kupiec test:

$H_0$: proportion of observed exceedance = proportion of expected exceedance

$H_a$: proportion of observed exceedance ≠ proportion of expected exceedance

The respective confidence levels remain the same as used in the computations performed in excel.

The results of the respective test will reflect if the backtest model has failed or passed.

# 5 FINDINGS

This Section of the research intends to state and analyzing the results. The findings are presented in a multi-segment approach so as to facilitate simplified presentation and reading of results. The author analyses the results obtained from the test of autocorrelation which was performed in RStudio. Next, the results obtained by running the VaR model that is constructed in Microsoft Excel and RStudio, both with and without Put options and floor, are discussed. Then, the results of the F-test which is conducted to test if the variances are constant through time or not and ultimately, to determine if the assumptions of backtest models are fulfilled or not. Lastly, the results of the backtest: Z-Test and Kupiec Test obtained through Microsoft Excel and RStudio are analyzed.

## 5.1 AUTOCORRELATION

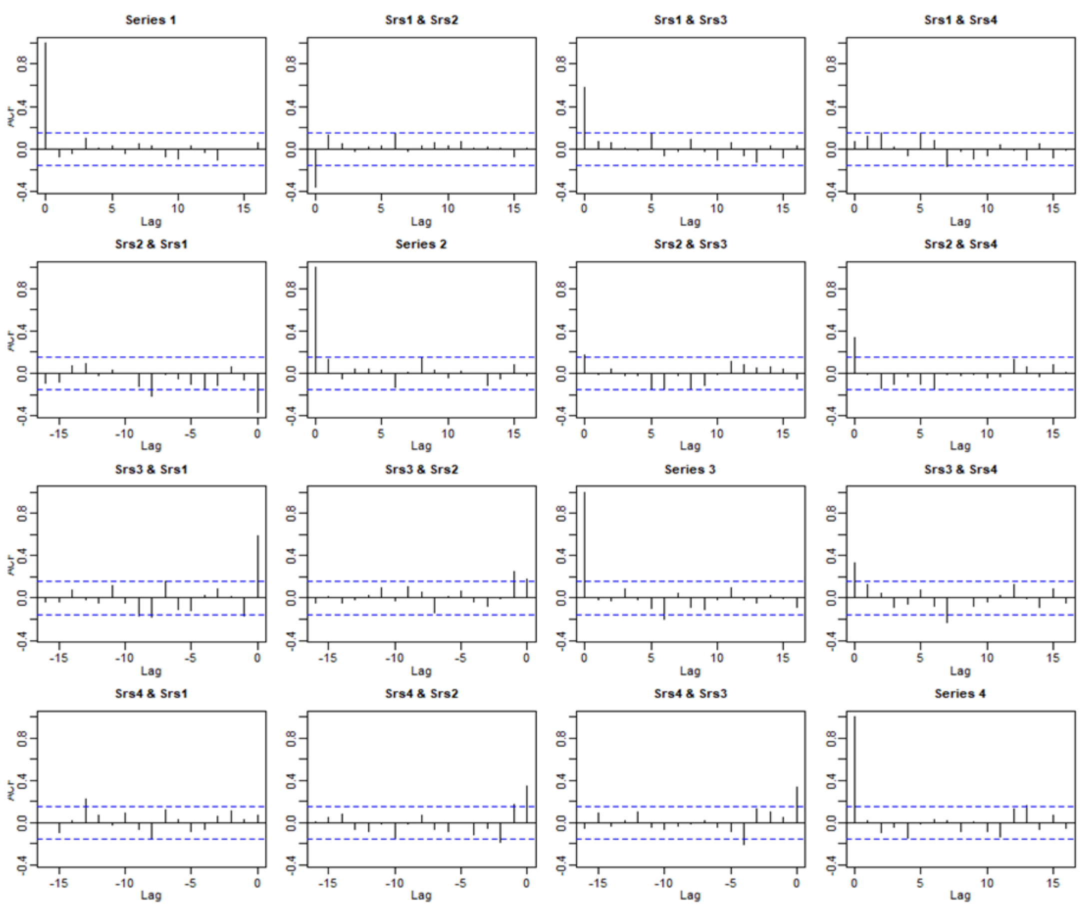



*Figure 5: Autocorrelograms generated in RStudio depicting autocorrelation of the risk factors.*

One of the possible criticism of the VaR model that is constructed could be that the monthly changes are not independent which if true, it means that there exists serial correlation and we need to be able to correct for it. The above-seen figure is the output derived in R when the autocorrelation function is run. In the figure, the series 1 reflects the history of discount yield changes, series 2 reflects the history of equity price changes, series 3 reflects the history of bund yield changes and series 4 reflects the history of inflation rate changes. In the given figure, the focus remains on the diagonals from left to right because the author aims to know the correlation of a given variable with itself at a certain lag.

In all the 4 autocorrelograms it is noted that at 0 lag all the four variables have a correlation with itself of 100% which is as expected because any given variable will tend to be 100% correlated with itself. But it is seen, in case of all 4 variables, that with the incorporation of even a 1-day lag the autocorrelation coefficient somewhat stays with the dotted blue lines which means that the autocorrelation is not significant. In series 3 which is the history of bund yield changes they are a slight outliner at lag 6 and in the case of series 4 that is a history of inflation rate change at lag 13 there is a slight crossing of the blue line. Though overall none of the variables have a significant level of autocorrelation.

Because there is no serial autocorrelation, the author can state that change in one variable on a given day is not correlated to the change in another variable on some other given day. To conclude, it can be said that as there exists no significant autocorrelation and therefore the approach adopted by the author of simulating with replacement is valid.

## 5.2 Model building: Monte Carlo Simulation using non-parametric bootstrapping

The scope of the study is to identify the potential change in the value of the pension scheme and in order to do that author has effectively built a VaR model based on the steps discussed in the preceding chapter. The constructed model simulates the potential change in the value of a pension scheme for the purposes of capital attribution in a financial institution.

The model computes the VaR value which is the estimated change in the value of the pension scheme and likewise, the pension scheme will lose the said amount of money. An important aspect of pension scheme for any given financial institution is that even if the pension scheme is in surplus it is not deemed to contribute to the capital of the financial institution whereas if there is pension scheme with deficit then that will directly have implications on the capital.

As stated earlier, Author has made a fair assumption that the pension scheme is fully funded from an IAS 19 valuation perspective in so far as the value of the assets are exactly matched by the value of the liabilities. There is no loss of generality in the made assumption, which means, for instance if the initial value of the pension scheme was zero then the model stands valid, secondly if the initial value of the pension scheme is € -100,000,000 still the model stand valid and lastly, if the initial value of the pension scheme is €100,000,000 the model still has validity.

### 5.2.1 VaR model built in Excel

**Comparison: VaR computation without any Floor and Put options v/s VaR computation with Floor and Put options.**

## A. CHANGE IN VALUE OF PENSION SCHEME

- The VaR model built without floor and put option simulates the change in the value of the pension scheme to be € **234,566,527** and that is the amount of money that the pension scheme will lose at 99% confidence level. Whereas, the VaR model constructed with incorporation of the effective floor to the discount yield and Put option to the equity price simulates the change in the value of the pension scheme to be € **229,504,947** and that is the amount of money that the pension scheme will lose at 99% confidence level. The VaR value falls because the floor on discount yields and the put option on Equity prices restrict it from falling.
- The Histograms seen below are constructed in Microsoft Excel which shows the distribution of the potential changes and the VaR being measure and presented is € **234,566,527** and € **229,504,947** in figure 6 and in figure 7 respectively.

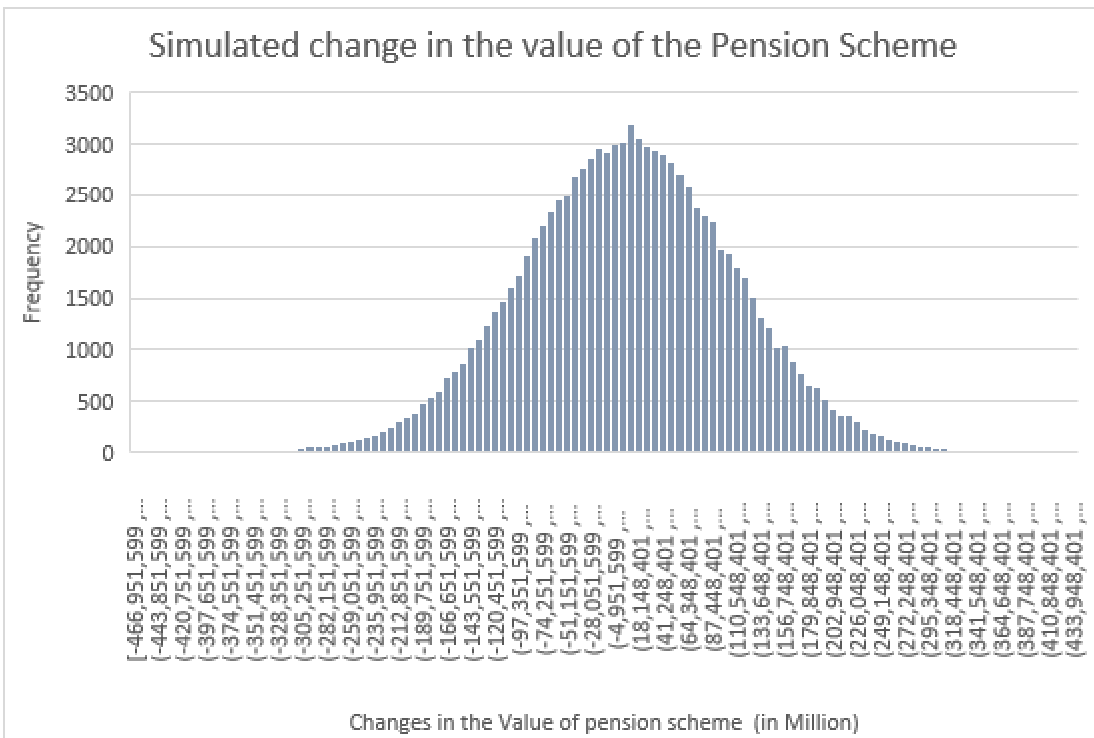



*Figure 6: Histogram constructed in Microsoft Excel depicting the simulated change in the value of the pension scheme.*

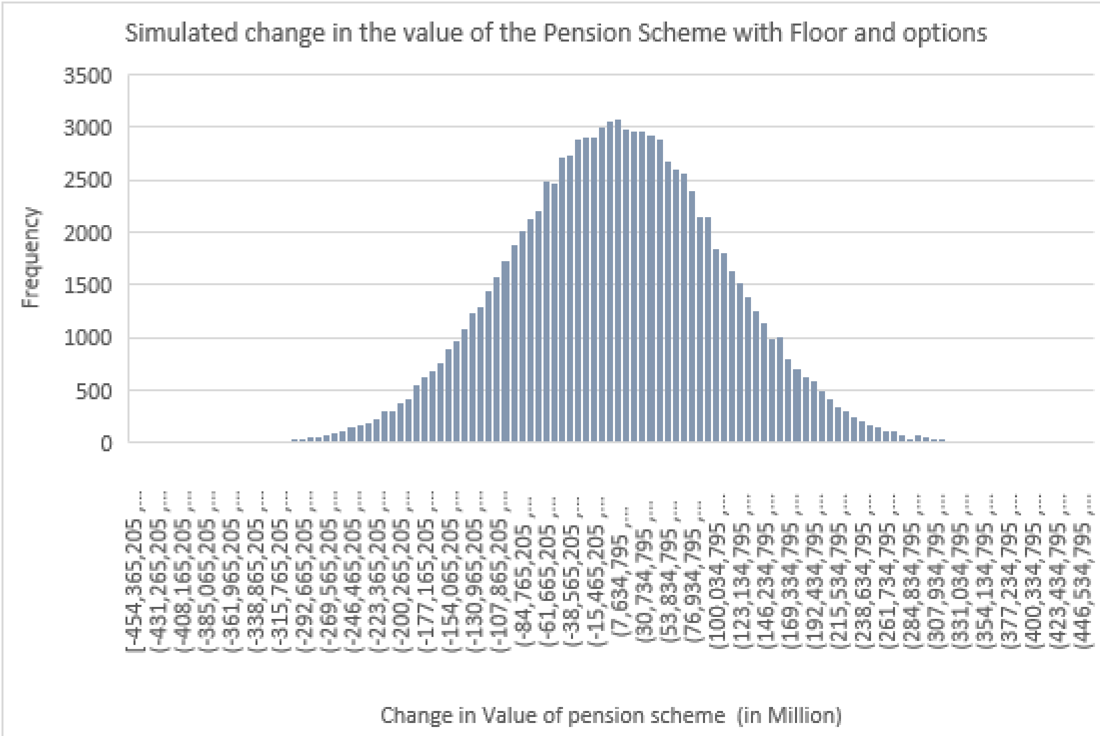



*Figure 7 : Histogram constructed in Microsoft Excel depicting the simulated change in the value of the pension scheme with incorporation of floor and put option.*

## B. CHANGE IN DISCOUNT YIELDS

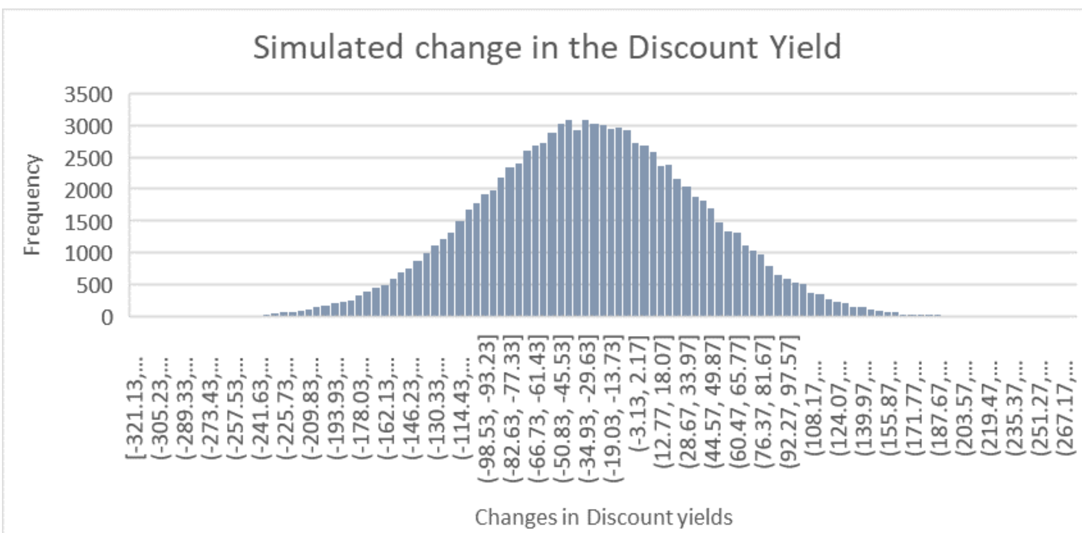



*Figure 8: Histogram depicting the simulated change in the discount yield without floor.*

The two given figures show the changes in the distribution of the discount yield with and without floor. In the above figure, it is noted that discount rate is not restricted from falling as there is no floor applied to the discount yield and thereby unrestrictedly the discount rate falls to -321.13 basis points. On contrary the figure below reflects bunching on the left edge, which is because, by apply floor of -200 basis points on the discount yield the author restricts the discount yield from falling below -200 basis points.

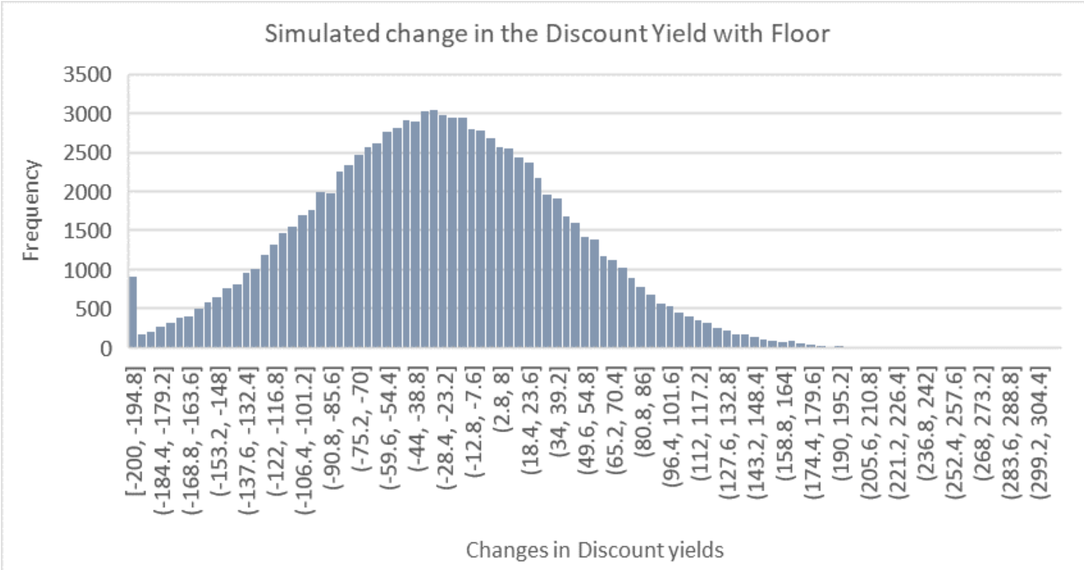



*Figure 9: Histogram depicting the simulated change in the discount yield with floor of -200 basis points.*

## C. CHANGE IN EQUITY PRICES

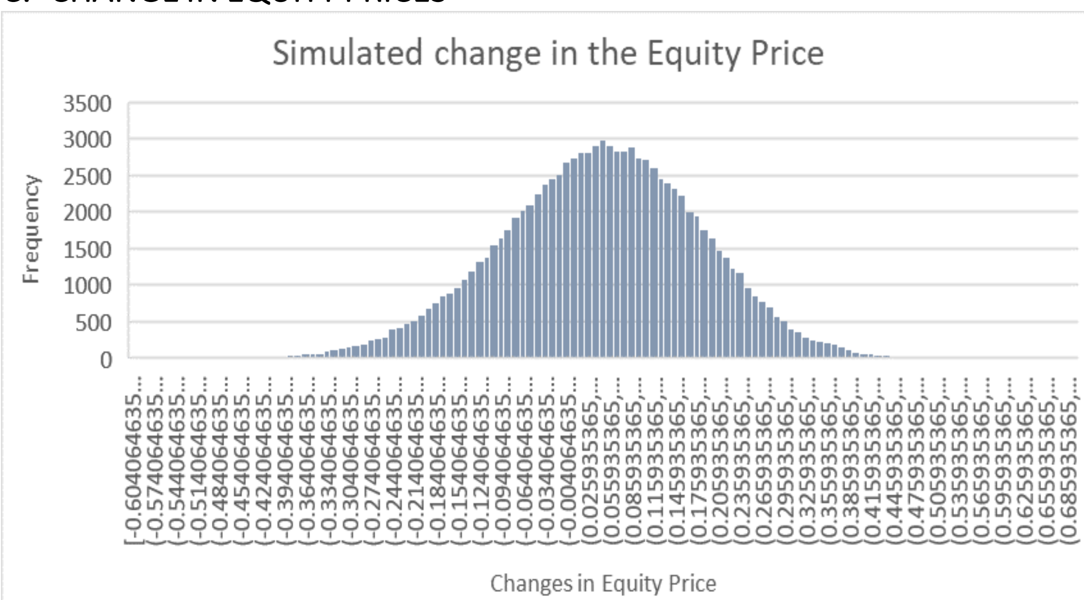



*Figure 10: Histogram depicting the simulated change in the equity price without Put option .*

The two given figures show the changes in the distribution of the equity price with and without put option. In the above figure, it is noted that no put option has been incorporate to the equity price. Whereas the figure given below reflects bunching on the left edge which is around 25% and the same is because the put option with strike of 75% has been applied which does not allow the equity price to fall below 25%.

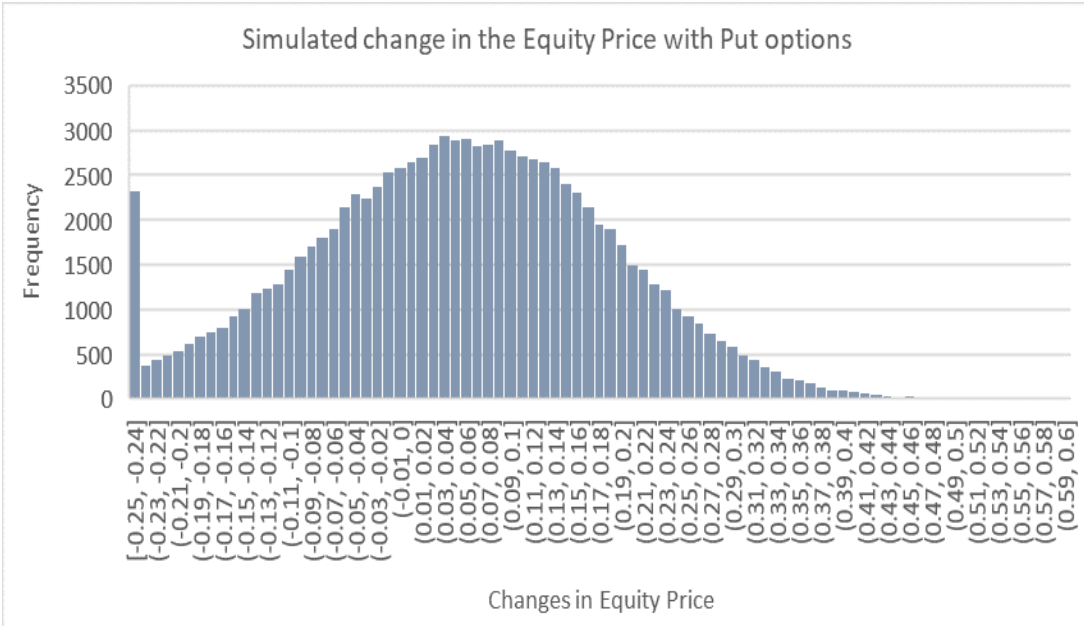



*Figure 11: Histogram depicting the simulated change in the equity price with Put option of 75%.*

## D. CHANGE IN BUND YIELDS AND CHANGE IN INFLATION RATE

The figures below show that there is almost no change in the distribution of the risk factors that is the Bund yield and the Inflation rate when floor is incorporated in the pension discount yield and Put option is embedded to the Equity.

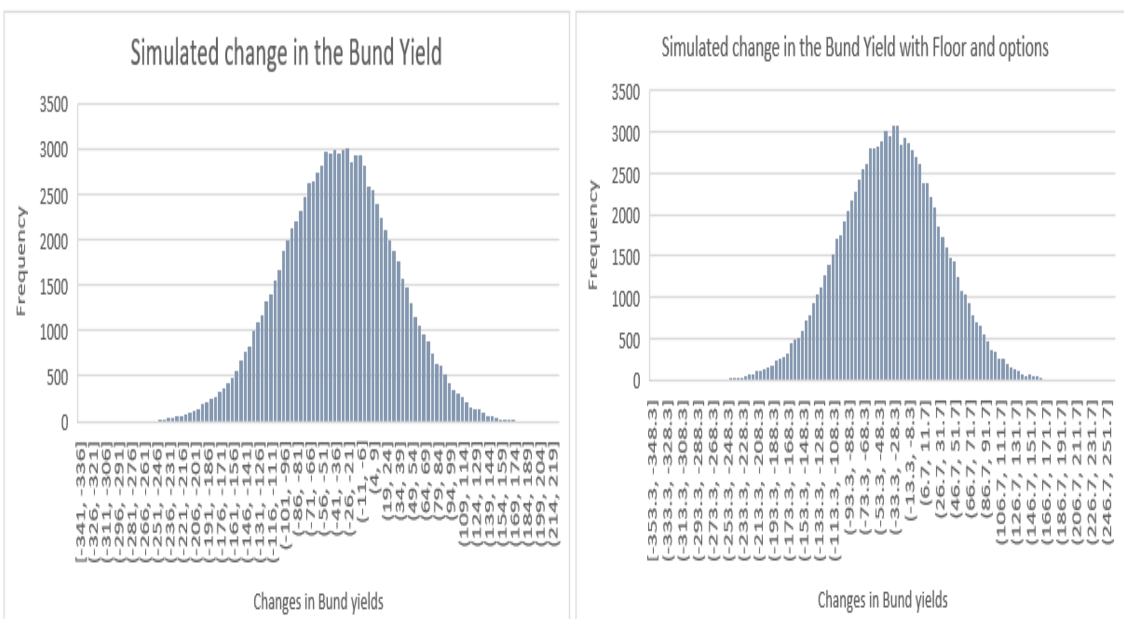



*Figure12: Histograms constructed in Microsoft Excel showcasing the simulated change in the Bund yields.*

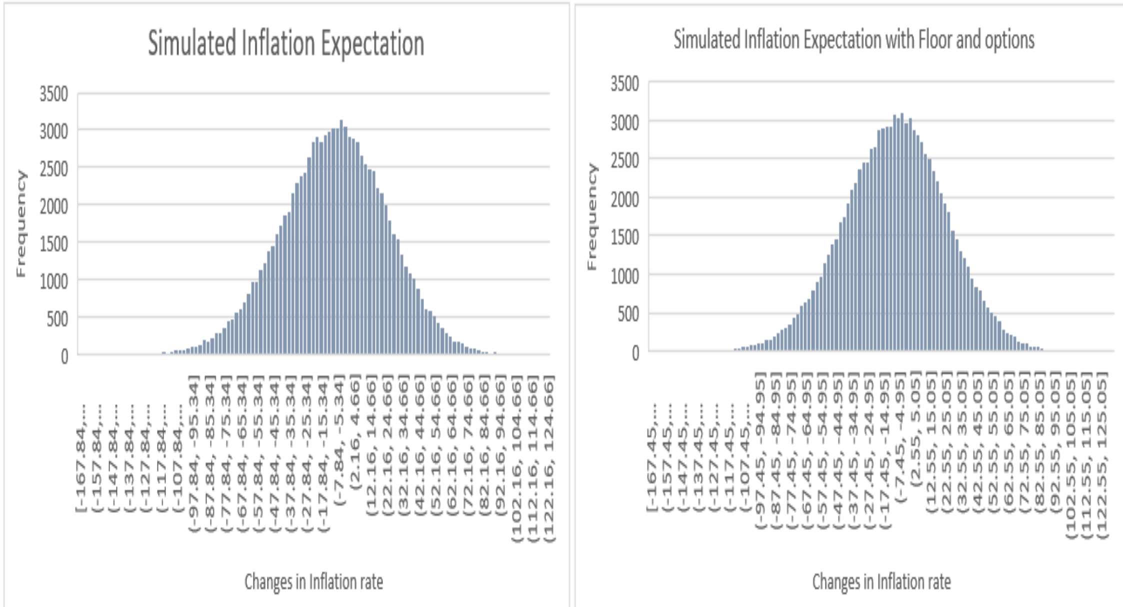



*Figure 13: Histogram constructed in Microsoft Excel showcasing the simulated change in the Inflation rate.*

### 5.2.2. VaR Model Building in RStudio

COMPARISON: VAR COMPUTATION WITHOUT ANY FLOOR AND PUT OPTIONS V/S VAR COMPUTATION WITH FLOOR AND PUT OPTIONS.

#### A. CHANGE IN VALUE OF PENSION SCHEME

- The VaR model built without floor and put option simulates the change in the value of the pension scheme to be € **234.3291** million and that is the amount of money that the pension scheme will lose at 99% confidence level. Whereas, the VaR model constructed with incorporation of the effective floor to the discount yield and Put option to the equity price simulates the change in the value of the pension scheme to be € **229.4298** and that is the amount of money that the pension scheme will lose at 99% confidence level. The VaR value falls because the floor on discount yields and the put option on Equity prices restrict it from falling.
- The Histograms seen below are constructed in RStudio that shows the distribution of the potential changes and the VaR being measure and presented is € **234,566,527** and € **229,504,947** in figure 14 and in figure 15 respectively.

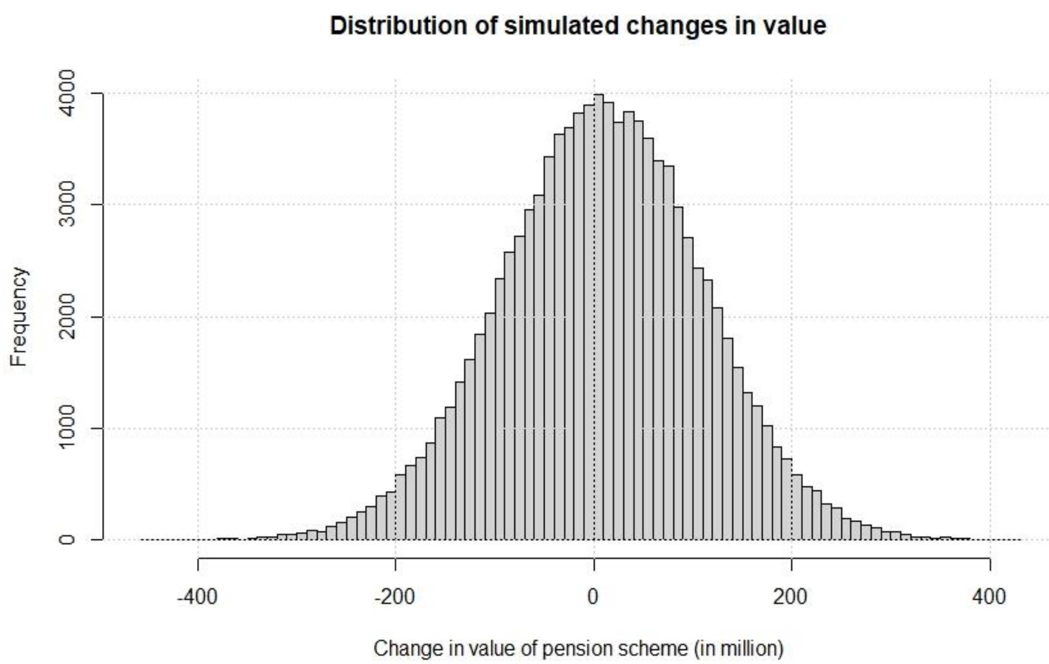



*Figure 14: Histogram constructed in RStudio depicting the simulated change in the value of the pension scheme.*

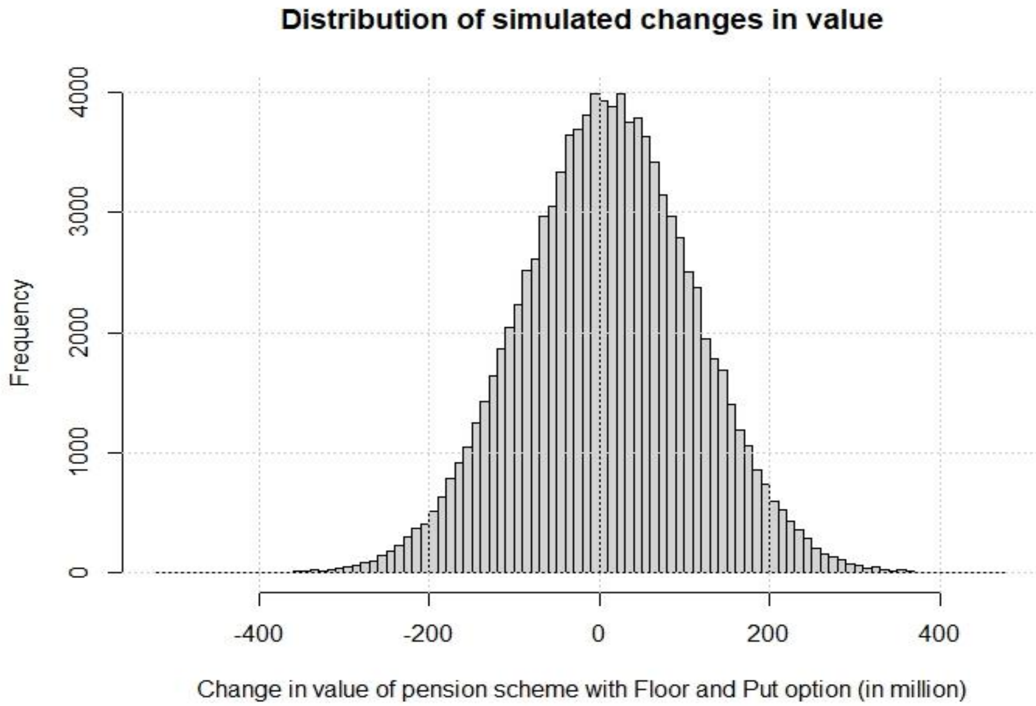



*Figure 15: Histogram constructed in RStudio depicting the distribution of simulated change in the value of the pension scheme with incorporation of floor and put option.*

## B. CHANGE IN DISCOUNT YIELDS

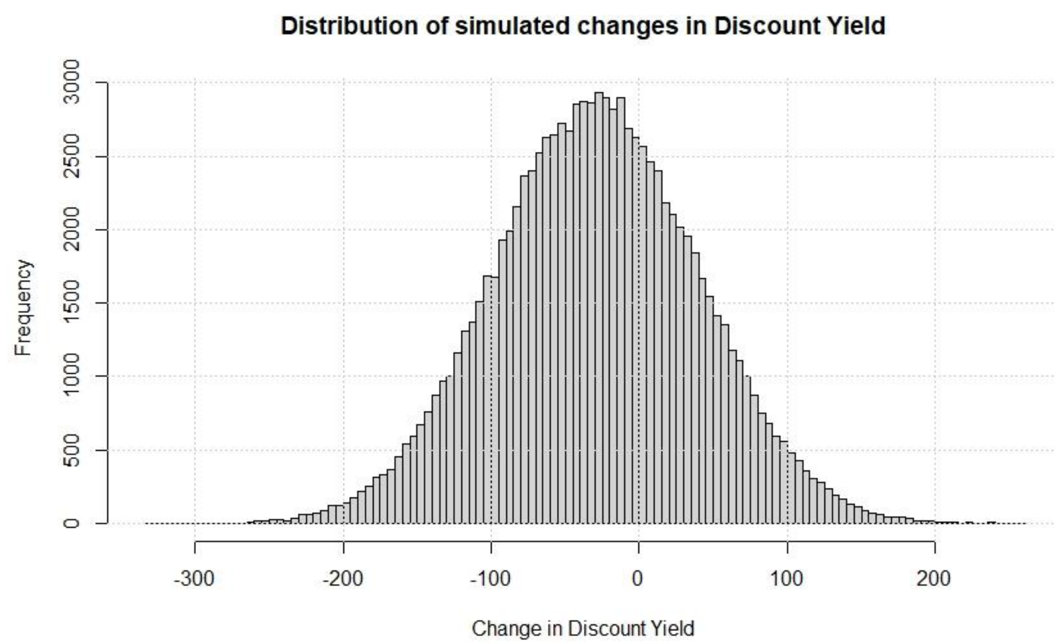



*Figure 16: Histogram depicting the distribution of simulated change in the discount yield without floor.*

The two given figures show the changes in the distribution of the discount yield with and without floor. In the above figure, it is noted that discount rate is not restricted from falling as there is no floor applied to the discount yield and thereby unrestrictedly the discount rate falls to -321.13 basis points. On contrary the figure below reflects bunching on the left edge, which is because, by apply floor of -200 basis points on the discount yield the author restricts the discount yield from falling below -200 basis points.

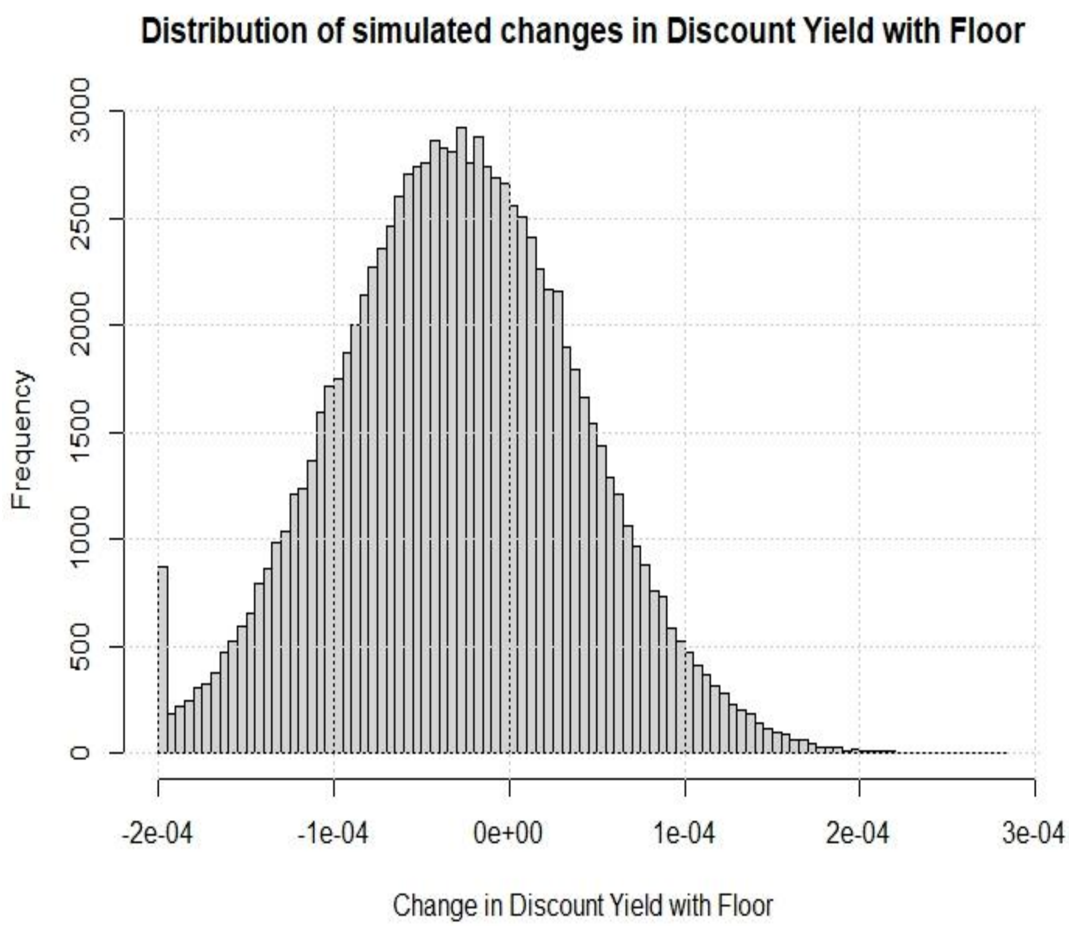



*Figure 17: Histogram constructed in RStudio depicting the simulated change in the discount yield with floor of -200 basis points.*

## C. C HANGE IN EQUITY PRICES

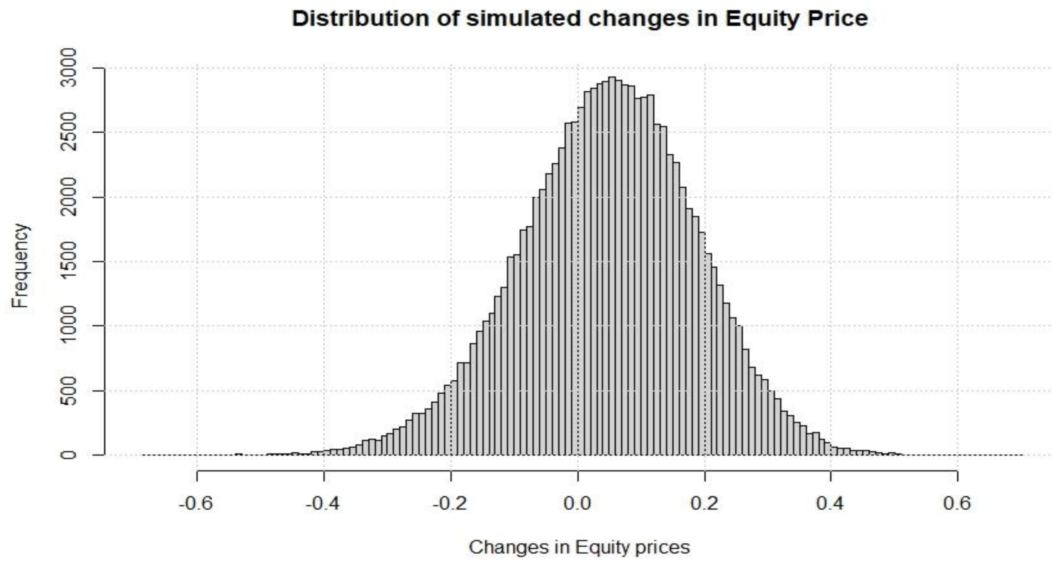



*Figure 18: Histogram depicting the simulated change in the equity price without Put option .*

The two given figures show the changes in the distribution of the equity price with and without put option. In the above figure, it is noted that no put option has been incorporate to the equity price. Whereas the figure given below reflects bunching on the left edge which is around 25% and the same is because the put option with strike of 75% has been applied which does not allow the equity price to fall below 25%.

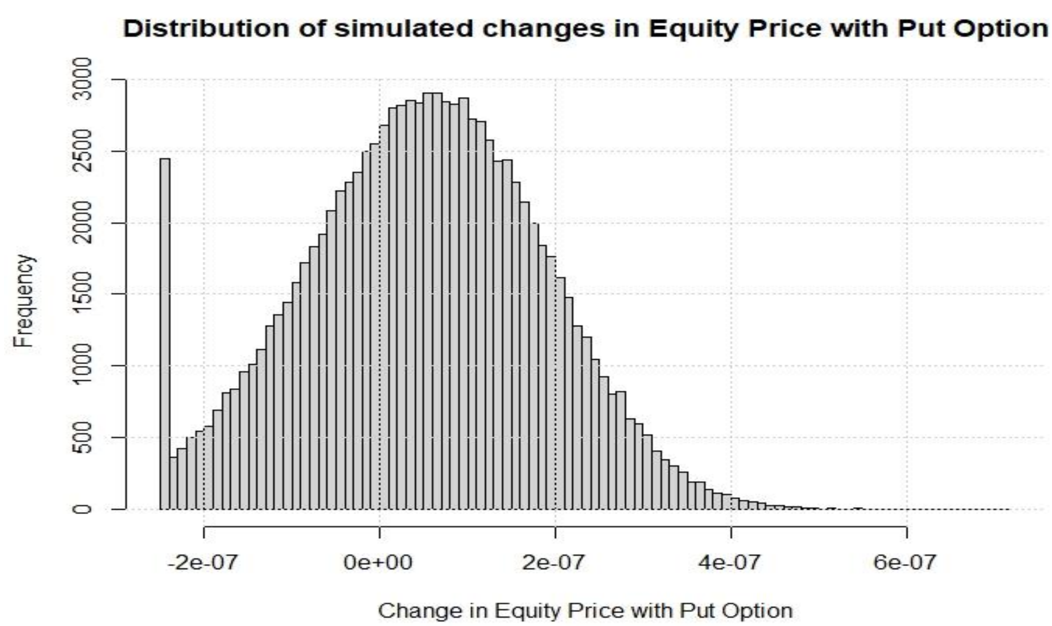



*Figure 19: Histogram depicting the simulated change in the equity price with Put option having strike of 75%.*

## D. CHANGE IN BUND YIELDS

The figures below show that there is almost no change in the distribution of the Bund yield when floor is incorporated in the pension discount yield and Put option is embedded to the Equity.

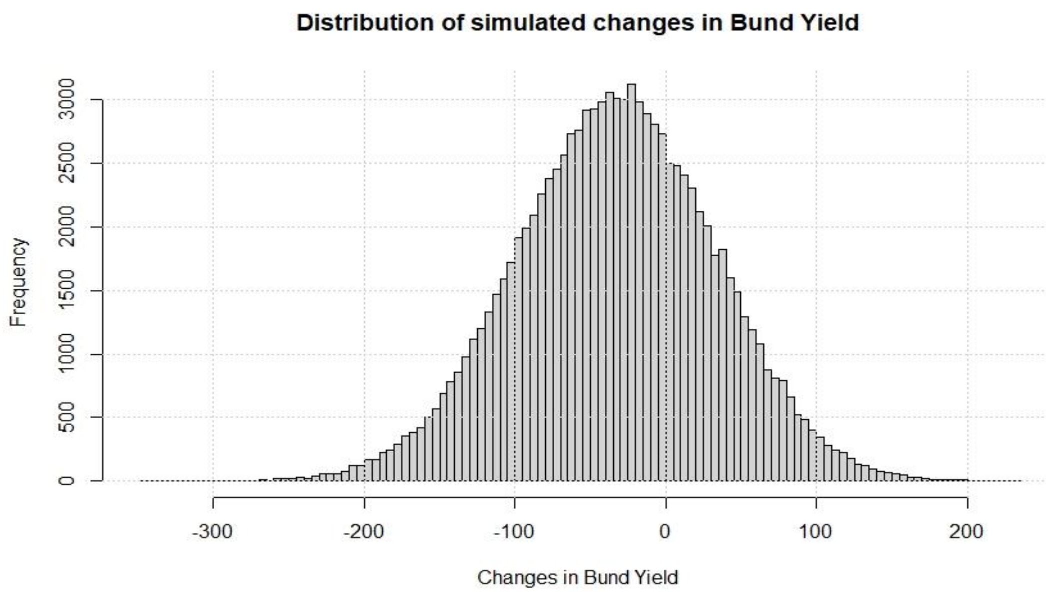



*Figure 20: Histogram constructed in RStudio showcasing the simulated change in the Bund yields.*

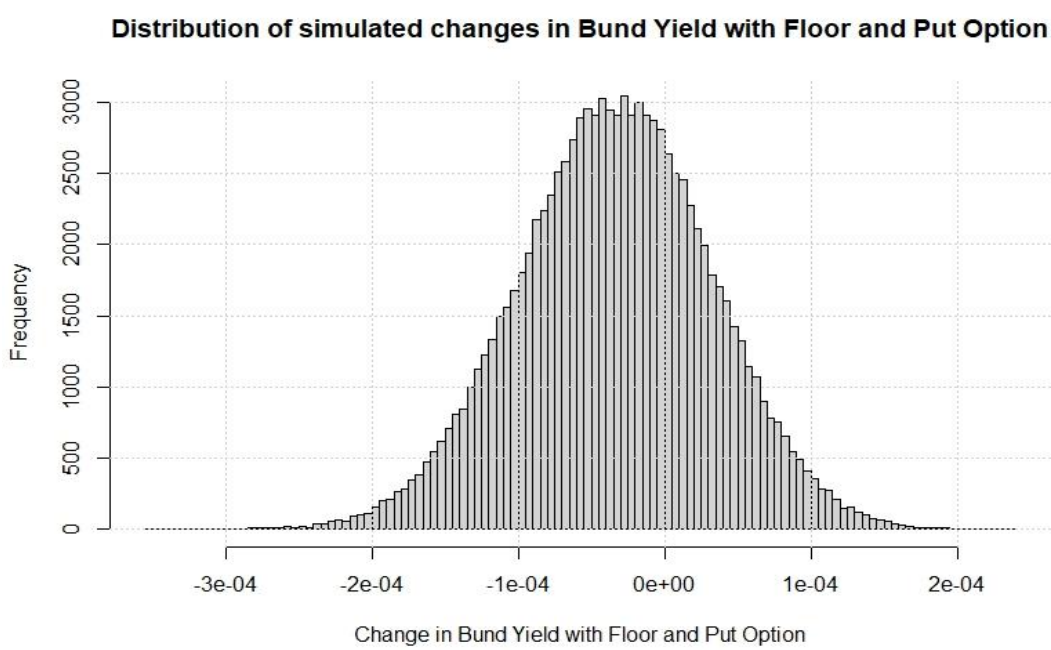



*Figure 21: Histogram constructed in RStudio showcasing the simulated change in the Bund yields.*

## E. CHANGE IN INFLATION RATE

The figures below show that there is almost no change in the distribution of the Inflation rate when floor is incorporated in the pension discount yield and Put option is embedded to the Equity.

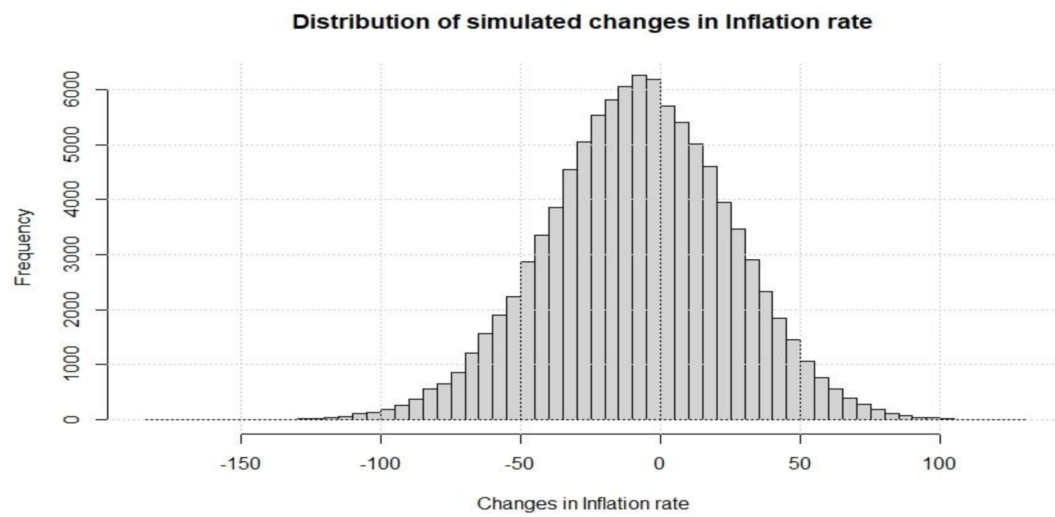



*Figure 22: Histogram showcasing the simulated change in the Inflation rate without floor or put option.*

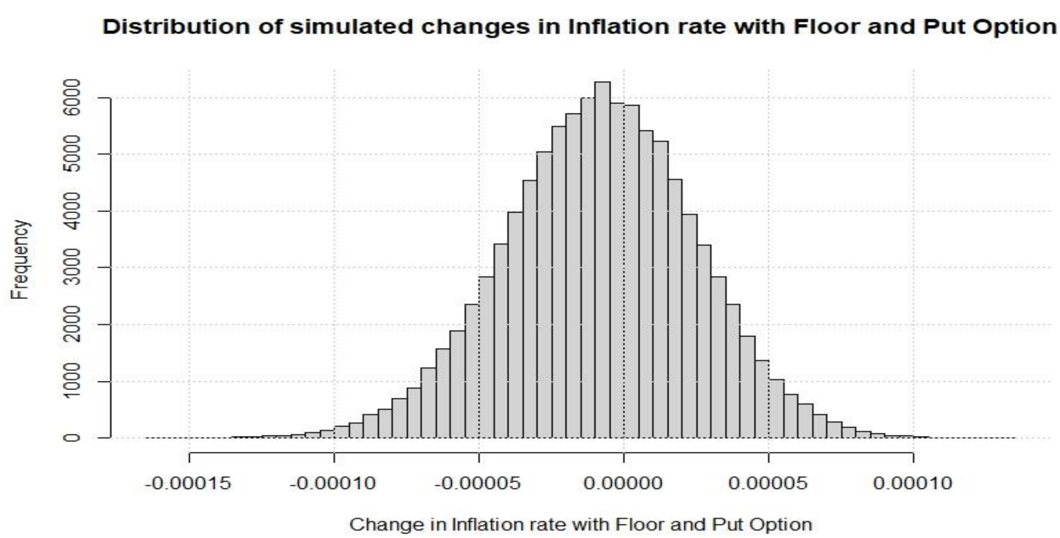



*Figure 23: Histogram showcasing the simulated change in the Inflation rate with floor and option.*

## 5.3 BACKTESTING

### Results of F-test:

The hypothesis that is being tested is:

$H_0$: $\sigma^2_1 = \sigma^2_2$

$H_a: \sigma^2_1 \neq \sigma^2_2$

$\alpha = 5\%$

| F Value | F Critical one-tail value | Result |
|---|---|---|
| 1.35264 | 1.448438 | Fail to reject the null hypothesis |
| 1.20559 | 1.448438 | Fail to reject the null hypothesis |
| 2.35626 | 1.448438 | Reject the null hypothesis |
| 1.73568 | 1.448438 | Reject the null hypothesis |

*Table: 1 – F-test result*

Based on the above computations it can be concluded that not all the variances are statistically constant through time based on the F-test results.

The results above show a potential weakness of the backtest model. The backtesting model implicitly assumes that the risk factors are independent identically distributed random variables and the same form the basis on which the use of the square root of time can be made. Now, the variables are proven to be independent but not identically distributed because they vary through time resulting from the feature of the financial time series.

The VaR model built by the author is a 1-year VaR model and the same is produced at a 99% confidence level which means the expected waiting time to observe an exceedance is a 100 year which is quite unreasonable and not practical so instead the researcher has opted to use a semi-heuristic approach to backtesting whereby the author backtests a related model and the related model opted uses the one month VaR which is consistent with the bootstrapping model.

Backtesting in Microsoft Excel:

### Results of Z-test:

The hypothesis that is being tested is:

$H_0$: Number of observed exceedance = Number of expected exceedance

$H_a$: Number of observed exceedance ≠ Number of expected exceedance

Results of Z-Test in Microsoft Excel

| Incorporating Floor and Put Option | | | |
|---|---|---|---|
| Backtesting - confidence level | 95% | 97.5% | 99% |
| Observed Exceedences | 2 | 1 | 1 |
| expected | 5 | 2.5 | 1 |
| sd | 2.18 | 1.56 | 0.99 |
| z Value | **-1.37649** | **-0.96077** | **0** |
| z Critical Value | **1.65** | **1.96** | **2.33** |

Table 2: Z-test result

Results of Z-Test in Microsoft Excel

| Without Incorporating Floor and Put Option | | | |
|---|---|---|---|
| Backtesting - confidence level | 95% | 97.5% | 99% |
| Observed Exceedences | 2 | 1 | 1 |
| expected | 5 | 2.5 | 1 |
| sd | 2.18 | 1.56 | 0.99 |
| z Value | **-1.37649** | **-0.96077** | **0** |
| z Critical Value | **1.65** | **1.96** | **2.33** |

Table 3: Z-test result

Based on the above results it can be concluded that the model passes the backtest because in all the cases the z value is less than the z critical value. Thereby, we fail to reject the null hypothesis

Z-test Backtesting results plotted - When incorporating floor and option:

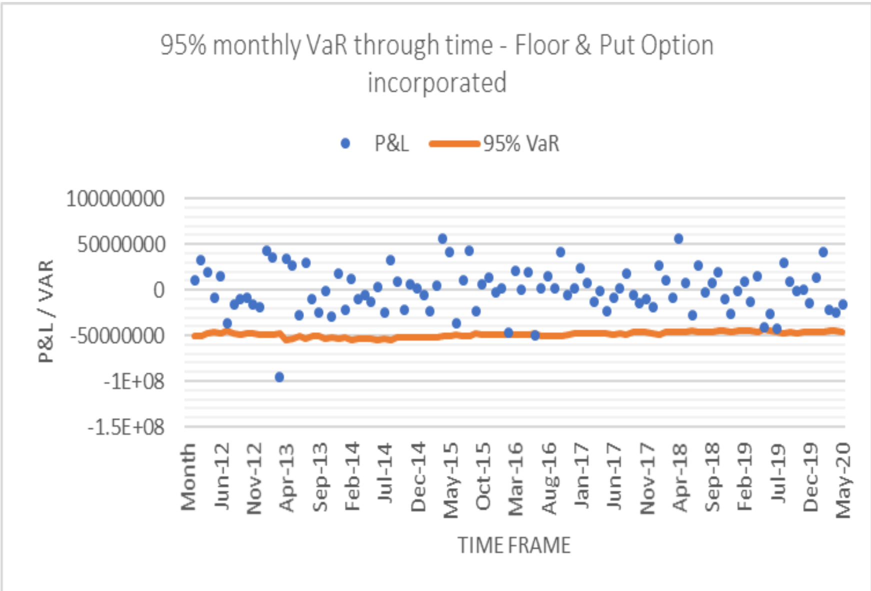



*Figure 24: Backtesting – 95% Confidence level – With floor and option*

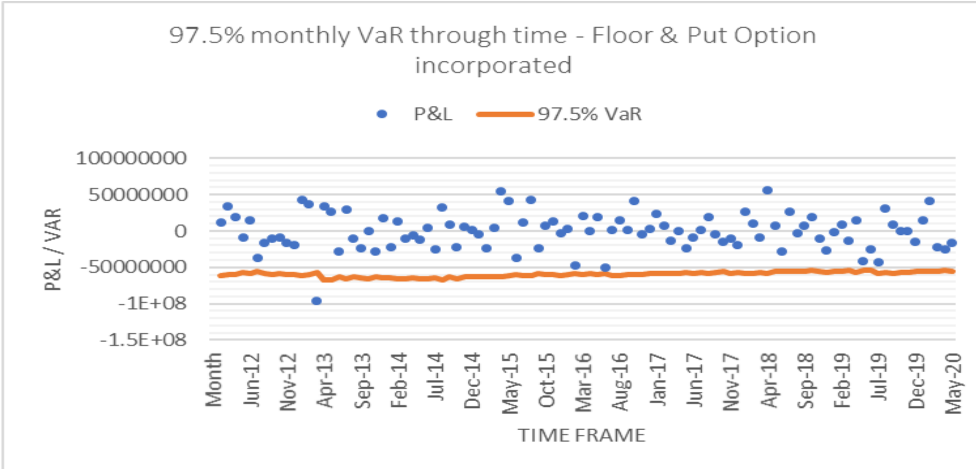



*Figure 25: Backtesting – 97.5% Confidence level – With floor and option*

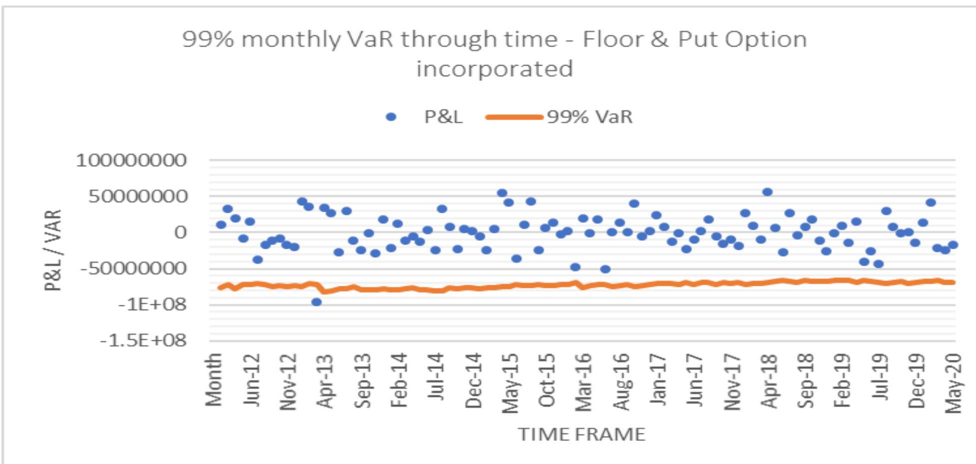



*Figure 26: Backtesting – 99% Confidence level – With floor and option*

Z-test Backtesting results plotted - without floor and option:

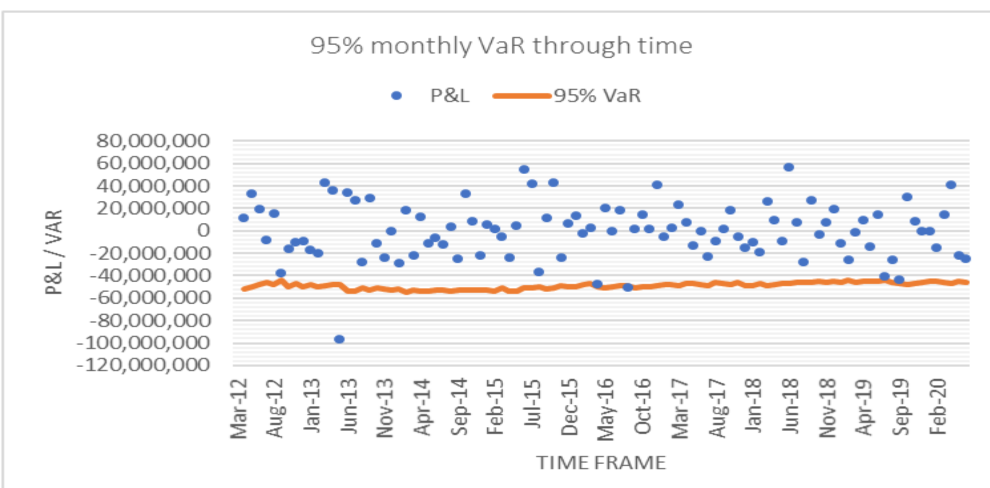



*Figure 27: Backtesting – 95% Confidence level – Without floor and option*

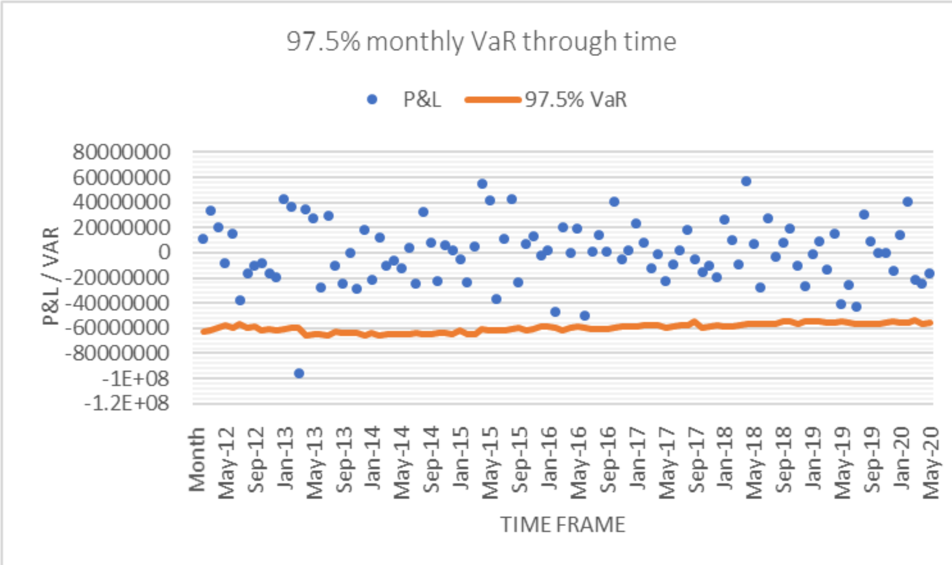



*Figure 28: Backtesting – 97.5% Confidence level – Without floor and option*

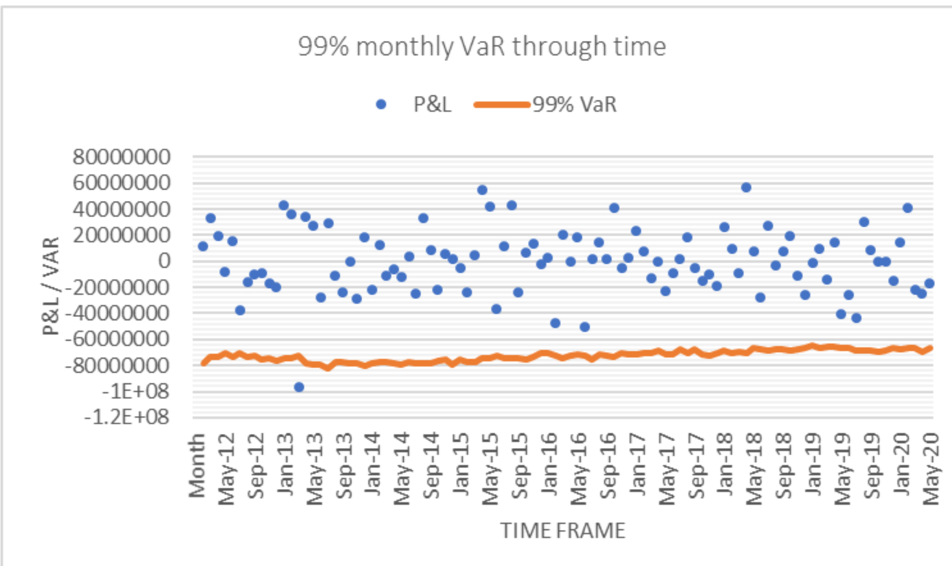



*Figure 29: Backtesting – 99% Confidence level – Without floor and option*

## Results of Kupiec test:

The hypothesis that is being tested is:

$H_0$: proportion of observed exceedance = proportion of expected exceedance

$H_a$: proportion of observed exceedance ≠ proportion of expected exceedance

Results of Kupiec Test in Microsoft Excel

| Incorporating Floor and Put Option | | | |
|---|---|---|---|
| VaR - Confidence level | 95% | 97.5% | 99% |
| Backtesting - confidence level | 99% | 99% | 99% |
| Kupiec Statistic | **2.428592** | **1.190378** | **0** |
| Kupiec Critical Value | **6.634897** | **6.634897** | **6.634897** |

Table 4: Kupiec test result

Results of Kupiec in Microsoft Excel

| Without Incorporating Floor and Put Option | | | |
|---|---|---|---|
| VaR - Confidence level | 95% | 97.5% | 99% |
| Backtesting - confidence level | 99% | 99% | 99% |
| Kupiec Statistic | **0.976859** | **1.190378** | **0** |
| Kupiec Critical Value | **6.634897** | **6.634897** | **6.634897** |

Table 5: Kupiec test result

Based on the above results it can be concluded that the model is passing the backtest as in all the cases the Kupiec statistic is less than the Kupiec critical value. Thereby, we fail to reject the null hypothesis.

Backtesting in RStudio:

The hypothesis that is being tested in Z-test is:

$H_0$: Number of observed exceedance = Number of expected exceedance

$H_a$: Number of observed exceedance ≠ Number of expected exceedance

The hypothesis that is being tested in Kupiec test is:

$H_0$: proportion of observed exceedance = proportion of expected exceedance

$H_a$: proportion of observed exceedance ≠ proportion of expected exceedance

The confidence level of respective test remains the same as applied while computing the tests in Microsoft Excel.

The results of the tests:

| Results of Z-Test and Kupiec test in RStudio | | | |
|---|---|---|---|
| Incorporating Floor and Put Option | | | |
| VaR_Model | Statistic | Value | Critical Values |
| 95% VaR | Num Exceedences | 2.0000000 | 5 |
| 95% VaR | Z Statistic | -1.3764944 | 1.65 |
| 95% VaR | Kupiec Statistic | 2.4285921 | 6.634897 |
| 97.5% VaR | Num Exceedences | 1.0000000 | 2.5 |
| 97.5% VaR | Z Statistic | -0.9607689 | 1.96 |
| 97.5% VaR | Kupiec Statistic | 1.1903780 | 6.634897 |
| 99% VaR | Num Exceedences | 1.0000000 | 1 |
| 99% VaR | Z Statistic | -1.3764944 | 2.33 |
| 99% VaR | Kupiec Statistic | 2.4285921 | 6.634897 |

Table 6: Backtest result in RStudio

| Results of Z-Test and Kupiec test in RStudio | | | | |
|---|---|---|---|---|
| Without Incorporating Floor and Put Option | | | | |
| | VaR_Model | Statistic | Value | Critical Values |
| 1 | 95% VaR | Num Exceedences | 1.0000000 | 5 |
| 2 | 95% VaR | Z Statistic | -1.8353259 | 1.65 |
| 3 | 95% VaR | Kupiec Statistic | 4.9472300 | 6.634897 |
| 4 | 97.5% VaR | Num Exceedences | 1.0000000 | 2.5 |
| 5 | 97.5% VaR | Z Statistic | -0.9607689 | 1.96 |
| 6 | 97.5% VaR | Kupiec Statistic | 1.1903780 | 6.634897 |
| 7 | 99% VaR | Num Exceedences | 1.0000000 | 1 |
| 8 | 99% VaR | Z Statistic | -1.8353259 | 2.33 |
| 9 | 99% VaR | Kupiec Statistic | 4.9472300 | 6.634897 |

Table 7: Backtest result in RStudio

The results as seen from the table above state that in all the cases the critical value is greater than the respective Z value or the Kupiec statistic value which means that the model has passed the backtest and in all cases researcher fails to reject the null hypothesis.

To conclude, the author has performed backtesting of a related model and the related model is the one-month VaR model which is consistent with the bootstrapping method in modelling. It is valid to extrapolate the backtest of one-month model to a one-year model if the risk factors are i.i.d random variables but it is untrue in case of this study. However, though the risk factors are not i.i.d random variables still the fact remains that the related VaR model did pass its backtest and therefore is a valid and accurate model.

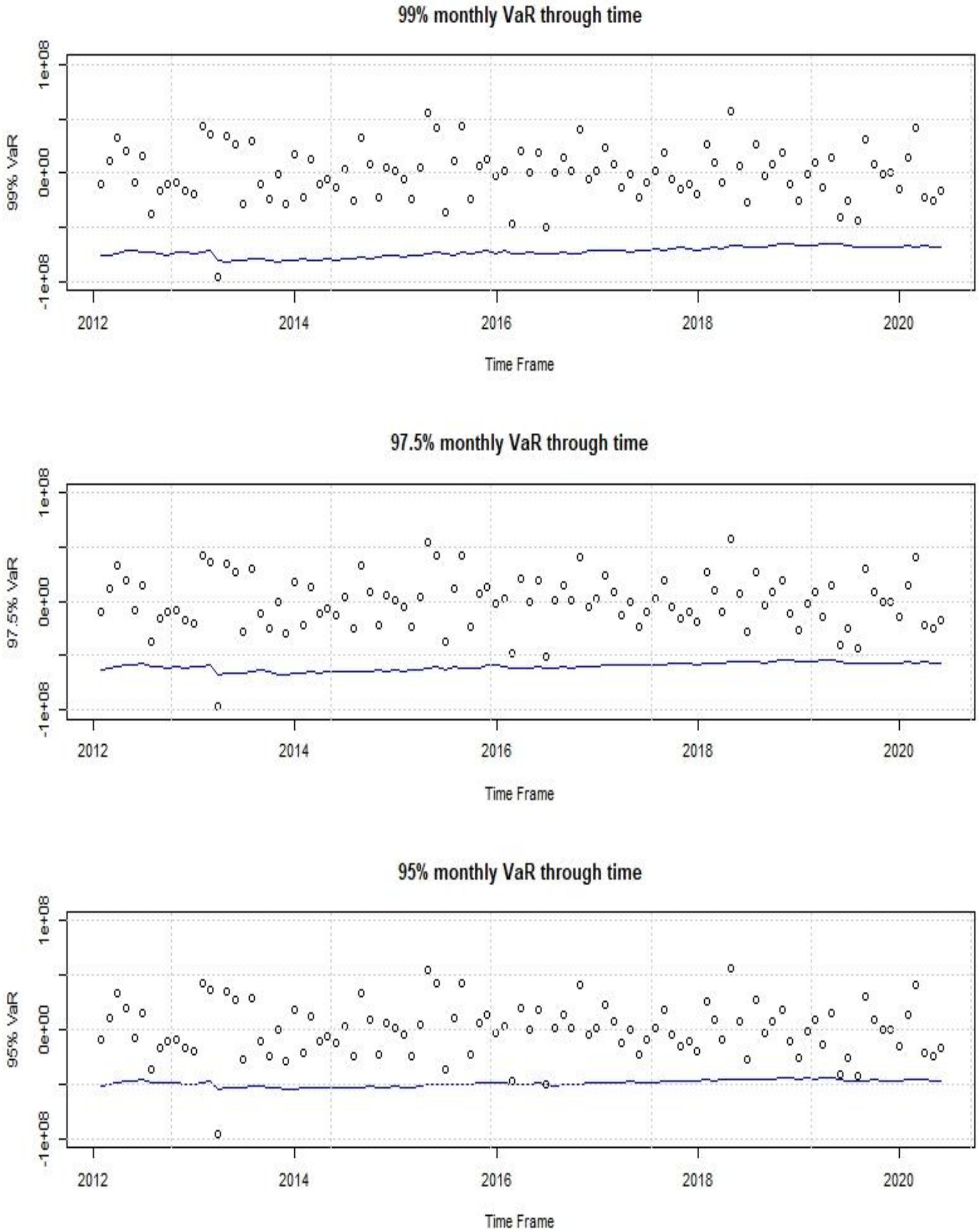



*Figure 30: Backtest Plot generated in RStudio where the line depicts the VaR and set of points representing the hypothetical P&L of the portfolio. (without Floor and Put options).*

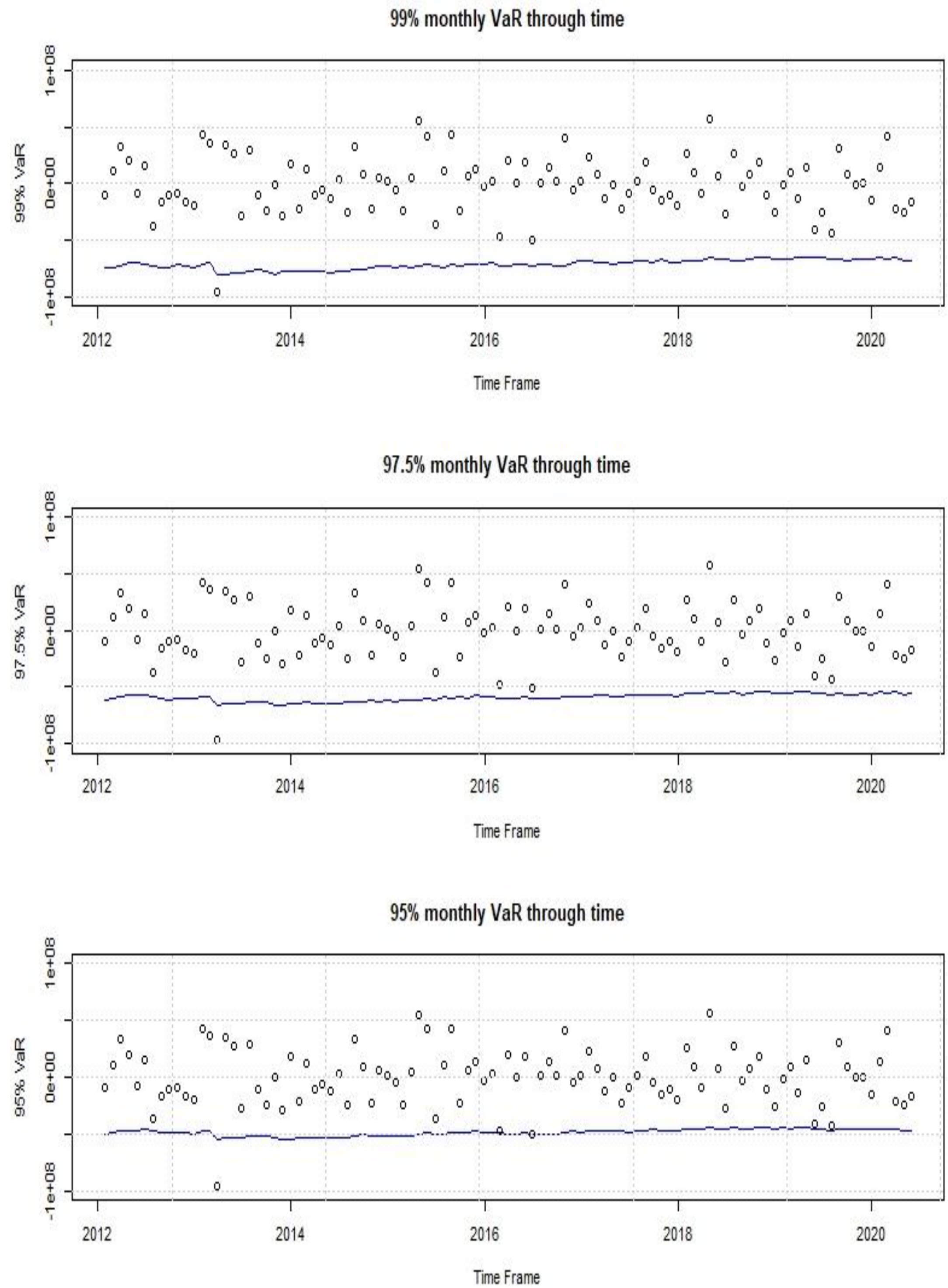



*Figure 31: Backtest Plot generated in RStudio where the line depicts the VaR and set of points representing the hypothetical P&L of the portfolio. (with Floor and Put options).*

# 6 DISCUSSION

The results obtained from the constructed VaR model in form of the potential change in the value of the pension scheme seem quite reasonable and the model used for computation has been effectively backtested and because the model did pass the accuracy test, it can be said that the model is valid. Although the study has accomplished the aim that was put out, which was to attempt at modeling and backtesting the Value at Risk model with a longer time horizon and high confidence level, there exist a range of concerns which have to be addressed so as to recognize the shortcomings around the model constructed and backtested. This section views the limitation of the model as well as the scope for the research to be further expanded.

A particular limitation observed is regarding the technique of computation, while the model is non-parametric and does not assume normality, its projections are strictly constrained by history, the model can only generate simulated rate changes from a combination of rate changes that have happened.

The standard approach used while modeling long horizon risk is to use a GARCH model but the author intends at choosing an alternative to that so overcome the complexity involved with a GARCH model. An issue with the GARCH model is pertaining to the hyper-reactive feature associated with it and by their nature, such a model adapts to sharp changes in market volatility. In this study, if the GARCH model was constructed for pension risk and applied in the year 2020 then there would be a sharp increase would have been highlighted and followed by a sharp decrease accounting to volatility in the month of March. Though this model is suited better as per author, there is scope to explore computation using the GARCH model and the GARCH model might prove superior in one aspect that is GARCH type volatility may deal better with volatility clustering.

The author has restricted the computation of VaR to Microsoft Excel and R statistical programming whereas the same can be extended for computation in MATLAB, SPSS, SAS, and Python and there are past studies which show that effective results can be obtained using other statistical programming tools (Gonzalez, 2020; Preacher and Hayes, 2004).

The backtest model constructed by the author is based on two backtesting methods: Z-test and Kupiec test. Backtest models like traffic light approach, independence test – Christoffersen test, and Joint test – Christoffersen's interval forecast test can be used to backtest the accuracy of the model.

Further research can be directed towards the development of a more robust backtesting technique. The failing of the F-test conveyed that the variances are not constant through time. A robust backtesting approach will engage in correcting and adjusting such that the variance is constant through time and that there are independent identically distributed variables. The author has researched producing 1-year projection on the basis of monthly VaR projections and future study can be undertaken pertaining to examination of the square root of time extension of daily/ weekly VaR projections to produce 1-year projection. Lastly, an examination of the use of overlapping data to estimate the standard deviation and correlation coefficients of the changes in the market rate can be explored.

# 7 CONCLUSION

In this paper, the author demonstrates an effective method for computing VaR which in today's time is gaining popularity as a risk measure for the long-term horizon, in general, and pension schemes, in particular. This thesis aimed to address an issue from the industry by constructing a VaR model that is Valid and provides accurate results. The first aspect of the study which is model building is effectively achieved by the author by constructing a Monte Carlo Simulation VaR model using the non-parametric bootstrapping to compute the change in the value of the pension scheme. As desired, the model is successfully constructed in both Microsoft Excel and RStudio. The results of the backtest are treated as invaluable feedback regarding the validity and accuracy of the built model. The author attempted to backtest the model by applying 2 backtesting techniques: Z-test and the Kupiec test. The results of the backtest were positive even at different confidence levels which proves that the model constructed is an accurate one and can be put to use in practical and industry scenarios.

Overall, the model is fit for the purpose based on the statistical test performed by the author.

The study leaves avenues for further investigation which if explored would complement this research. The potential scope for further research: first, the author detects a shortcoming in the backtest assumption about independent identically distributed random variables. The author successfully proves that variables are independent and not identically distributed even though the model built passes the backtest. Future studies can engage in correcting or adjusting for the identical distribution of the random variables. Second, apart from the backtest methods applied in the study, various other backtesting techniques can be explored in a similar context.

# REFERENCES


Abbasi, B. and Guillen, M. (2013) 'Bootstrap control charts in monitoring value at risk in insurance', *Expert Systems with Applications, 40*(15), pp.6125-6135.

Albrecht, P., Bährle, H.F. and König, A. (1996) 'Value-at-Risk: A Risk Theoretical Perspective with Focus on Applications in Insurance', *Manuskript/Institut für Versicherungswissenschaft, Universität Mannheim, 90*, pp.3-24.

Alexander, C. (2008) *Value-At-Risk Models*. 4th ed. Chichester: Wiley, pp.13 - 245.

Alexander, G.J. and Baptista, A.M. (2008) 'Active portfolio management with benchmarking: Adding a value-at-risk constraint', *Journal of Economic Dynamics and Control*, *32*(3), pp.779-820.

Antolín, P., Blome, S., Karim, D., Payet, S., Scheuenstuhl, G., Yermo, J. (2009) 'Investment Regulations and Defined Contribution Pensions*', OECD Working Papers on Insurance and Private Pensions*, No. 37, OECD publishing, © OECD. doi:10.1787/222771401034

Antonelli, S. and Iovino, M. G. (2002) 'Optimization of Monte Carlo procedures for Value at Risk estimates', *Economic Notes*, 31(1), pp. 59-78.

Basu, A.K. and Drew, M.E. (2007) 'Appropriateness of Default Investment Options in Pension Plans: The Australian Evidence', *7th Global Conference on Business & Economics*.

Beder, T. S. (1995) 'VAR: Seductive but dangerous', *Financial Analysts Journal*, 51(5), pp.12-24.

Berrar, D. (2019) 'Introduction to the non-parametric bootstrap', *Encyclopedia of Bioinformatics and Computational Biology*, pp.766-773.

Berstein, S. M. and Chumacero, R. A. (2010) 'VaR limits for pension funds: an evaluation', *Quantitative Finance*, 12(9), pp.1315-1324.

Biadoo Jnr, M., Andoh, C. and Bokpin, G. (2019) 'Optimal asset allocation of defined contribution pension funds in Ghana', *Journal of Global Business and Technology*, 15(2), pp.17 - 29.

Brown, A. (2008) 'Private profits and socialized risk–Counterpoint: Capital Inadequacy', *Global Association of Risk Professionals.*

Bryman, A. (2012) '*Social research method'*.

Campbell, S. (2005) 'A Review of Backtesting and Backtesting Procedures', *Finance and Economics Discussion Series*, 2005(21), pp.1-23.

Chen, Y., Sun, X. and Li, J. (2017) 'Pension fund asset allocation: A mean variance model with CVaR constraint', *Procedia Computer Science*, 108, pp. 1302-1307.

Coronado, M. (2000) 'Comparing Different Methods for Estimating Value-at-Risk (VaR) for Actual Non-Linear Portfolios: Empirical Evidence', Working paper, *Facultad de Ciencias Economicas y Empresariales, ICADE*.

Damodaran, A. (2007) 'Strategic Risk Taking: A Framework for Risk Management', *Pearson Education*, New Jersey.

Danielsson, J. and De Vries, C. G. (2000) 'Value-at-Risk and extreme returns', *Annales d'Économie et de Statistique*, (60), pp.239-270.

Dowd, K. (1998) 'Beyond Value at Risk: The new Science of Risk Management', John Wiley & Sons, England.

Dowd, K. (2005) *Measuring Market Risk,* John Wiley & Sons.

Dowd, K., Blake, D. and Cairns, A. (2004) 'Long-term Value at Risk', *The Journal of Risk Finance*, 5(2), pp.52-57.

Efron, B. (1979) 'Bootstrap Methods: Another Look at the Jackknife', *The Annals of Statistics,* 7(1), pp.1-26.

Efron, B. (2000) 'The Bootstrap and Modern Statistics', *Journal of the American Statistical Association,* 95(452), pp.1293-1296.

Einhorn, D. and Brown, A. (2008) 'Private profits and socialized risk', *Global Association of Risk Professionals, 42,* pp.10-26.

Estrella, A., Hendricks, D., Kambhu, J., Shin, S. and Walter, S. (1994) 'The price risk of options positions: Measurement and capital requirements', *Federal Reserve Bank of New York Quarterly Review*, 19(2), pp. 27-43.

Glyn, A. (2002) 'History of Value-at-Risk: 1922-1998', *Working papers*.

Gonzalez, J. (2020) 'A Stochastic Approach to Portfolio Optimization Using Competing Risk Metrics'.

Gnamassou, Y. T. (2010) 'Value at Risk Prediction: A comparison of Alternative Techniques Applied to a Large Sample of Individual Stock Data', Paper, *HEC Montreal*.

Halilbegovic, S. and Vehabovic, M. (2016) 'Backtesting Value at Risk forecast: The case of Kupiec Pof-Test', *European Journal of Economic Studies*, 17(3), pp. 393-404.

Holton, G. A. (2002) '*History of Value-at-Risk: 1922-1998*', Available at: https://econwpa.ub.uni-muenchen.de/econ-wp/mhet/papers/0207/0207001.pdf

Hull, J. (2006) *Options, futures, And other derivatives*. 6th edn. New Jersey: Pearson.

Hull, J. (2015) *Options, futures, And other derivatives*. 9th edn. New Jersey: Pearson.

Hull, J. and White, A. (1998) 'Incorporating volatility updating into the historical simulation method for Value-at-Risk', *The Journal of Risk*, 1(1), pp. 5-19.

Jorion, P. (2007) *Value at Risk: The New Benchmark for Managing Financial Risk*. 3rd edn. New York: McGraw-Hill.

Jorion, P. (2003) 'Portfolio optimization with tracking-error constraints', *Financial Analysts Journal*, 59(5), pp. 70-82.

Jorion, P. (2001) *Value at Risk: The New Benchmark for Managing Financial Risk*. 2nd ed. New York: McGraw-Hill.

Katsenga, G.Z. (2013) *Value at risk (VaR) backtesting - Evidence from a South African market portfolio'*. Doctoral dissertation. Africa: University of Witwatersrand Business School.

Kollar, B. and Kliestik, T. (2014) 'Simulation Approach in Credit Risk Models', *Advances in Education Research*, (51), pp.150-155.

Kupiec, P. (1995) 'Techniques for verifying the accuracy of risk measurement models), *The J. of Derivatives*, *3*(2).

Linsmeier, T.J. and Pearson, N.D. (1996) ‘Risk measurement: An introduction to value at risk’, *University of Illinois at Urbana - Champaign*.

Mancini, L. and Trojani, F. (2011) ‘Robust value at risk prediction’, *Journal of financial econometrics*, *9*(2), pp.281-313.

Markowitz, H. M. (1952) ‘Portfolio selection’, *Journal of Finance*, *7*(1), pp.77-91. p.5-16.

Micocci, M., Gregoriou, G. and Masala, G. (2010) *Pension Fund Risk Management: Financial and actuarial modelling*. Boca Raton: Chapman & Hall/CRC.

Nieppola, O. (2009) *Backtesting Value-at-Risk Models*. MSc Thesis. Finland: Helsinki School of Economics -Department of Economics. Available at: https://pdfs.semanticscholar.org/5fbb/f20d7f3e03c4da3a9d33aa618fdfe5d0a9d4.pdf [Accessed 15 June 2020]

Payne, G. & Payne, J. (2004) ‘*Key Concepts in social Research’*.

Preacher, K.J. and Hayes, A.F. (2004) ‘SPSS and SAS procedures for estimating indirect effects in simple mediation models’, *Behavior research methods, instruments, & computers*, *36*(4), pp.717-731.

Rahman, M. (2016) ‘The Advantages and Disadvantages of Using Qualitative and Quantitative Approaches and Methods in Language’, “Testing and Assessment” Research: A Literature Review. *Journal of Education and Learning*, 6(1).

Raychaudhuri, S. (2008) ‘Introduction to Monte Carlo Simulation’, *Proceedings of the 2008 Winter Simulation Conference*.

Roy, A.D. (1952) ‘Safety first and the holding of assets’, *Econometrica: Journal of the econometric society*, pp.431-449.

Sharma, M. (2012) ‘The historical simulation method for Value-at-Risk: A research-based evaluation of the industry favorite’, *SSRN Electronic Journal*.

Srinivasan, A. and Shah, A. (2001) ‘Improved techniques for using Monte Carlo in VaR estimation’, University of California, Santa Barbara. IGIDR Bombay,(1-30), p.3.

Turner J.A. and Belier D.J. (1992) ‘Trends in Pensions 1992’, *US Government Printing Office*, Washington.

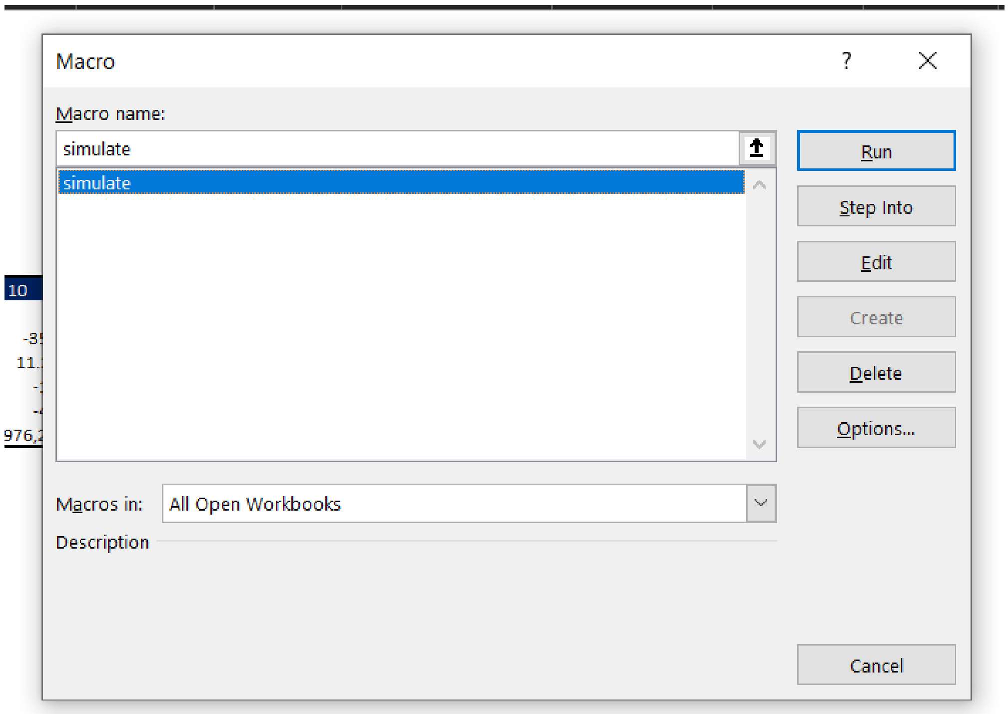
Valášková, K., Spuchľáková, E. and Adamko, P. (2015) ‘Non-parametric Bootstrap Method in Risk Management’, *Procedia Economics and Finance*, 24, pp.701-709.

Watkins, D. and Gioia, D. (2015) ‘Mixed methods research’. *OUP Us*.

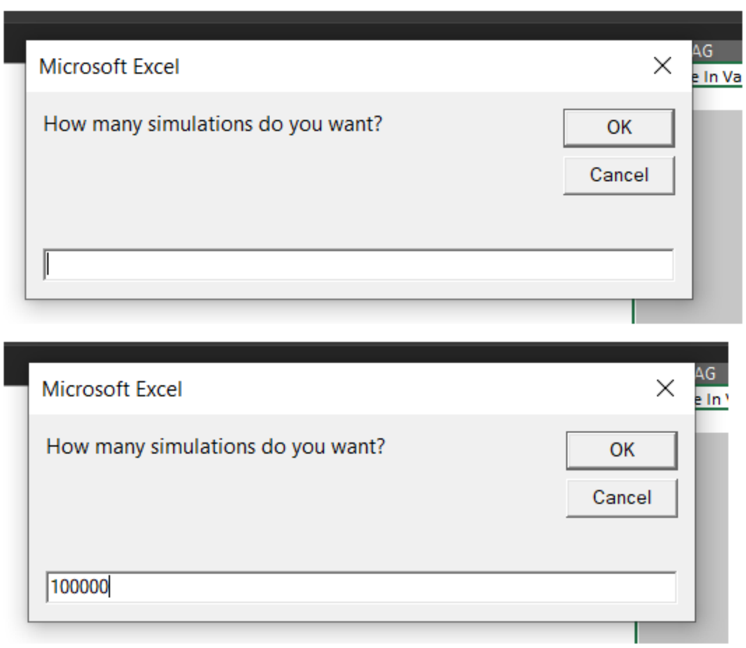
Wong, C.M. and So, M.K. (2003) ‘On conditional moments of GARCH models, with applications to multiple period value at risk estimation’, *Statistica Sinica*, pp.1015-1044.

# APPENDIX

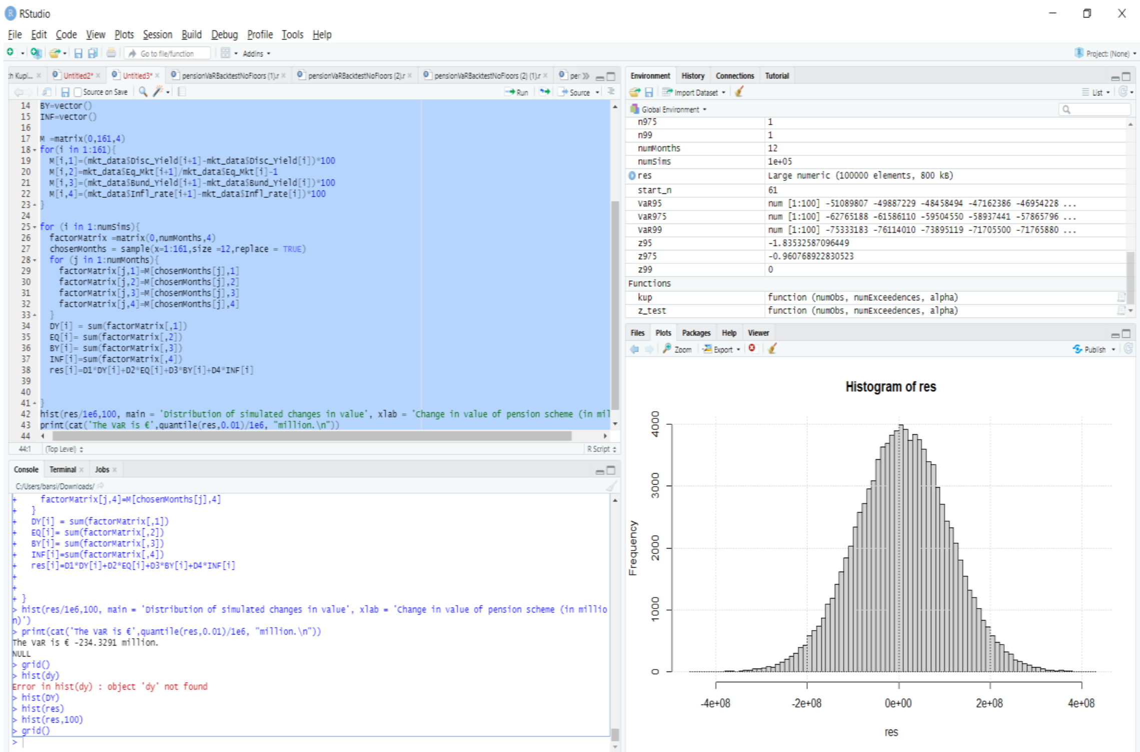
Appendix 1: Snapshot of Pop window 1



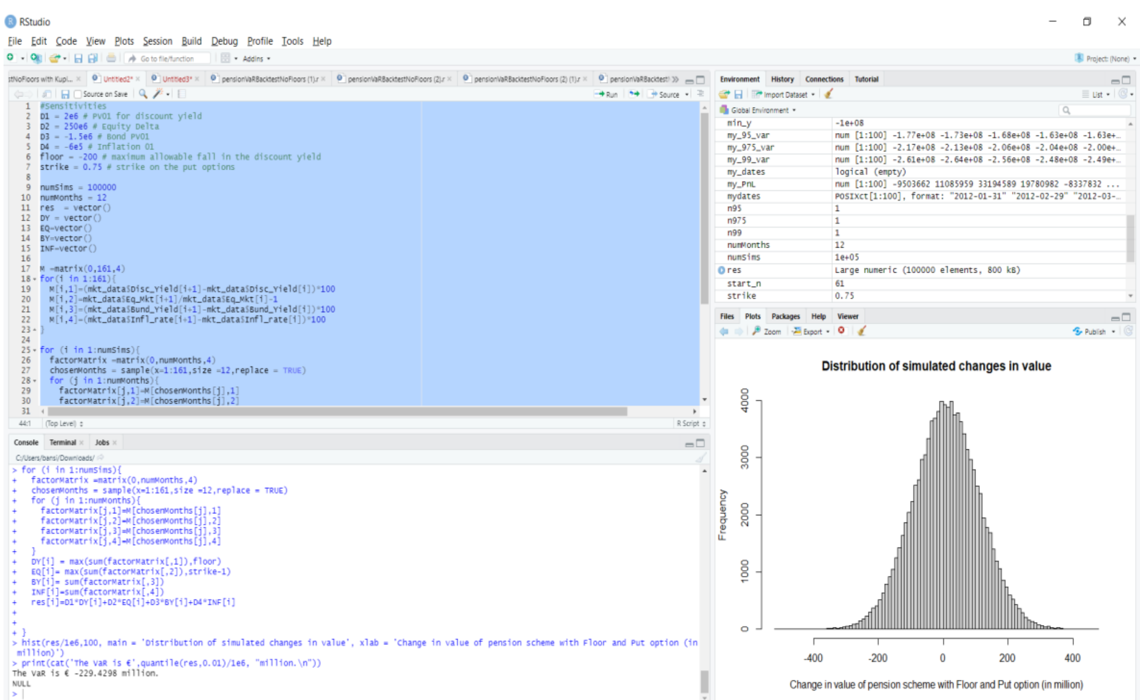

Appendix 2: Snapshot of Pop window 2



Appendix 3: Snapshot of RStudio without Floor and Option




Appendix 4: Snapshot of RStudio with Floor and Option



Appendix 5: VBA code backtesting VaR model – Z-test

```
Sub simulate()
Range("AG2:AK10000").Select
    Selection.ClearContents
    Range("AG2").Select

n = 10000

For i = 1 To n
    ActiveSheet.Calculate
    dValue = ActiveSheet.Range("delta_value").Value
    dDiscYield = ActiveSheet.Range("delta_discount_rate").Value
    dEqValue = ActiveSheet.Range("delta_equity_value ").Value
    dBundYield = ActiveSheet.Range("delta_bund_yield").Value
    dInflation = ActiveSheet.Range("delta_inflation").Value

    ActiveSheet.Range("ag1").Offset(i, 0).Value = dValue
    ActiveSheet.Range("ah1").Offset(i, 0).Value = dDiscYield
    ActiveSheet.Range("ai1").Offset(i, 0).Value = dEqValue
    ActiveSheet.Range("aj1").Offset(i, 0).Value = dBundYield
    ActiveSheet.Range("ak1").Offset(i, 0).Value = dInflation
Next i
End Sub
```

```
Sub accumulateVaRData()
counter = 1
For i = 61 To 161
    ActiveSheet.Range("o8").Value = i
    simulate
    my_99_var = ActiveSheet.Range("ad2").Value
    my_975_var = ActiveSheet.Range("ad3").Value
    my_95_Var = ActiveSheet.Range("ad4").Value
    my_pnl = ActiveSheet.Range("v8").Value
    mydate = ActiveSheet.Range("u8").Value
    ActiveSheet.Range("as1").Offset(counter, 0).Value = my_99_var / Sqr(12)
    ActiveSheet.Range("as1").Offset(counter, 1).Value = my_975_var / Sqr(12)
    ActiveSheet.Range("as1").Offset(counter, 2).Value = my_95_Var / Sqr(12)
    ActiveSheet.Range("as1").Offset(counter, 3).Value = my_pnl
    ActiveSheet.Range("ar1").Offset(counter, 0).Value = mydate
counter = counter + 1
Next i
MsgBox ("Done")
End Sub
```

Appendix 6: Backtesting VaR model in RStudio (following is the R-Command) – Z-test and Kupiec test without floor and options

```
#Sensitivities
D1 = 2e6
D2 = 250e6
D3 = -1.5e6
D4 = -6e5

z_test <- function(numObs,numExceedences,alpha){
  mu <- numObs*alpha
  x <- numExceedences
  sig = sqrt(numObs*alpha*(1-alpha))
  z <- (x-mu)/sig
}
kup <- function(numObs,numExceedences,alpha){
```

```
  Pi_obs <- numExceedences/numObs
  n1 <- numObs-numExceedences
  n0 <- numExceedences
  LL_Null <- log((alpha^n0)*(1-alpha)^n1)
  LL_Alt <- log((Pi_obs^n0)*(1-Pi_obs)^n1)
  kup <- -2*(LL_Null-LL_Alt)
}

numSims = 10000
numMonths = 12
res  = vector()
DY = vector()
EQ=vector()
BY=vector()
INF=vector()
my_99_var <- vector()
my_975_var <- vector()
my_95_var <- vector()
my_PnL <- vector()
my_dates <- vector()

start_n <- 61
end_n <- 160

for (k in start_n:end_n){
  set.seed(42)
  M =matrix(0,k,4)
  for(i in 1:k){
    M[i,1]=(mkt_data$Disc_Yield[i+1]-mkt_data$Disc_Yield[i])*100
    M[i,2]=mkt_data$Eq_Mkt[i+1]/mkt_data$Eq_Mkt[i]-1
    M[i,3]=(mkt_data$Bund_Yield[i+1]-mkt_data$Bund_Yield[i])*100
    M[i,4]=(mkt_data$Infl_rate[i+1]-mkt_data$Infl_rate[i])*100
  }

  for (i in 1:numSims){
    factorMatrix =matrix(0,numMonths,4)
    chosenMonths = sample(x=1:k,size =12,replace = TRUE)
    for (j in 1:numMonths){
      factorMatrix[j,1]=M[chosenMonths[j],1]
      factorMatrix[j,2]=M[chosenMonths[j],2]
      factorMatrix[j,3]=M[chosenMonths[j],3]
      factorMatrix[j,4]=M[chosenMonths[j],4]
    }

    DY[i] = sum(factorMatrix[,1])
    EQ[i]= sum(factorMatrix[,2])
    BY[i]= sum(factorMatrix[,3])
    INF[i]=sum(factorMatrix[,4])
    res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i] #Simulated P&L

  }
    my_99_var[k+1-start_n] <- quantile(res,0.01)
  my_975_var[k+1-start_n] <- quantile(res,0.025)
  my_95_var[k+1-start_n] <- quantile(res,0.05)
  my_PnL[k+1-start_n] <- D1*M[k,1]+D2*M[k,2]+D3*M[k,3]+D4*M[k,4]
}

max_y = max(my_PnL)
max_y = round(max_y,-8)
min_y = min(my_PnL)
min_y = min(my_PnL,min(my_99_var)/sqrt(12))
```

```
min_y = round(min_y,-8)

par(mfrow = c(3,1))
plot(mkt_data$Date[61:160],my_99_var/sqrt(12),main='99% monthly VaR through time',ylim =
c(min_y,max_y),type='l',col='blue', ylab = '99% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()
plot(mkt_data$Date[61:160],my_975_var/sqrt(12),main='97.5% monthly VaR through
time',ylim=c(min_y,max_y),type ='l',col='blue', ylab = '97.5% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()
plot(mkt_data$Date[61:160],my_95_var/sqrt(12),main='95% monthly VaR through time',ylim=c(min_y,max_y),type
='l',col ='blue',ylab = '95% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()

mydates <- mkt_data$Date[61:160]
VaR95 <- my_95_var/sqrt(12)
VaR975 <- my_975_var/sqrt(12)
VaR99 <- my_99_var/sqrt(12)
backtest_data <- data.frame(mydates,my_PnL,VaR95,VaR975,VaR99)

n95 <- 0
n975 <- 0
n99 <- 0
for (i in 1:100){
  if(my_PnL[i] < VaR95[i])
  {n95 <- n95+1}
    if(my_PnL[i] < VaR975[i])
  {n975 <- n975+1}
    if(my_PnL[i] < VaR99[i])
  {n99 <- n99+1}

}

z95 <-  z_test(100,n95,0.05)
k95 <- kup(100,n95,0.05)
z975 <-  z_test(100,n975,0.025)
k975 <- kup(100,n975,0.025)
z99 <-  z_test(100,n99,0.01)
k99 <- kup(100,n99,0.01)
#Print the Backtest Statistics to the
print('95% VaR')
sprintf("Num Exceedences = %1.0f",n95)
sprintf("Z = %f",z95)
sprintf("Kupiec Statistic = %f",k95)
print('975% VaR')
sprintf("Num Exceedences = %1.0f",n975)
sprintf("Z = %f",z975)
sprintf("Kupiec Statistic = %f",k975)
print('99% VaR')
sprintf("Num Exceedences = %1.0f",n99)
sprintf("Z = %f",z99)
sprintf("Kupiec Statistic = %f",k99)

VaR_Model <- vector()
Statistic <- vector()
Value <- vector()
VaR_Model[1] <- '95% VaR'
Statistic[1] <- 'Num Exceedences'
Value[1] <- n95
```

```
VaR_Model[2] <- '95% VaR'
Statistic[2] <- 'Z Statistic'
Value[2] <- z95
VaR_Model[3] <- '95% VaR'
Statistic[3] <- 'Kupiec Statistic'
Value[3] <- k95
VaR_Model[4] <- '97.5% VaR'
Statistic[4] <- 'Num Exceedences'
Value[4] <- n975
VaR_Model[5] <- '97.5% VaR'
Statistic[5] <- 'Z Statistic'
Value[5] <- z975
VaR_Model[6] <- '97.5% VaR'
Statistic[6] <- 'Kupiec Statistic'
Value[6] <- k975
VaR_Model[7] <- '99% VaR'
Statistic[7] <- 'Num Exceedences'
Value[7] <- n99
VaR_Model[8] <- '99% VaR'
Statistic[8] <- 'Z Statistic'
Value[8] <- z95
VaR_Model[9] <- '99% VaR'
Statistic[9] <- 'Kupiec Statistic'
Value[9] <- k95
backTestStatistics <- data.frame(VaR_Model,Statistic,Value)
```

## Appendix 7: Backtesting VaR model in RStudio (following is the R-Command) – Z-test and Kupiec test with floor and options

```
#Sensitivities
D1 = 2e6
D2 = 250e6
D3 = -1.5e6
D4 = -6e5
my_floor = -200
my_strike = 0.75

z_test <- function(numObs,numExceedences,alpha){
  mu <- numObs*alpha
  x <- numExceedences
  sig = sqrt(numObs*alpha*(1-alpha))
  z <- (x-mu)/sig
}
kup <- function(numObs,numExceedences,alpha){
  Pi_obs <- numExceedences/numObs
  n1 <- numObs-numExceedences
  n0 <- numExceedences
  LL_Null <- log((alpha^n0)*(1-alpha)^n1)
  LL_Alt <- log((Pi_obs^n0)*(1-Pi_obs)^n1)
  kup <- -2*(LL_Null-LL_Alt)
}
numSims = 10000
numMonths = 12
res  = vector()
DY = vector()
EQ=vector()
BY=vector()
INF=vector()
my_99_var <- vector()
```

```
my_975_var <- vector()
my_95_var <- vector()
my_PnL <- vector()
my_dates <- vector()

start_n <- 61
end_n <- 160

for (k in start_n:end_n){
 set.seed(42)
 M =matrix(0,k,4)
 for(i in 1:k){
  M[i,1]=(mkt_data$Disc_Yield[i+1]-mkt_data$Disc_Yield[i])*100
  M[i,2]=mkt_data$Eq_Mkt[i+1]/mkt_data$Eq_Mkt[i]-1
  M[i,3]=(mkt_data$Bund_Yield[i+1]-mkt_data$Bund_Yield[i])*100
  M[i,4]=(mkt_data$Infl_rate[i+1]-mkt_data$Infl_rate[i])*100
 }

 for (i in 1:numSims){
  factorMatrix =matrix(0,numMonths,4)
  chosenMonths = sample(x=1:k,size =12,replace = TRUE)
  for (j in 1:numMonths){
   factorMatrix[j,1]=M[chosenMonths[j],1]
   factorMatrix[j,2]=M[chosenMonths[j],2]
   factorMatrix[j,3]=M[chosenMonths[j],3]
   factorMatrix[j,4]=M[chosenMonths[j],4]
  }

  DY[i] = max(sum(factorMatrix[,1]),my_floor)
  EQ[i]= max(sum(factorMatrix[,2]),my_strike-1)
  BY[i]= sum(factorMatrix[,3])
  INF[i]=sum(factorMatrix[,4])
  res[i]=D1*DY[i]+D2*EQ[i]+D3*BY[i]+D4*INF[i] #Simulated P&L
 }
 my_99_var[k+1-start_n] <- quantile(res,0.01)
 my_975_var[k+1-start_n] <- quantile(res,0.025)
 my_95_var[k+1-start_n] <- quantile(res,0.05)
 my_PnL[k+1-start_n] <- D1*M[k,1]+D2*M[k,2]+D3*M[k,3]+D4*M[k,4]
}
max_y = max(my_PnL)
max_y = round(max_y,-8)
min_y = min(my_PnL)
min_y = min(my_PnL,min(my_99_var)/sqrt(12))
min_y = round(min_y,-8)

par(mfrow = c(3,1))
plot(mkt_data$Date[61:160],my_99_var/sqrt(12),main='99% monthly VaR through time',ylim =
c(min_y,max_y),type='l',col='blue', ylab = '99% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()
plot(mkt_data$Date[61:160],my_975_var/sqrt(12),main='97.5% monthly VaR through
time',ylim=c(min_y,max_y),type ='l',col='blue', ylab = '97.5% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()
plot(mkt_data$Date[61:160],my_95_var/sqrt(12),main='95% monthly VaR through time',ylim=c(min_y,max_y),type
='l',col ='blue',ylab = '95% VaR')
points(mkt_data$Date[61:160],my_PnL[1:100])
grid()

mydates <- mkt_data$Date[61:160]
VaR95 <- my_95_var/sqrt(12)
```

```
VaR975 <- my_975_var/sqrt(12)
VaR99 <- my_99_var/sqrt(12)
backtest_data <- data.frame(mydates,my_PnL,VaR95,VaR975,VaR99)

n95 <- 0
n975 <- 0
n99 <- 0
for (i in 1:100){
  if(my_PnL[i] < VaR95[i])
  {n95 <- n95+1}
  if(my_PnL[i] < VaR975[i])
  {n975 <- n975+1}
  if(my_PnL[i] < VaR99[i])
  {n99 <- n99+1}
}

z95 <-  z_test(100,n95,0.05)
k95 <- kup(100,n95,0.05)
z975 <-  z_test(100,n975,0.025)
k975 <- kup(100,n975,0.025)
z99 <-  z_test(100,n99,0.01)
k99 <- kup(100,n99,0.01)
#Print the Backtest Statistics to the
print('95% VaR')
sprintf("Num Exceedences = %1.0f",n95)
sprintf("Z = %f",z95)
sprintf("Kupiec Statistic = %f",k95)
print('975% VaR')
sprintf("Num Exceedences = %1.0f",n975)
sprintf("Z = %f",z975)
sprintf("Kupiec Statistic = %f",k975)
print('99% VaR')
sprintf("Num Exceedences = %1.0f",n99)
sprintf("Z = %f",z99)
sprintf("Kupiec Statistic = %f",k99)

VaR_Model <- vector()
Statistic <- vector()
Value <- vector()
VaR_Model[1] <- '95% VaR'
Statistic[1] <- 'Num Exceedences'
Value[1] <- n95
VaR_Model[2] <- '95% VaR'
Statistic[2] <- 'Z Statistic'
Value[2] <- z95
VaR_Model[3] <- '95% VaR'
Statistic[3] <- 'Kupiec Statistic'
Value[3] <- k95
VaR_Model[4] <- '97.5% VaR'
Statistic[4] <- 'Num Exceedences'
Value[4] <- n975
VaR_Model[5] <- '97.5% VaR'
Statistic[5] <- 'Z Statistic'
Value[5] <- z975
VaR_Model[6] <- '97.5% VaR'
Statistic[6] <- 'Kupiec Statistic'
Value[6] <- k975
VaR_Model[7] <- '99% VaR'
Statistic[7] <- 'Num Exceedences'
Value[7] <- n99
VaR_Model[8] <- '99% VaR'
```

```
Statistic[8] <- 'Z Statistic'
Value[8] <- z95
VaR_Model[9] <- '99% VaR'
Statistic[9] <- 'Kupiec Statistic'
Value[9] <- k95
backTestStatistics <- data.frame(VaR_Model,Statistic,Value)
```