A COMPARATIVE ANALYSIS OF STOCK MARKET VOLATILITY

MSc Research Project
Masters in Fintech

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MSc Project Submission Sheet

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A COMPARATIVE ANALYSIS OF STOCK MARKET VOLATILITY

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Abstract

The importance and estimation of stock market volatility cannot be overemphasized, as it helps in risk management, asset allocation, option pricing and portfolio management, and as such, several attempts have been made by various scholars to build forecasting model that can give accurate predictions of stock market volatility and returns. The primary objective of this study is to compare stock market volatility using the developed stock market index, while the secondary objectives are to investigates the presence of volatility clustering, conditional volatility and leptokurtosis distribution in the stock market index and compare the forecasting ability of symmetry and asymmetry GARCH. The datasets used for this comprises of S & P 500, NSADAQ Composites and DOWJONES covering the period from January, 2015 to June 2019. The symmetry and asymmetry GARCH models adopted for this study are GARCH (1,1), EGARCH (1,1) and GJRGARCH (1,1) and the models were evaluated through Information Criterion such as (AIC), (BIC), (SIC) and (HQIC). The findings, the study reveals that S & P 500, NSADAQ, DOWJONES possesses the same attributes such as high returns, high risk, presence of volatility clustering, serial correlation, leptokurtosis distribution and conditional volatility. The findings also revealed that there is ARCH and GARCH effect in each of the models were positively significant and there exists the presence of leverage or asymmetry effect on S & P 500, NSADAQ Composites and DOWJONES. The study concludes that BIC of and GARCH (1,1) model has the smallest values, and as such, GARCH (1,1) gives the best forecasting ability than EGACRH (1,1) and GJRGACRH (1,1) models.

1.0 Introduction

Among the key concerns of stakeholders in the stock market is the ability to appropriately model and forecast stock market volatility, as it is of key importance in forecasting future returns and risk. The stock market crash of 1987 reportedly triggered massive anxiety for investors which eventually led to a huge investment loss. This incidence undoubtedly led to a major deleveraging in the stock market. Therefore, to come up with appropriate investment and financing decisions including but not limited to asset allocation, risk and portfolio management, asset and options pricing, and hedging strategy, investors and stakeholders to a significant extent would rely on forecasted stock returns, risk and volatilities to avoid huge losses (Bollerslev, Pattona & Quaedvlieg, 2016).

The volatility of stock market returns often occur upon the arrival of new information. For example, buying and selling decisions are mostly influenced by news concerning corporate profits, interest rates, dividends or the economy (Oyelami and Ademola, 2014). It is as well important to stress that factors such as government policies, political instability, positive or negative shocks, international capital flights likewise influence the volatility clustering, asymmetry effect, leptokurtosis distributions, conditional and time varying volatility of stock market returns. Oyelami and Ademola (2004) further emphasized that updates in the above-mentioned factors usually bring about positive or negative news which hastily spreads across markets. In addition, the leverage or asymmetry effect of stock market returns that often ensue from the movements in stock returns are negatively related to changes in volatility while, volatility clustering on the other hand emerge from substantial changes in stock returns and vice versa (Black, 1976; Taylor, 1986).

Similarly, Meia, Liu, Ma and Chenc (2017) acknowledged that the examination of volatility features such as leverage effect, time varying volatility, volatility clustering and leptokurtosis are of paramount importance in ascertaining accurate forecast of stock market volatility. In recap, uncertainties and fluctuations in the prices of securities as well as the fear of a potential huge loss instilled massive fear on the investors thereby leading to the stock market crash of 1987. The crash influenced the emergence of forecasting models such as the Autoregressive Conditional Heteroscedasticity (ARCH) model which was developed by Engle (1982) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model which was subsequently developed by Bollerslev in 1986 (Andersen, Bollerslev, and Diebold, 2003; Bollerslev, 1986)

1.1 Motivation for the study

The ARCH and GARCH models were developed to process historical information of stock market returns for the prediction of future returns since such information holds the characteristics of heteroskedasticity (Ching and Siok, 2013). Regardless of the benefits of these models, they have been reported as vulnerable to significant setbacks based on their ability to accurately estimate volatility asymmetric and leverage effect, thereby rendering their contributions in ensuring appropriate forecasting less effective. At corollary, volatility asymmetric is a phenomenon that occurs when stock market returns persistently grows particularly when market performance is at low-ebb as opposed to when it is highly-peaked (Meia, Liu, Ma and Chenc, 2017).

Furthermore, the downsides of these models (ARCH and GARCH) prompted the need for more improvement in attaining more accuracy, thereby leading to the advent of other variants of the GARCH models such as the Exponential Generalized Autoregressive Conditional Heteroscedasticity (EGARCH), Non-Linear Asymmetric Generalized Autoregressive Conditional Heteroscedasticity (NGARCH), GARCH-in-mean (GARCH-M), Threshold GARCH (TGARCH), Quadratic GARCH (QGARCH), Integrated Generalized Autoregressive Conditional heteroskedasticity (IGARCH), and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) (Nelson, 1991; Higgins and Bera, 1992; Glosten, Jagannathan & Runkle, 1993; Zakoian, 1994; Sentana, 1995; Qifa, Zhongpu, Cuixia, and Yezheng, 2019).

1.2 Research Objectives

Brandt and Kinsley (2003) reported that even though the advantages of symmetry and asymmetry GARCH models have been shown in various studies, these models have been said to sometimes yield conflicting and inconsistent outcomes, not only because the studies are conducted on different stock market indices or for the varying temporal scope, but more importantly the heterogeneity of stock markets world over, since they are susceptible to different exposures such as macroeconomic and political instabilities, capital flight, external shocks amongst several others (Gong and Lin, 2019). This phenomenon varies across countries. In respect of this, this study therefore is aimed at:

(1) estimating and comparing the volatility clustering, conditional volatility, leverage effect and volatility estimator of developed stock markets index and;

(2) estimating and comparing the performance and forecasting ability of symmetry and asymmetry GARCH models of developed stock market index.

To evaluate and compare the performance and forecasting ability of symmetry and asymmetry GARCH models, this study employed information criterion estimates such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) and Hannan Quinn Information Criterion (HQIC) for model selection and evaluation. The rest of the study is structured as follows: Section two discusses the review of related work conducted on stock market volatility with evidence from emerging and developed stock market index. The methodology is explained in section three with special focus to methodological approach, modelling, data sourcing and model evaluation. Section four, explains how the findings was implemented, while section five and six provide a detailed evaluation and discussions of the study's findings and section seven provides conclusion and suggestions for future work.

2. 0 Related Work

According to Robert *et al.*, (1987), GARCH-M model was developed to determine the risk-return relationship of time series. The EGARCH model was also introduced by Nelson in 1991 to estimate the logarithmic feature of conditional volatility for the purpose of identifying asymmetric effects in the time series. In addition, Zakoian (1994) founded the TGARCH model to help investigate the affiliation between return on asset and its asymmetric volatility.

As emphasized by Wei (2012), the Quadratic GARCH (QGARCH) model is much more efficient in estimating and forecasting stock market volatility compared to GARCH model whereas, Yeh and Lee (2000) in a study carried out in forecasting Chinese stock market returns argued that GJR-GARCH model performs better than other GARCH models. As the argument persists, Awartani and Corradi (2005) also emphasized that symmetry GARCH model such as GARCH (1,1) is less effective in the forecasting stock market returns and volatilities compared to asymmetry GARCH models.

However, the findings of McMillan, Speight and Apgwilym (2000) contradicts that of Awartani and Corradi (2005) which revealed that GARCH model gives a better forecast of stock market volatilities and returns in comparison to EGARCH model. In addition, Lin (2008) revealed that SSE composite index are characterised with clustering volatility, time-varying volatility, and leptokurtosis distribution with ARCH and GARCH effects. The study further revealed that EGARCH (1,1) model provides more reliable and accurate stock market returns and volatilities.

In addition, the study of Wong and Kok (2005) revealed that ARCH-M model is superior than GARCH model in forecasting the stock market volatility of Thailand, Malaysia and Singapore stock market returns, while random walk model is superior than ARCH and GARCH models in forecasting the volatility of Indonesia and Philippines stock market, the study of Omar and Halim (2015) revealed that there is presence of volatility clustering, leverage and persistence effects on Malaysian stock market index and EGARCH (1,1) model is superior than other GARCH model in forecasting the Malaysian stock market returns and volatilities.

3.0 Research Methodology

The aim of the study was to compare stock market volatility using the developed stock market index as a case study. This study adopted the KDD approach. The data used for the study were retrieved from Yahoo Finance from June 2015 to June 2019 and it comprises of developed stock market index such as S & P 500, NASDAQ Composites and DOWJONES. Various pre-possessing tests such as checking for missing values in the datasets, descriptive statistics, unit root test and serial correlation test were carried out. The datasets were also transformed into a time series format in order to prepare the data ready for exploration and mining. The data mining entails data modelling, while the last procedure that was executed under the KDD approach is the interpretation and evaluation of the modelling.

The KDD diagram below shows the sequential methodological steps which comprising of data selection, data pre-processing, data transformation, data mining, interpretation and evaluation. This approach also ensures that the datasets chosen for this study are processed to ensure meaningful insight, analysis, interpretation and evaluation are drawn from the datasets.

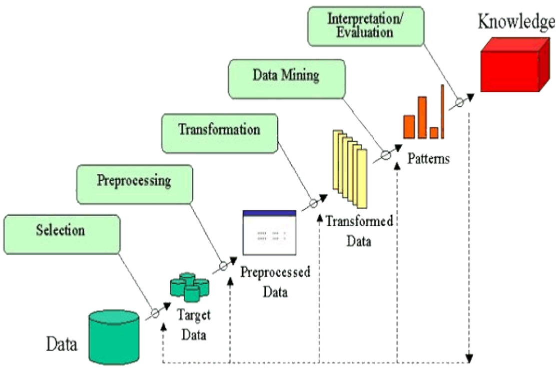

Figure 1 KDD

3.1 Sourcing of Data

Three datasets comprising of top stock market index including Standard and Poor's 500, NASDAQ Composites and Dow Jones Industrial Average were adopted for the study. This study covers the period between January 2015 to June 2019. These periods were considered for the study because the stock market experienced sell-off in 2015 and the Brexit vote which started in 2016. These incidents inevitably caused a global decline in the value of stock prices. The datasets were sourced from Yahoo Finance, and it consist of 1130 observations.

3.2 Modelling

The study adopted the symmetry and asymmetry GARCH models, which consist of GARCH (p, q), GJRGARCH (p, q), EGARCH (p, q). This model was adopted based on the study of Omar and Halim (2015). The justification for choosing these models were based on the objectives of the study, which was aimed at investigating the volatility clustering, conditional volatility and leverage effect of stock market index and to estimates and compare the forecasting ability of symmetry and asymmetry GARCH models of developed stock market index. The models are specified below:

(i). GARCH Model: The GARCH (p, q) model introduces conditional variance with the ARCH (q) model. The GARCH (p, q) uses the order of previous conditional variance and previous residual to determine conditional variance. The GARCH (p, q) model is specified in order (p, q).

$$\sigma_t^2 = \lambda o + \Sigma^q/_{i=1}\ \alpha i\ \varepsilon^2/_{t-i} + \Sigma^p/_{j=1}\ \beta j\ \sigma^2/_{t-j} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where:

$\sigma t2$ = current volatility

αi = parameter measuring the effect of previous residual of $\varepsilon^2/_{t-i}$

βj measures the effect of change in its lagged value, of $\sigma^2/_{t-j}$.

(ii). The Glosten-Jagannathan-Runkle (GJR) Model: The GJRGARCH (p, q) models explains the asymmetry and leverage effect of volatility. The model also shows the influence of positive and negative incidences on volatility by using a_{t-1} as a threshold. The GJR-GARCH (1,1) model is generally expressed as

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m} \alpha_i a_{t-i}^2 + \sum_{i=1}^{m} I(a_{t-i})\alpha_i^* a_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where $I(a_{t-i}) = 1$ if $a_{t-i} < 0$ and $I(a_{t-i}) = 0$ if $a_{t-i} \geq 0$.

(iii). EGARCH Model: The EGARCH (p, q) model also explains the asymmetric effect of news or shocks on the conditional volatility. The EGARCH (p, q) model is generally expressed as:

$$\ell n(\sigma_t^2) = \alpha_0 + \alpha_1(|\varepsilon_{t-1}| - \gamma\ \varepsilon_{t-1}) + \cdots + \alpha_m(|\varepsilon_{t-m}| - \gamma\ \varepsilon_{t-m}) + \beta_1 \ell n(\sigma_{t-1}^2) + \cdots + \beta_r \ell n(\sigma_{t-r}^2) \quad \ldots\ldots (3)$$

$$\ell n(\sigma_t^2) = \alpha_0 + \alpha_1 g(\varepsilon_{t-1}) + \alpha_2 g(\varepsilon_{t-2}) + \cdots + \alpha_m g(\varepsilon_{t-m}) + \beta_1 \ell n(\sigma_{t-1}^2) + \cdots + \beta_r \sigma_{t-r}^2 \quad \ldots\ldots\ldots\ldots\ldots (4)$$

where $\varepsilon_t = a_t / \sigma_t$ and $\alpha_1 = 1$. Nelson choose the function $g(\varepsilon_t)$ to be a linear combination of ε_t and $|\varepsilon_t|$ such as

$$g(\varepsilon_t) = \delta\varepsilon_t + \lambda[|\varepsilon_t| - E(|\varepsilon_t|)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where δ and λ are constant terms to be estimated. In the above function, both ε_t and $|\varepsilon_t| - E(|\varepsilon_t|)$ are i.i.d variable with zero mean.

3.3 Model Selection and Evaluations

The symmetry and asymmetry GARCH models of S & P 500, NASDAQ Composites and DOWJONES daily returns were evaluated using information criterion such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Shibata Information Criterion (SIC) and Hannan Quinn Information Criterion. These model selection criterions were used to compare the forecasting ability of GARCH (1,1), GJRGARCH (1,1) and EGARCH (1,1) models and the choice of this model evaluation techniques was based on the study of Omar and Halim (2015).

4.0 Implementation

The implementation of the data modelling, analysis, interpretation and evaluation were carried out in the following steps:

(i). Step one: The datasets chosen for this study were retrieved from Yahoo Finance. The datasets consist of S & P 500, NASDAQ Composites and DOWJONES. The datasets cover the period from January 2015 to June 2019, and it comprises of 1130 observations.

(ii). Step two: The second step is the installation of different packages such as ("zoo"), ("xts"), (curl), (TTR), (quantmod), (lattice), (timeDate), (timeSeries), (parallel), (rugarch), (aTSA), (forecast), (ggplot2), (FinTS), (pdfetch), (rmgarch),(e1071), (MLmetrics), (tseries), (psycho) and (Metrics) for the purpose of data exploration, analysis, interpretation and evaluation.

(iii). Step three: The third step in the implementation stage was feature engineering, which entails: checking for missing values, checking for the summary, dimensions, descriptive statistics of the datasets.

(iv). Step four: The fourth step entails the estimation of the daily returns of stock market index and checking for the descriptive statistics such as minimum, maximum, mean, median, variance, standard deviations, kurtosis and skewness of daily returns. The volatility clustering, autocorrelation functions and partial autocorrelation functions of daily returns was also examined.

(v). Step five: In this stage, pre-estimation tests such as unit root test including Augment Dickey Fuller test (ADF), serial correlation test such as Ljung box test and ARCH test were conducted.

(vi). Step six: The sixth step in the implementation state is the data modelling, at this stage, GARCH (1,1) EGARCH (1,1) and GJRGARCH (1,1) was estimated. The data modelling entails estimation of coefficient, measurement of value at risk, estimation of conditional variance and squared residuals, plotting of estimation of conditional variance and squared residuals and model forecasting and volatility estimator.

5.0 Evaluation

This section categorically analyzed, interpret and compare the data used for the study. The data used for the study comprises of S& P 500, Dow Jones Industrial Average and NASDAQ and based on the acronyms from Yahoo Finance were the data were retrieved. S & P 500 was proxied as GSPC, Dow Jones Industrial Average was proxied as DJI while, NASDAQ was proxied as IXIC. Furthermore, this sections also used the information criterion such as (AIC), (BIC), (SIC) and HQIC) to compare the forecasting ability of symmetry and asymmetry GARCH models.

5.1 Standard and Poor's 500 (S & P 500)

Table 1: Descriptive Statistics of Daily Returns (S & P 500)

Statistics	**Values**
Minimum	-0.04097923
Maximum	0.04959374
Mean	0.0003522956
Median	0.0003934001
Skewness	-0.3931364
Kurtosis	3.840201
Variance	7.283338e-05
Standard Deviations	0.008534247

Source: Author's Computation, (2019)

The table above shows that there is a significant difference between the maximum value of daily returns of S & P 500 (0.04959374) and minimum value of daily returns of S& P 500 (-0.04097923). The standard deviation of the daily returns of S & P 500 is also high, with a value of 0.008534247. The standard deviation was considered high because, it is far from the mean value which is 0.0003522956. The mean value of daily return is positive (0.0003522956), and this implies that S & P 500 offers high average returns, however, these returns are subjected to high volatility.

In addition, the skewness of S & P 500 shows a negative value of (-0.3931364), which indicates an asymmetric tail that exceeds more towards negative values rather than positive values. Thus, it shows that S & P 500 has non-symmetric returns. The kurtosis statistics shows a value of (3.840201), which exceeds the normal value of three thus indicating that the return distribution is fat-tailed and a heavier tail than a standard normal distribution. The daily returns of S & P 500 also reveal that its kurtosis distribution is leptokurtic.

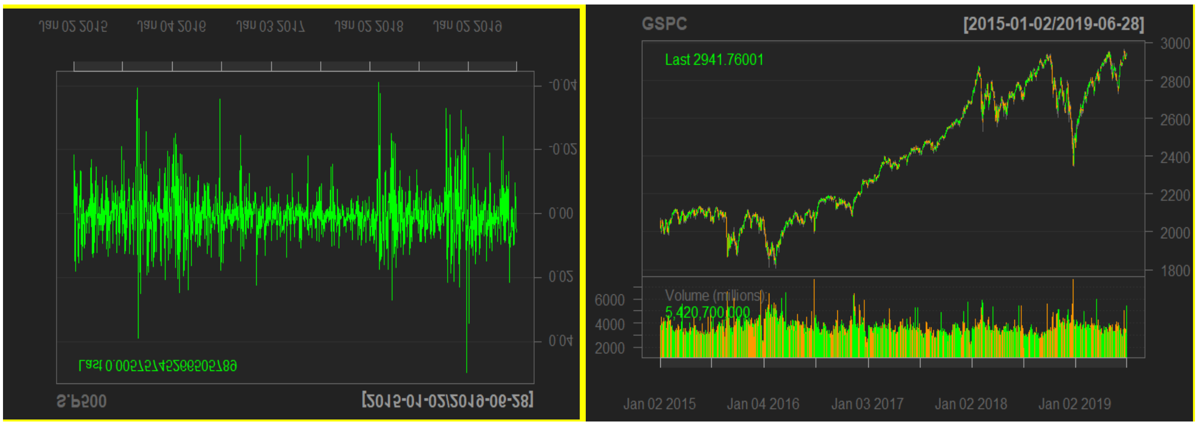

Figure 2: Volatility Clustering and Chart Series of S & P 500

The figure 2 as shown above explains the volatility clustering of daily returns and the chart series of closing returns. The graph reveals that volatility of the daily returns of S & P 500 changes over time, and a such tends to cluster with periods of low volatility and periods of high volatility. This turbulence and tranquillity suggest the existence of volatility clustering in the graph of daily returns. The implication of this therefore is that that large price fluctuations are followed by large price fluctuations and small price fluctuations are followed by small price fluctuations of both signs (positive and negative).

Table 3: Stationary Test: Augment Dickey Fuller Test

Parameters	**S & P 500**	**Nasdaq Composites**	**DOWJONES**
x-squared	-10.969	-11.152	-10.564
P-value	0.01	0.01	0.01

Source: Author's Computation, (2019)

The Augment Dickey Fuller test was employed to measure the stationarity of S & P 500, Nasdaq Composites and DOWJONES. The hypothesis for this test is as follows:

Ho: There is presence of unit root, which means that the series is nonstationary.

Hi: There is no presence of unit root, which means that the series is stationary.

The ADF presented above shows that there is presence of unit root test in the series. The probability value of ADF test is less than 0.05, thus, S & P 500, Nasdaq Composites and DOWJONES is stationary, and as such, the null hypothesis which states that there is unit root in the series should be rejected.

Table 4: Serial Correlation Test: Ljung Box Test

Parameters	**S & P 500**	**Nasdaq Composites**	**DOWJONES**
x-squared	11.53	15.086	9.83
P-value	0.3177	0.1289	0.4555

Source: Author's Computation, (2019)

The Ljung-Box Q-test was employed to investigate the presence of serial correlation in the series. The hypotheses for this test are as follows:

Ho: There is no serial correlation in the series

Hi: There is serial correlation in the series

The Ljung box-test presented above reveals that are no serial autocorrelations up to lags 10. From the table above, The Ljung Box Q-test revealed that we cannot reject the null hypothesis of no serial correlation at 5% level of significance.

Table 5: ARCH LM-test

Parameters	**S & P 500**	**Nasdaq Composites**	**DOWJONES**
x-squared	167.32	147.73	166.65
P-value	2.2e-16	2.2e-16	2.2e-16

Source: Author's Computation, (2019)

The ARCH LM- test was employed to investigate the presence of ARCH effect in the time series. The hypotheses for this test are as follows.

Ho: There is no ARCH effect.

Hi: There is ARCH effect.

The ARCH LM-test presented above revealed that there is presence of conditional heteroskedasticity in the time series, and as such, the null hypothesis of no ARCH effect was rejected. This therefore implies that the volatility of S & P 500, Nasdaq Composites and DOWJONES daily returns was serially correlated. This also indicate that the conditional mean model is needed to be used for return series. The Engle's ARCH test rejects the null hypothesis, to agree that there is an ARCH effect in the series, with a low p-value at 5% level of significance, and as such, we proceed with the GARCH models.

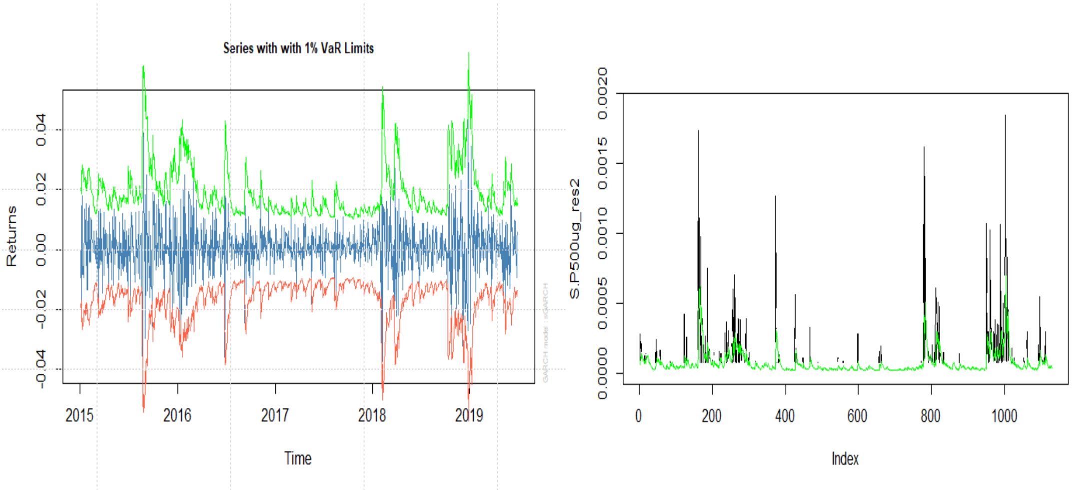

Figure 3: Value at Risk with Estimated Returns and Squared Residuals and the Estimated Conditional Variances

Table 6: Estimated Coefficient of GARCH models (S & P 500)

Parameters	**GARCH (1,1)**	**EGARCH (1,1)**	**GJRGARCH (1,1)**
Mu	6.181447***	0.0003303576	0.000455***
ar1	9.5622***		0.969487***
ma1	-9.83797***		-0.988027***
Omega	4.10454	-0.611530520***	0.000004***
alpha1	1.893505***	-0.211560769***	0.000000
beta1	7.56766***	0.9374269492***	0.794099**
Gamma		0.1749959016***	0.270189***

Source: Author's Computation, (2019)

Note: * represent 1%, ** represent 5% and * represent 10%**

The table above revealed that the ARCH (α) and GARCH (β) coefficient in GARCH (1,1) are (1.893) and (7.567) respectively. These coefficients are positive and statistically significant, and as such, the significance of the alpha (α) and beta (β) reveals that the lagged conditional variance and lagged squared disturbance have an impact on the conditional variance, and as such, it implies that the information about volatility from the previous periods have an explanatory power on current volatility. Furthermore, the addition of the estimated ARCH and GARCH coefficients in the GARCH (1,1) model also indicated that the volatility shocks have a persistent effect on the conditional variance.

The result shown in the table above as well reveal that ARCH (α) and GARCH (β) coefficient in EGARCH (1,1) model are (-0.211560769) and (0.9374269492), which are smaller than 1. This therefore implies that conditional variance is not volatile and there is no atypical increase or decrease in prices, but a gradual movement is observed. The results also revealed that the coefficient of the leverage effect (0.1749959016) of EGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of 0.1749959016 is significant at 5% level.

In addition, the ARCH (α) and GARCH (β) coefficient in GJRGARCH (1,1) model are (0.000000) and (0.794099), which are smaller than 1. The ARCH (α) is not statistically significant in GJRGACRCH, while the GARCH (β) is positively significant. The leverage effect (0.270189) of GJRGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of 0.270189 is statistically significant at 5% level. The positive sign indicates that positive shocks imply a higher next period conditional variance than the negative shocks. This therefore implies that there is a presence of leverage effect in S & P 500

Table 7: Model Selection and Evaluation of S & P 500

Information Criterion	GARCH (1,1)	EGARCH (1,1)	GJRGARCH (1,1)
Akaike	-6.9819	-7.0414	-7.0237
Bayes	-6.9552	-7.0192	-6.9925
Shibata	-6.9820	-7.0415	-7.0237
Hannan-Quinn	-6.9718	-7.0330	-7.0119

Source: Author's Computation, (2019)

The table above shows the AIC, BIC, SIC and HQIC of the GARCH model and bearing in mind that the smaller value of the Information Criteria provides a better fit for the daily return series. From the table, it can be inferred that Bayesian information criterion reveals the smallest value from each of the models. In addition, the Bayesian value for GARCH (1,1) is also lower than that of EGARCH and GJRGARCH.

5.2 NASDAQ Composites

The daily returns of Nasdaq Composite were used to show the trend and descriptive statistics. The descriptive statistics consists of minimum, maximum, mean, median, variance, standard deviation, kurtosis and skewness of daily returns of Nasdaq Composite.

Table 8: Descriptive Statistics of NASDAQ Composites Daily Returns

Statistics	**Values**
Minimum	-0.0442539
Maximum	0.05836341
Mean	0.0005133327
Median	0.0008546306
Skewness	-0.3855541
Kurtosis	3.117142
Variance	0.0001060718
Standard Deviations	0.01029912

Source: Author's Computation, (2019)

The table above shows that there is a large difference between the maximum value of daily returns of NASDAQ Composites (0.05836341) and minimum value of daily returns of NASDAQ Composites (-0.0442539). The standard deviation of the daily returns of NASDAQ Composites is also high, with a value of 0.01029912. The standard deviation was considered high because, it is far from the mean value which is 0.0005133327. The mean value of daily return is positive (0.0005133327), and this implies that NASDAQ Composites offers high average returns, however, these returns are subjects to high volatility.

In addition, the skewness of NASDAQ Composites shows a negative value of (-0.3855541), which indicates an asymmetric tail that exceeds more towards negative values rather than positive values. Thus, it shows that NASDAQ Composites has non-symmetric returns. The kurtosis statistics shows a value of (3.117142), which exceeds the normal value of three, indicating that the return distribution is fat-tailed and a heavier tail than a standard normal distribution. The daily returns of NASDAQ Composites also reveal that its kurtosis distribution is leptokurtic.

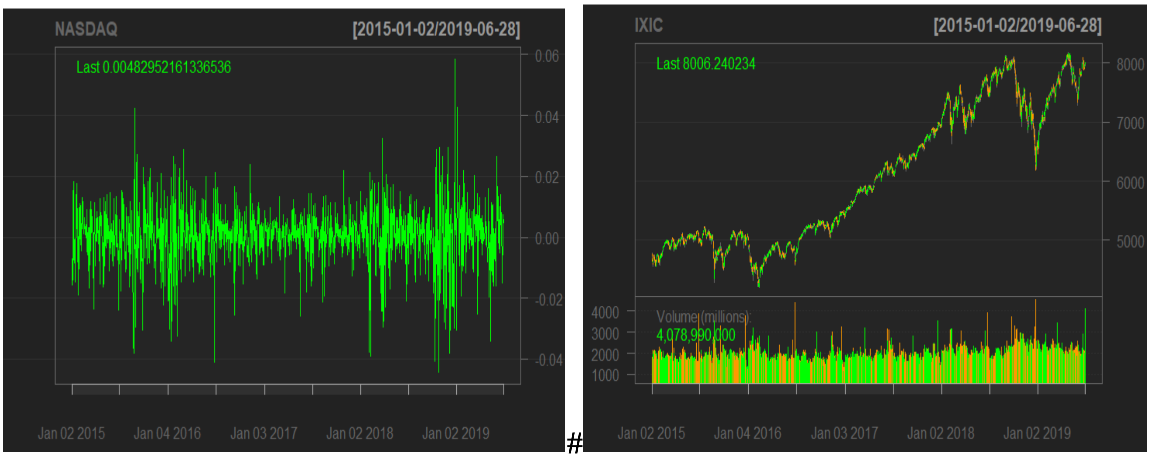

Figure 4: Volatility Clustering and Chart Series of Nasdaq Composite

The figure 4 shown above explains the volatility clustering of daily returns and the chart series of closing returns. The graph reveals that volatility of the daily returns of NASDAQ Composites changes over time, and as such, it tends to cluster with periods of low volatility and periods of high volatility. This turbulence and tranquillity suggest the existence of volatility clustering in the graph of daily returns. This therefore implies that that large price fluctuations are followed by large price fluctuations and small price fluctuations are followed by small price fluctuations of both signs (positive and negative).

Table 9: Estimated Coefficient of GARCH models (NASDAQ Composites)

Parameters	**GARCH (1,1)**	**EGARCH (1,1)**	**GJRGARCH (1,1)**
Mu	8.32617***	0.0003782108	0.000652***
ar1	9.45726***	0.00000***	0.954148s***
ma1	-9.7891***	0.00000***	-0.981145***
Omega	6.48816***	-0.551925***	0.000007***
alpha1	1.52588***	-0.208756***	0.000000
beta1	7.84922***	0.9411141***	0.799946***
Gamma		0.0856786***	0.229477***

Source: Author's Computation, (2019)

Note: * represent 1%, ** represent 5% and * represent 10%**

The table above revealed that the ARCH (α) and GARCH (β) coefficient in GARCH (1,1) are (1.525) and (7.849) respectively. These coefficients are positive and statistically significant, and as such, the significance of the alpha (α) and beta (β) reveals that the lagged conditional variance and lagged squared disturbance have an impact on the conditional variance and as such it implies that the information about volatility from the previous periods have an explanatory power on current volatility. Furthermore, the addition of the estimated

ARCH and GARCH coefficients in the GARCH (1,1) model also indicated that the volatility shocks have a persistent effect on the conditional variance.

Furthermore, the result shown in the table above reveals that ARCH (α) and GARCH (β) coefficient in EGARCH (1,1) model are (-0.208756) and (0.9411141), which are smaller than 1. This therefore implies that conditional variance is not volatile and there is no atypical increase or decrease in prices, but a gradual movement is observed. The results also reveal that the coefficient of the leverage effect (0.0856786) of EGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of 0.0856786 is statistically significant at 5% level.

In addition, the ARCH (α) and GARCH (β) coefficient in GJRGARCH (1,1) model are (0.000000) and (0.799946), which are smaller than 1. The ARCH (α) is not statistically significant in GJRGACRCH, while the GARCH (β) is positively significant. The leverage effect (0.270189) of GJRGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of 0.229477 is statistically significant at 5% level. The positive sign indicates that positive shocks imply a higher next period conditional variance than the negative shocks. This therefore implies that there is a presence of leverage effect in NASDAQ Composites.

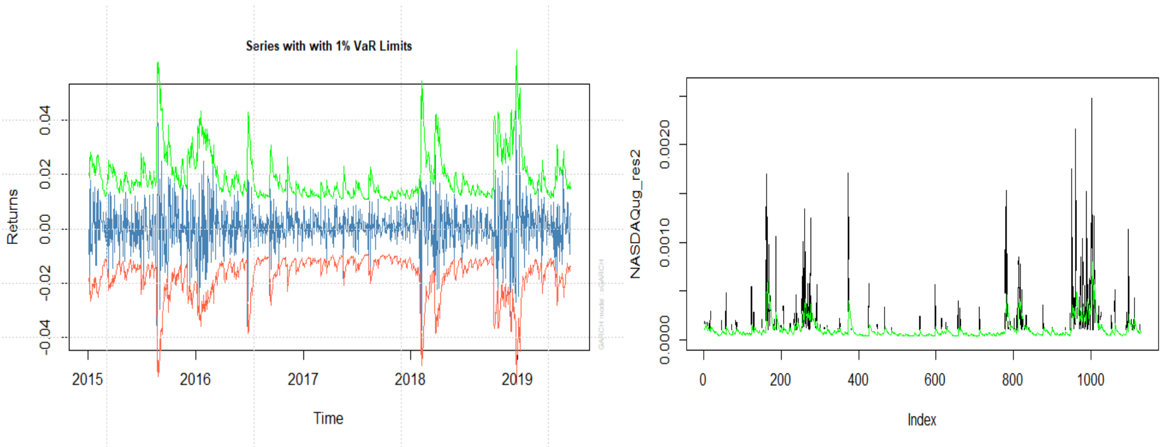

Figure 5: Value at Risk with Estimated Returns and Squared Residuals and the Estimated Conditional Variances

Table 10: Model Selection and Evaluation of NASDAQ Composites

Information Criterion	GARCH (1,1)	EGARCH (1,1)	GJRGARCH (1,1)
Akaike	-6.5330	-6.5931	-6.5756
Bayes	-6.5063	-6.5708	-6.5445
Shibata	-6.5330	-6.5931	-6.5757
Hannan-Quinn	-6.5229	-6.5847	-6.5639

Source: Author's Computation, (2019)

The table above shows the AIC, BIC, SIC and HQIC of the GARCH model and bearing in mind that the smaller value of the Information Criteria provides better fit for the daily return series. From the table, it can be deduced that Bayesian information criterion reveals the smallest value from each of the models. In addition, the Bayesian value for GARCH (1,1) is also lower than that of EGARCH and GJRGARCH.

5.3 Dow Jones Industrial Average

The daily closing price of Dow Jones Industrial Average was used to show the trend and descriptive statistics of the stock market index. The descriptive statistics comprises minimum, maximum, mean, median, variance, standard deviation, kurtosis and skewness of Dow Jones Industrial Average.

Table 11: Descriptive Statistics of Dow Jones Industrial Average

Statistics	Values
Minimum	-0.04604885
Maximum	0.04984582
Mean	0.0003916169
Median	0.0004887411
Skewness	-0.4045638
Kurtosis	3.79531
Variance	7.431192e-05
Standard Deviations	0.008620436

Source: Author's Computation, (2019)

The table above shows that there is a large difference between the maximum value of daily returns of DOWJONES (0.04984582) and minimum value of daily returns of DOWJONES (-0.04604885). The standard deviation of the daily returns of DOWJONES is also high, with a value of 0.01029912. The standard deviation was considered high because, it is far from the mean value which is 0.008620436. The mean value of daily return is positive (0.0003916169), and this implies that DOWJONES offers high average returns, however, these returns are subjects to high volatility.

In addition, the skewness of DOWJONES shows a negative value of (-0.4045638), which indicates an asymmetric tail that exceeds more towards negative values rather than positive values. Thus, it shows that DOWJONES has non-symmetric returns. The kurtosis statistics shows a value of (3.79531), which exceeds the normal value of three and this indicate that the return distribution is fat-tailed and a heavier tail than a standard normal distribution. The daily returns of DOWJONES also reveal that its kurtosis distribution is leptokurtic.

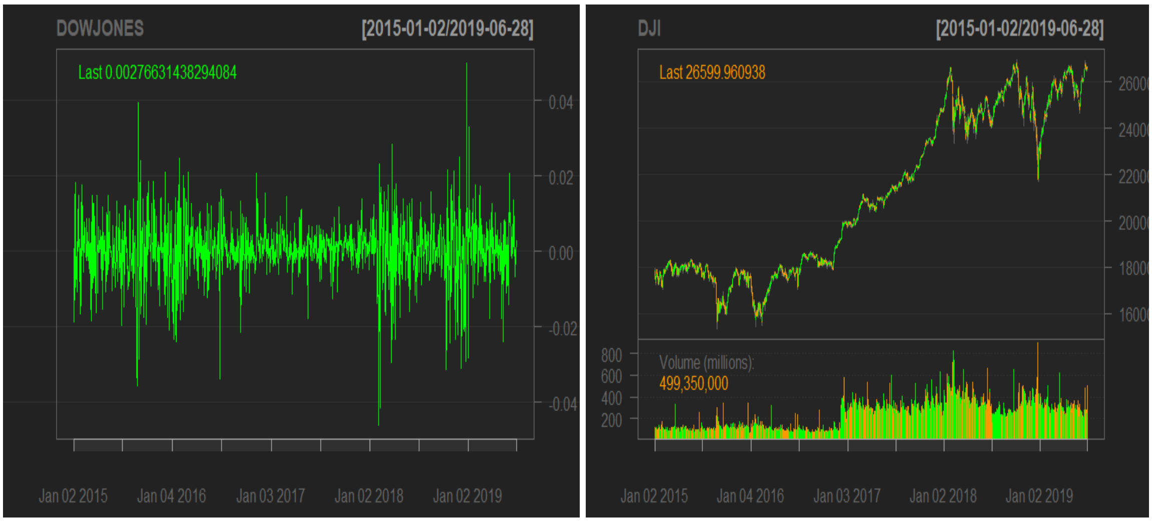

Figure 6: Volatility Clustering and Chart Series of Dow Jones Industrial Average

The figure 6 shown above explains the volatility clustering of daily returns and the chart series of closing returns. The graph reveals that volatility of the daily returns of DOWJONES changes over time, and as such, it tends to cluster with periods of low volatility and periods of high volatility. This turbulence and tranquillity suggest the existence of volatility clustering in the graph of daily returns. This therefore implies that that large price fluctuations are followed by large price fluctuations and small price fluctuations are followed by small price fluctuations of both signs (positive and negative).

Table 12: Estimated Coefficients of GARCH Models (DOWJONES)

Parameters	**GARCH (1,1)**	**EGARCH (1,1)**	**GJRGARCH (1,1)**
Mu	7.63803***	0.0004601**	0.000531
ar1	-2.92030		-0.518388
ma1	2.50776		0.463692
Omega	3.69938	-0.595251***	0.000003***
alpha1	1.77319***	-0.175885***	0.000005
beta1	7.73765***	0.9389629***	0.824942***
Gamma		0.1887279***	0.243439***

Source: Author's Computation, (2019)

Note: * represent 1%, ** represent 5% and * represent 10%**

The table above revealed that the ARCH (α) and GARCH (β) coefficient in GARCH (1,1) are (1.773) and (7.737) respectively. These coefficients are positive and statistically significant, and as such, the significance of the alpha (α) and beta (β) reveals that the lagged conditional variance and lagged squared disturbance have an impact on the conditional variance and as such it implies that the information about volatility from the previous periods have an explanatory power on current volatility. Furthermore, the addition of the estimated ARCH and GARCH coefficients in the GARCH (1,1) model also indicated that the volatility shocks have a persistent effect on the conditional variance.

Furthermore, the result shown in the table above reveals that ARCH (α) and GARCH (β) coefficient in EGARCH (1,1) model are (-0.175885) and (0.9389629), which are smaller than 1. This therefore implies that conditional variance is not volatile and there is no atypical increase or decrease in prices, but a gradual movement is observed. The results also reveal that the coefficient of the leverage effect (0.1887279) of EGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of 0.1887279 is statistically significant at 5% level.

In addition, the ARCH (α) and GARCH (β) coefficient in GJRGARCH (1,1) model are (0.000005) and (0.824942), which are smaller than 1. The ARCH (α) is not statistically significant in GJRGACRCH, while the GARCH (β) is positively significant. The leverage effect (0.243439) of GJRGARCH (1,1) model is positive and significant at 5% level as the *p*-value is less than 0.05. The study implies that negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news because the value of

0.243439 is statistically significant at 5% level. The positive sign indicates that positive shocks imply a higher next period conditional variance than the negative shocks. This therefore implies that there is a presence of leverage effect in DOWJONES.

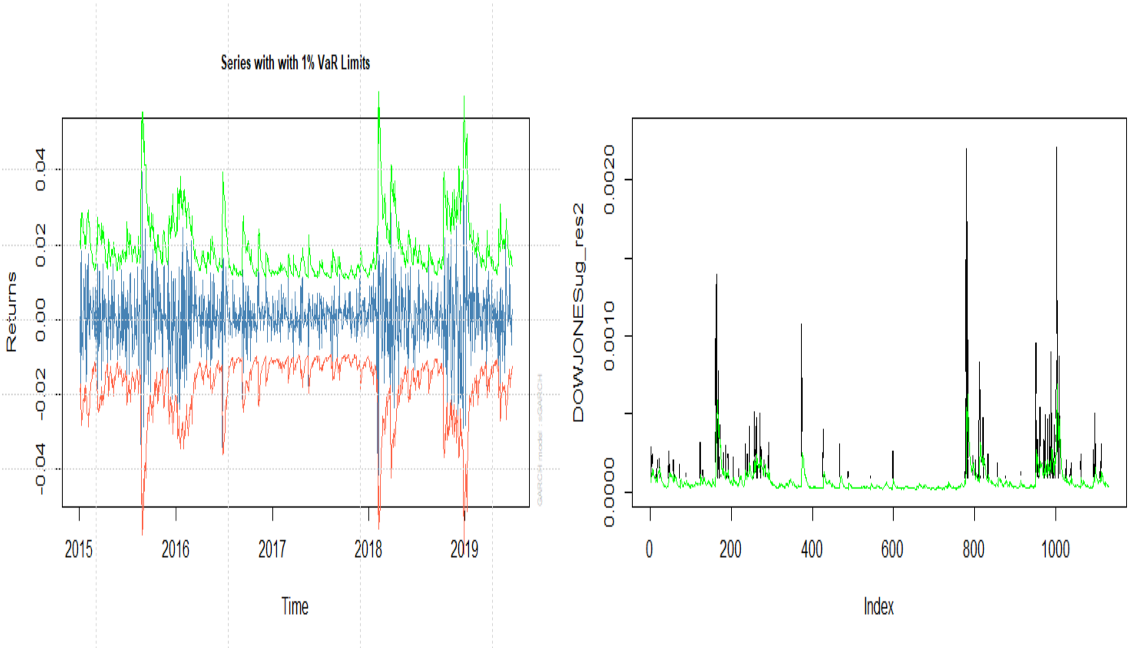

Figure 7: Value at Risk with Estimated Returns and Squared Residuals and the Estimated Conditional Variances

Table 13: Model Selection and Evaluation of DOWJONES

Information Criterion	**GARCH (1,1)**	**EGARCH (1,1)**	**GJRGARCH (1,1)**
Akaike	-6.5330	-7.0123	-6.9583
Bayes	-6.5063	-6.9900	-6.9316
Shibata	-6.5330	-7.0123	-6.9583
Hannan-Quinn	-6.5229	-7.0039	-6.9482

Source: Author's Computation, (2019)

The table above shows the AIC, BIC, SIC and HQIC of the GARCH model and bearing in mind that the smaller value of the Information Criteria provides better fit for the daily return series. From the table, it can be deduced that Bayesian information criterion reveals the smallest value from each of the models. In addition, the Bayesian value for GARCH (1,1) is also lower than that of EGARCH and GJRGARCH.

6.0 Discussion

The findings of the study revealed that the mean value of S & P 500, NASDAQ Composites and DOWJONES daily returns is positive and the standard deviation for these stock market index such as S & P 500, NASDAQ Composites and DOWJONES is also high compared to their mean value. This therefore implies that S & P 500, NASDAQ Composites and DOWJONES offers high average returns which are subjects to high volatility. The findings also revealed that S & P 500, NASDAQ Composites and DOWJONES are negatively skewed, while the kurtosis distribution of these stock market index is leptokurtic, thus showing fat-tailed and heavier tail distribution.

Furthermore, the series of S & P 500, NASDAQ and DOWJONES daily returns are stationarity. The volatility of the daily returns of S & P 500, NASDAQ and DOWJONES changes over time, and as such tends to cluster with periods of low volatility and periods of high volatility. The findings of the study also show the presence of serial correlation, conditional heteroskedasticity and volatility clustering in S & P 500, NASDAQ and DOWJONES. Consequently, the ARCH (α) and GARCH (β) coefficient of S & P 500, NASDAQ and DOWJONES in the three model such as GARCH (1,1), EGARCH (1,1) and GJRGARCH (1,1) are positively significant. The information about volatility from the previous periods have an explanatory power on current volatility on each of the stock market index.

The findings also revealed that the volatility shocks of S & P 500, NASDAQ and DOWJONES have a persistent effect on the conditional variance. The EGARCH (1,1) and GJRGARCH (1,1) was used to measure the presence of the leverage effect in the series and it was revealed that there exists a significant and positive influence of leverage effect on S & P 500, NASDAQ and DOWJONES daily returns, and as such, the negative shocks or bad news have a greater effect on the conditional variance than the positive shocks or good news. The findings of this study align with the study of Omar and Halim (2015) which reveals the presence of volatility clustering, leverage effect and fat tailed distributions in the Malaysian stock market index. However, the study of Omar and Halim (2015) revealed that EGARCH (1,1) is superior than other GARCH models and this contradicts with the findings of this study.

Furthermore, the findings of McMillan, Speight, Apgwilym (2000) agrees with the result of this study which revealed that GARCH (1,1) model gives a better forecast of stock market volatilities and returns than GARCH model. However, the study of Yeh and Lee (2000) and Awartani and Corradi (2005) revealed that GJR-GARCH and GARCH (1,1) model performs better than other GARCH models, and this contradicts with the findings of the study. In addition, the information criterion such as AIC, BIC, SIC and HQIC was used for model selection and evaluation and the findings revealed that in all of the models such as GARCH

(1,1), EGARCH (1,1) and GJRGARCH (1,1) examined in the study, the Bayesian Information Criterion (BIC) reveals the smallest value from each of the models, and as such, GARCH (1,1) gives the best forecasting ability that EGARCH and GJRGARCH.

7.0 Conclusion and Future Work

The primary objective of this study was to compare stock market volatility using the developed stock market index such S & P 500, NSADAQ, DOWJONES as a point of reference. The secondary objective was to investigate the presence of volatility clustering, conditional volatility and leptokurtosis distribution in the stock market index; to estimates and compare the forecasting ability of symmetry and asymmetry GARCH models such as GARCH (1,1), EGARCH (1,1) and GJRGARCH (1,1) using the information criterion such as Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), Shibata Information Criterion (SIC) and Hannan Quinn Information Criterion (HQIC) and to estimate the volatility estimator of each of the stock market index examined in the study.

Based on the findings, the study concludes that S & P 500, NASDAQ, DOWJONES possesses the same attributes such as high returns which was offset by high risk, presence of volatility clustering, serial correlation, leptokurtosis distribution and conditional volatility. The study also concludes that there exists the presence of leverage or asymmetry effect on the stock market index examined in the study. The ARCH and GARCH effect in each of the models examined were positively significant and this implies that previous information of returns can be used to predict the future returns. The study also concludes that the GARCH (1,1) models gives the best forecasting ability than EGACRH (1,1) and GJRGACRH (1,11) models.

In conclusion, this study recommends that further studies can be conducted in the following areas:

(1). To compare stock market volatility using developed and emerging stock market index as a case study.

(2). To compare the forecasting ability of symmetry and asymmetry GARCH models of developed and emerging stock market index using other model evaluation techniques such as Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE).

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